CALCULATION OF EARTHQUAKE RESPONSE TO THE EMBEDDED PIPELINE LAID THROUGH DIFFERENT MEDIA

Xu XIE and Yu-Ao HE

1Department of Civil Engineering, Tianjin University, Tianjin, China
2Department of Civil Engineering, Tianjin University, Tianjin, China

SUMMARY

This paper gives an approach for the analysis of the earthquake response to the embedded pipeline laid through different media by the theory of elastic surface wave propagation. Through analysing practic examples and comparing several aspects, the authors have reached the same results as the earthquake hazards investigation has reached. The results contain that the deformation of pipeline laid through different media is obviously larger than through identical medium.

INTRODUCTION

The Embedded pipeline is a sort of particular underground structure under control of soil medium. Free vibration doesn't occur easily; and the effect of the inertial forces are negligible. The pipeline is also a kind of structure of rather long distribution. Ground conditions which they laid through are quite complicated. It is well known that the rate of seismic damage of pipeline laid through different media is much higher than those that laid through identical medium with the aid of recent theoretical research results and on-the-spot observation, this paper, is of the opinion that surface wave is the major factor for pipeline deformations; and that seismic response of embedded pipeline laid through different media can be calculated by using the theory of surface wave. Since the components of Love wave and Rayleigh wave within the two-dimensional plane produce no effect on each other, their deformation effects generated respectively can be considered separately in analysis, and repeated addition can be carried out according to space geometrical relationship within time domain.

For the sake of convenience, marks below are meant as follows:

\[ r_s = \begin{cases} 
\left(\frac{C}{V_s}\right)^2 - 1 \frac{k}{2} & C > V_s \\
- i \left[ 1 - \left(\frac{C}{V_s}\right)^2 \right]^{\frac{1}{2}} & C < V_s 
\end{cases} \]

\[ r_p = \begin{cases} 
\left(\frac{C}{V_p}\right)^2 - 1 \frac{k}{2} & C > V_p \\
- i \left[ 1 - \left(\frac{C}{V_p}\right)^2 \right]^{\frac{1}{2}} & C < V_p 
\end{cases} \]

\( V_s \) is the wave velocity of S-wave; and \( V_p \) is the wave velocity of P-wave. \( k \) is wave number, \( k = \omega / C \); \( \omega \) is frequency; \( C \) wave velocity; \( G \) shear modulus of soil; \( d(1) \) thickness of layer of L-layer.

Dispersity of Love Wave in the Layered Uniform Semi-space In the process
of propagation, Love wave contains dispersive that body wave doesn't.

According to the equation of motion of Love wave, solution to the third (M) layer is as follows:

\[ U_m = \exp[i(\omega t-kx)] \cdot [\Delta_1 \exp(-ikr_{sm}Z_m) + \Delta_2 \exp(ikr_{sm}Z_m)] \]  

(1)

Shearing stress component can be expressed as:

\[ T_m = G_m \partial U_m / \partial Z_m \]  

(2)

As wave velocity \( C \) has the same value in all the layers, continuity of displacement and stress on the discontinuous plane between the higher and lower layers are equal to the continuance of \( \psi \), \( \tau \), and recurrence relation can be obtained as:

\[ \left\{ \begin{array}{c}
\frac{\hat{U}_m}{C} \\
T_m
\end{array} \right\} = \left[ a_{m-1} \cdot [a_{m-2}] \cdots [a_1] \right] \left\{ \begin{array}{c}
\frac{\hat{U}_1}{C} \\
T_1
\end{array} \right\} \]  

(3)

The above formula is the relation formula between the surface displacement \( U_m \), stress \( T_m \) of the M layer and the ground surface \( U_1, T_1 \). In which,

\[ [a_1] = \left[ \begin{array}{c}
\frac{(\exp(-Q_1)+\exp(Q_1))/2}{\exp(Q_1)-\exp(-Q_1))/2r_{s1}G_1} \\
\frac{\exp(Q_1)-\exp(-Q_1))/2r_{s1}/2}{\exp(Q_1)+\exp(-Q_1))/2}
\end{array} \right] \]

\[ Q_1 = ikr_{s1}d_1 \]

According to the condition of surface wave, the dispersive equation can be obtained as:

\[ -L_{21}(n-1) - G(n) \cdot r_{sn} \cdot L_{11}(n-1) = 0 \]  

(4)

\( L_{ijm} \) stands for the \((i,j)\) element of matrix \([a_m] \cdot [a_{m-1}] \cdots [a_1]\). As the ground surface amplitude which corresponds w frequency is known for \( F(w) \), the solution within frequency domain is as follows:

\[ U(w,x,z) = F(w) \cdot \phi(w,z) \cdot \exp(-ikx) \]  

(5)

In which, the formula of \( \phi(w,z) \) within M layer is as follows (the unit value of ground surface is used as the reference value):

\[ \phi_m(w,Z_m) = \frac{(L_{11}(m-1)-L_{21}(m-1))/(C_m \cdot r_{sm})}{\exp(-ikr_{sm}Z_m)/2} + \frac{(L_{11}(m-1)+L_{21}(m-1))/(C_m \cdot r_{sm})}{\exp(ikr_{sm}Z_m)/2} \]  

(6)

Propagation of Love wave in different media. Suppose that there is one Love wave propagating towards medium 2 (Fig.1) along the first modal; the frequency is \( w \) and the amplitude value of the ground surface is a unit; wave produces reflection and transmission waves on the discontinuous plane. Displacement and the stress continuous condition can be obtained as

\[ \phi_1(w,Z) + \sum_{j=1}^{N} a_j \phi_j(w,Z) = M \sum_{j=1}^{M} b_j \phi_j(w,Z) \]

\[ G(Z)k_j \phi_j(w,Z) - G(Z) \sum_{j=1}^{N} a_j k_j \phi_j(w,Z) = G(Z) \sum_{j=1}^{M} b_j k_j \phi_j \]  

(7)

where, \( a_j \) is reflex coefficient of the reflection wave of j modal; and \( b_j \) is transmission coefficient of the transmission wave of j modal. According to L.E. Alsop's method, this paper employs the square of deformation difference
and minimum on the discontinuous plane of soil on both sides to replace the displacement continuous condition. By applying the orthogonality relation of Love wave, \( a_j \) and \( b_j \) can be obtained.

Then, we can go further to get the ground response of both I and II sides:

\[
U(x,t,z) = \frac{i}{2\pi} \int_{-\infty}^{+\infty} F(w) \left[ \phi_1^I(w,z) \exp(-i k_1 x) + \sum_{j=1}^{N} a_j \phi_j^I(w,z) \exp(i k_j x) \right] \exp(iwt) dw
\]

\[
U(x,t,z) = \frac{i}{2\pi} \int_{-\infty}^{+\infty} F(w) \left[ \sum_{j=1}^{N} b_j \phi_j^II(w,z) \exp(-i k_j x) \right] \exp(iwt) dw
\]

\( F(w) \) should be the component of Love wave which is recorded on the ground surface.

**Dispersivity of Rayleigh wave in the layered uniform semi-space** Rayleigh wave also has dispersive characteristics. It not only contains the horizontal component but also vertical component. This is quite different from the S-wave and the Love wave.

According to the solution of equation and the continuous, we can get the recurrence relation similar to Love wave, and according to the free surface and earthquake source conditions, the displacement formula of \( M \) layer within frequency domain is expressed as follows (suppose \( x = 0 \)):

the horizontal direction: \( U_{1m}(w,z_m) = i(v_{pm}^2/w^2)k[A_m^r \exp(-iP_m) + A_m^i \exp(iP_m)] \)

\[-2i(v_{sm}^2/w^2)k \cdot r_{sm}[\Delta_m^r \exp(-iQ_m) + \Delta_m^i \exp(iQ_m)] \]

the vertical direction: \( U_{2m}(w,z_m) = -i(v_{pm}^2/w^2)k \cdot r_{pm}[\Delta_m^r \exp(-iP_m) + \Delta_m^i \exp(iP_m)] \)

\[-2i(v_{sm}^2/w^2)k[w_m^r \exp(-iQ_m) + w_m^i \exp(iQ_m)] \]

in which,

\( P_m = k \cdot r_{pm} \cdot Z_m \)

\( Q_m = k \cdot r_{sm} \cdot Z_m \)

\( \Delta_m^r = (iw/C)J_{1m}U_1(1)(w,0) + (iw/C)J_{2m}U_2(1)(w,0) \)

\( w_m^r = (iw/C)J_{3m}U_1(1)(w,0) + (iw/C)J_{4m}U_2(1)(w,0) \)

**Propagation of Rayleigh Wave in Different Media** In order to avoid solving nonlinear equation group, this paper only takes the effect of first modal into consideration. The displacement and stress continuous conditions of Rayleigh wave on the discontinuous plane are as follows:

\[
\begin{align*}
\phi_1^I + \phi_1^II &= \phi_2^I \\
\int_0^\infty G^I(z)k_1 \phi_1^I dz &= \int_0^\infty G^II(z)ak_1 \phi_1^II dz
\end{align*}
\]

Where, \( \phi_2 \) is vertical vibration modal, I and II represent two kinds of media, "1" on the right corner above stands for the first modal.

Similarly, by using the mean-square integral of displacement difference

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on both sides and minimum condition, solution can be obtained as:

$$ b = 2\left[\frac{P_2}{P_1} \cdot S_{11} + T_{11}\right]/\left[(\frac{P_2}{P_1})^2 S_{11} + 2\left(\frac{P_2}{P_1}\right) T_{11} + T_{11}\right] \quad (12) $$

$$ a = \frac{P_1 - P_2 \cdot b}{P_1} $$

where,  $$ P_1 = \int_0^\infty G(Z) \phi_1 \phi_2 dZ; \quad P_2 = \int_0^\infty G(Z) k_1^2 \phi_2 \phi_2 dZ; $$

Meanings of $S_{11}$, $T_{11}$, $V_{11}$ are the same as Love wave.

According to values of $a$, $b$ obtained above, we can get the wave motions of the soil on I and II side. The form in the time domain is as follows:

horizontal direction:

$$ U_1 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(w) \left[ \phi_1 \exp(-ik_1x) + a \phi_2 \exp(ik_1x) \right] e^{i\omega t} dw \quad (13) $$

vertical direction:

$$ U_2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(w) \left[ \phi_2 \exp(-ik_1x) + a \phi_2 \exp(ik_1x) \right] e^{i\omega t} dw \quad (14) $$

where, $F(w) = \int f(t) e^{-i\omega t} dt$; $f(t)$ is vertical seismic records.

Practical Example  Applying the above-mentioned calculation theory, this paper analyses the earthquake response of the pipeline of a certain engineering.

According to the geological prospect results of ground, the conditions of the ground which pipeline cuts across can be divided into the four following cases:

1. gneiss — clay,
2. gneiss — silt,
3. silt — clay,
4. uniform silt overburden.

The diameter of the pipeline is 1.4 meter, the thickness of pipe-wall is 0.12cm, the modulus of elastic of pipe material $E = 20.58 \times 10^5$ N/m, the distance from the pipe axis to the ground surface is 3.0m, the size of others are in figure.

<table>
<thead>
<tr>
<th>KIND</th>
<th>$V_s$(m/s)</th>
<th>$V_p$(m/s)</th>
<th>$G$(N/m²)</th>
<th>$(kg/m^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLAY</td>
<td>145.0</td>
<td>301.8</td>
<td>33897450.0</td>
<td>1580.0</td>
</tr>
<tr>
<td>SILT</td>
<td>185.0</td>
<td>308.8</td>
<td>68450000.0</td>
<td>1960.0</td>
</tr>
<tr>
<td>GNEISS</td>
<td>500.0</td>
<td>935.414</td>
<td>700000000.0</td>
<td>2800.0</td>
</tr>
</tbody>
</table>

This paper calculates the earthquake response of embedded pipeline with the way of well-known earthquake wave record when strong earthquake happens. Before inputting at first we minimize and adjust displacement record in IMPERIAL VALLEY earthquake horizontal displacement amplitude value is 5.45cm, vertical displacement amplitude value is 2.8cm. Assume inputting 10.22 seconds before the whole record, at intervals of 0.02 second.

The pipeline is buried in four-kinds above-mentioned grounds. Through calculation, the results are as Table 2. In calculation, assum $x=5.0$ meter.
Table 2

<table>
<thead>
<tr>
<th>kinds of ground</th>
<th>GNEISS-CLAY</th>
<th>GNEISS-SILT</th>
<th>SILT-CLAY</th>
<th>SILT-SILT</th>
</tr>
</thead>
<tbody>
<tr>
<td>max tensile stress</td>
<td>$0.2351 \times 10^9$</td>
<td>$0.21385 \times 10^9$</td>
<td>$0.12875 \times 10^9$</td>
<td>$0.1122 \times 10^9$</td>
</tr>
<tr>
<td>max compression stress</td>
<td>$-0.1464 \times 10^9$</td>
<td>$-0.13490 \times 10^9$</td>
<td>$-0.871 \times 10^8$</td>
<td>$-0.6 \times 10^8$</td>
</tr>
</tbody>
</table>

Conclusion The calculation results show that the stress of pipeline laid through different media would be larger as this difference greater. The smallest is occurred in the condition of identical medium.

![Diagram](image)

Fig. 1 The embedded pipeline laid through different media

REFERENCES


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