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ATTENUATION ANALYSIS OF HIGH FREQUENCY SH WAVES IN RANDOM MEDIA BY FINITE DIFFERENCES

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SUMMARY

The paper presents finite-difference simulations of seismic wave attenuation in 2-D random media which are assumed to model the near-surface region of the earth crust. The aim is to investigate the dependence of the high frequency fall-off observed in Fourier acceleration spectra of real earthquakes on the elastic scattering and anelastic attenuation occurring in this region.

INTRODUCTION

A correct description of the high-frequency region of Acceleration Fourier Amplitude Spectra (AFAS) is crucial for predicting how earthquake motions scale with source strength (Ref. 1), and is useful in the design of rigid structures and mechanical components. A convenient model for the AFAS at high frequencies has the form (Ref. 2)

$$A(f) = A_o \exp(-\pi\kappa f) \qquad f > f_E \qquad (1)$$

where f is frequency in Hz and f_E the lower validity limit of (1). Spectra of both small and strong earthquakes show that the attenuation parameter κ increases with epicentral distance, and tends to a finite value κ_o at vanishing distance. This implies that κ_o, hereinafter denoted simply as κ, is a measure of attenuation in the few km below the recording site, which likewise affects distant and near earthquakes. In fact, assuming that crack openings in rocks are largest near the surface and tend to close at 2-3 km depth, elastic scattering and inelastic absorption are expected to increase with decreasing depth in this region. Observed κ values (Ref. 3) range from less than 0.01 s for small earthquakes recorded on competent rock, to about 0.07 s for strong motions on deep soil basins. Earthquake data recorded in deep donwhole arrays clearly show the strong high-frequency attenuation occurring in the upper 1-2 km of the earth crust and confirm the previous κ values (Ref. 4). To gain a better insight in "near-site" attenuation phenomena, we have propagated seismic signals in 2-D finite difference models of rock media with random fluctuations of seismic velocity. Our main aim is to interpret the high frequency spectral features of the numerical signals in terms of the simple model (1), and to compare the values of κ and of the cut-off frequency f_{max} (in log-log spectral plots) with observations. Among previous numerical studies of attenuation in random media (e.g. Ref. 5), only one is devoted to simulating near-site effects on surface ground motions (Ref. 6).

NUMERICAL MODEL

For simplicity numerical analyses are restricted to SH wave propagation, described by the equation

$$v_{tt} = \left[\beta^2 (x, z)\, v_x\right]_x + \left[\beta^2 (x, z)\, v_z\right]_z \tag{2}$$

where the subscripts denote differentiation, v is the horizontal displacement in y-direction, and the propagation velocity β (x, z) is taken as the sum of a deterministic component β_o, depending only on depth z, and of a stochastic fluctuation, i.e.

$$\beta (x, z) = \beta_o (z) + \tilde{\beta} (x, z). \tag{3}$$

The time derivative in (2) is replaced by a second-order, centered FD approximation with step Δt, while the space derivates are replaced by fourth-order approximations with step *h* both in x and z directions. Details on the numerical model and its accuracy are given in Ref. 6. For $1.2 \le \beta_o \le 3.0$ km/s, the typical values $h = 5$ m and $\Delta t = 0.0007$ s used in calculations are found to ensure stability and lack of dispersion below ~50 Hz.

The modeled region in 2-D analyses is typically 3 km deep and 2 km wide, with absorbing boundaries at the bottom and top edges in the form of paraxial approximations to the wave equation, so that no free-surface condition exists at the upper boundary. Although the use of vertically incident excitation tends to minimize spurious reflections from the lateral edges, we have introduced two side strips with a viscous term of the form bv_t added to the left-hand side of (2) to damp out such reflections (see Fig.4). Excitation is applied at the base as a plane wave, smoothed delta-function impulse having an essentially flat spectrum to 50 Hz: upward propagation of this signal to a selected receiver point provides the impulse-response function, which can be subsequently multiplied in the frequency domain with any desired excitation spectrum.

Generation of the discrete random media is described in Ref. 5. Based on extensive numerical testing with Gaussian, exponential and self-similar(SS) media, the latter were chosen because they generate constant-Q attenuation at high frequencies. The single-scattering theoretical attenuation curves for 1-D and 2-D SS media with constant β_o are shown in Fig.1, where $k = 2\pi f/\beta_o$ = wavenumber, *a* = correlation distance, and σ_{ref} = standard deviation of the reference discrete media characterized in this case by $a = 40$ m, $h = 5$ m (Ref. 5 discusses the determination of σ, an undefined quantity in continuous SS media). Fig.1 indicates that $Q^{-1} = C_1\sigma^2_{ref} / 2$ in 1-D for $ka \gtrsim 1$, whereas $Q^{-1} = C_2\sigma^2_{ref} / tg\ (\theta_{min}/2)$ (asymptotic value yielded by Eq.(C13) of Ref. 5) in 2-D for $ka \gtrsim 3$; C_1 and C_2 are factors depending on *a* and *h* in the reference discrete media. Hence, the theory predicts that apparent attenuation at high frequency in SS media is described by (1), with $f_E \simeq \beta_o/2\pi a$ in 1-D and $f_E \simeq 3\beta_o/2\pi a$ in 2-D.

Inelastic absorption is additionally introduced in the model through a complex propagation velocity with the imaginary part accounting for a frequency-independent inelastic factor Q^{1}. The resulting complex form of (2) splits then into two coupled equations for the real and imaginary parts of displacement v, and each of them is handled by the same FD scheme previously described. The required imaginary part of the excitation can be obtained through appropriate Fourier expansions of the real input. This approach, believed to be original and checked with exact solutions in 1-D problems, is especially useful in 2-D analyses (Ref. 7). The mean velocity β_o in (3) is chosen as a smoothly increasing function of depth, such that the transfer function of the vertically inhomogeneous half-space without fluctuations can be obtained in closed form. This leads to an essentially constant amplification factor $(\beta_{o,max} / \beta_{o,min})^{1/2}$ except at very low frequency. In agreement with previous studies (Ref. 5), a value $\sigma_{ref} = 0.10$ was used for the ratio of r.m.s. fluctuation amplitude to the mean value of β_o over the considered depth range.

RESULTS AND DISCUSSION

From the synthetic impulse-response over a given distance, the attenuation factor κ can be estimated following (1), i.e. by slope of the log (response / excitation spectral ratio) in the $f_E \div 50$ Hz interval, with f_E picked visually or as previously described. Upon convolution of the response with a seismic input represented by an attenuated ω-squared spectrum, the simulated AFAS can be obtained and its high frequency portion investigated, including the cut-off frequency f_{max}. A time window of 0.25 s containing the main impulse is used for the estimate of κ, while a longer window of 0.8 s is used for the AFAS. In most cases the previous estimates are obtained by averaging, either on random media representing different realizations of the same stochastic process (in 1-D analyses), or by using the synthetic seismograms of different receivers at the same distance from the base in a given medium (in 2-D analyses). The influence of the $\beta_o(z)$ profile on scattering attenuation for a given σ was tested first in layered (1-D) media such as those shown in Fig.2a,b, where $\beta_{o,min}=1.2$ km/s at $z=0$ and $\beta_{o,max}=3.0$ km/s at $z=3.0$ km. The transfer function of each medium was calculated exactly by Haskell propagator matrix method using a constant layer thickness of 5 m. The attenuation effectiveness of the two types of media is illustrated in Figs.3a and b, the former showing the average transfer function over 3 simulations on media as in Fig.2a, and the latter on media as in Fig.2b. The higher spectral decay obtaining in the first case is a consequence of the fact that the ratio of r.m.s. fluctuation amplitude to $\beta_o(z)$ is higher than in the second case over a large depth range. Profiles of $\beta_o(z)$ of the type of Fig.2a were therefore retained in subsequent analyses. Note that the value $\kappa \simeq 0.006$ is on the low side of observations of small earthquakes at competent rock sites. For SS media having a constant β_o equal to the mean value $\bar{\beta}_o=2.16$ km/s of Fig.2a, the theory would predict a scattering factor $Q^s=2/\sigma^2=200$, $\kappa=0.007$ and $f_E=8.6$ Hz, in agreement with the numerical results.

The geometry of a typical 2-D model is shown in Fig.4; the homogeneous layer between the base and the heterogeneous region is needed for proper working of the paraxial conditions. A 2-D SS medium was used having isotropic correlation distance $a = 40$ m and $\beta_o(z)$ increasing from 1.5 km/s to 3.0 km/s.

Fig.5a, from bottom to top, displays the incident impulse and the displacement response calculated at the two central receivers without inelastic absorption. The transfer of energy from the main pulse to the coda between 1.5 and 3.0 km, due to scattering, prevails in this case over the amplification caused by the decrease in $\beta_o(z)$. The results of an additional simulation on the same random medium but including an inelastic $Q^i=200$ are shown in Fig.5b. Although slightly different in form and duration, the excitation impulse has also in this case a flat spectrum out to 50 Hz. The smoothed onset of the main pulse at 1.5 km in Fig.5b is an artifact due to filtering of high-frequency noise. The depletion of high frequency content caused by inelastic absorption is immediately apparent. Fig.6a shows the spectral ratios for the central receiver at 3.0 km, and Fig.6b the average spectral ratios over the 5 receivers at the same distance. In both figures the solid lines refer to the simulation without inelastic absorption, whereas the stippled ones are for Q^i=200. An f_E=25 Hz was used in estimating the κ values of Fig.6, according to theory, and close inspection of the solid curves effectively reveals an increase in slope occurring between 20 and 30 Hz. The average κ=0.011 in Fig.6b agrees well with observations on rock sites, and is nearly double than in the 1-D case of Fig.3a, but the results are not directly comparable because f_E is different. An equivalent Q^s=120 results from the previous value of κ in a medium with constant mean velocity equal to $\bar{\beta}_0$ and 3 km travel path, which can be compared with the 2-D high frequency theoretical result Q^s=134 for θ_{min}=45° (Fig.1). The analysis with absorption requires a homogeneous base layer 1 km thick with Q^i=200, which contributes a factor κ=0.0017. After subtracting this artificial contribution from the average κ=0.019 in Fig.6b, scattering and inelastic attenuation are easily found to be approximately additive. Fig.7a displays for the central receiver at 3 km the AFAS (arbitrary scale) resulting from convolution of the impulse response with an ω-squared spectrum attenuated with Q=300 over a distance of 10 km and with a corner frequency f_c= 2 Hz. The spectra visually suggest an f_{max} ≈30 Hz. However, looking at the average AFAS over the 5 receivers at 3 km (Fig.7b), and comparing them with the AFAS for the smooth medium (i.e. without random fluctuations), it appears that f_{max}≈12 Hz would be an appropriate choice for curve 3. This value agrees quite well with those observed in strong-motion spectra. The previous example, and others that cannot be shown suggest that the spectral cut-off at f_{max} can indeed be a "near-site" effect depending on the interaction of scattering and inelastic attenuation in the upper 2-3 km of the crust with the incident spectrum. Other relevant conclusions of this study are: (a) based on the results of 2-D simulations, κ values contributed by scattering alone in realistic rock media should be ~0.01, which agree with observations of small earthquakes on competent sites; (b) matching the values κ=0.05-0.06 of observed strong-motion spectra requires inelastic absorption with very low values of Q^i; (c) reasonably accurate predictions of apparent attenuation, in agreement with spectral model (1), can be obtained from single-scattering theory, and (d) a useful new approach including inelastic absorption in FD propagation schemes has been satisfactorily applied.

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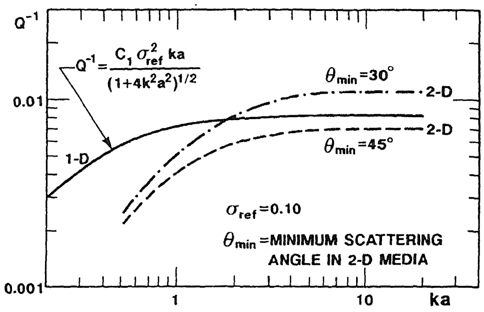


Fig. 1

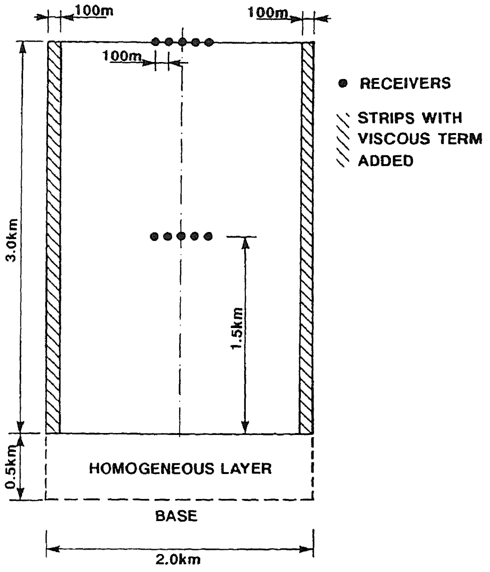
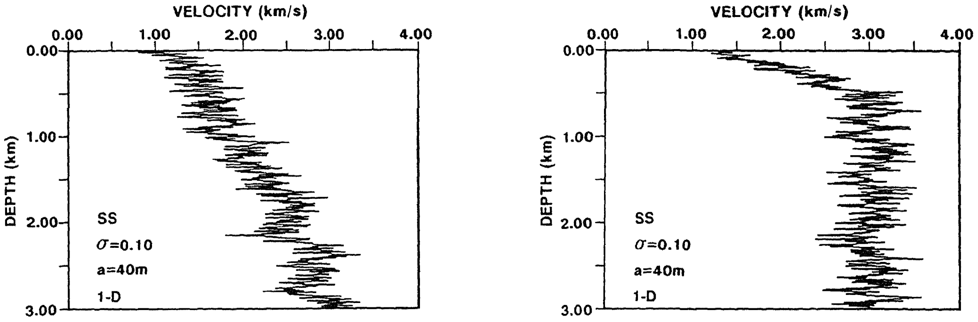
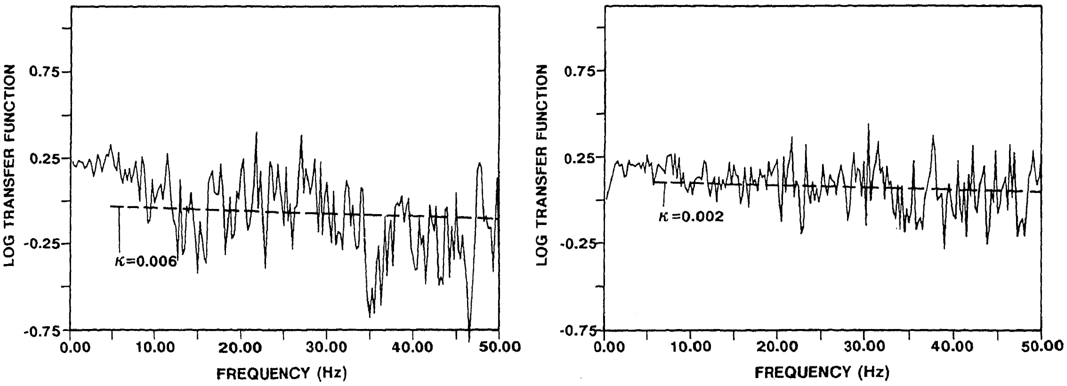


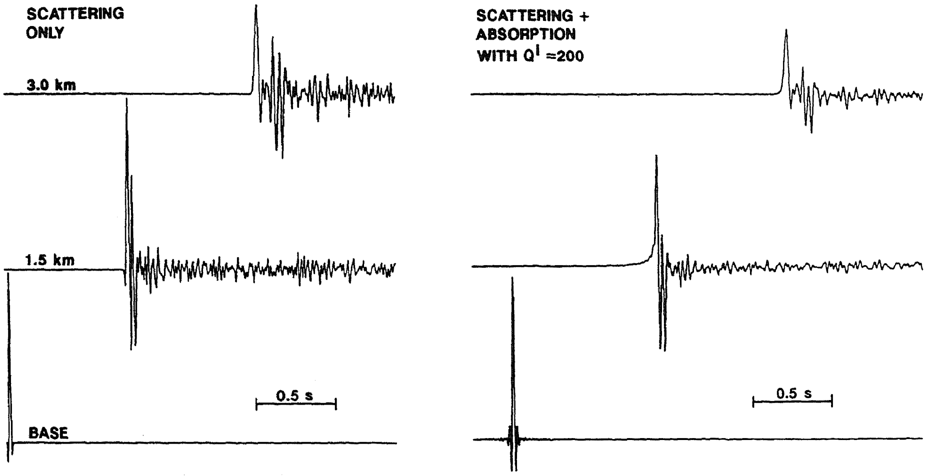
Fig.4



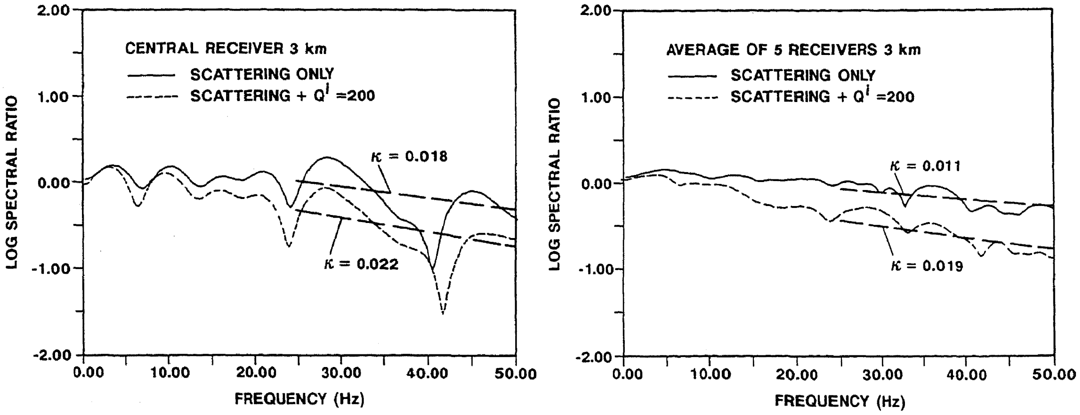
a Fig.2 b



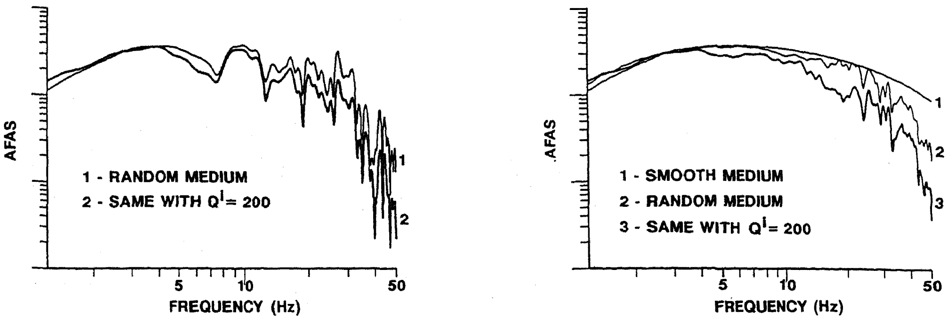
a Fig.3 b



a Fig.5 b



a Fig.6 b



a Fig.7 b