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A NEW DAMAGE MODEL FOR REINFORCED CONCRETE STRUCTURES

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SUMMARY

Many different damage models have been proposed for concrete in the past. Most of these are not well suited to predict the residual strength of damaged members. This paper reviews some basic facts about concrete damage and uses these to systematically model damage as a low-cycle fatigue phenomenon. Instead of the number of cycles to failure the energy dissipation capacity of a member is taken as the main variable, which depends on many different factors. The model is capable of simulating reasonably well the strength and stiffness degradation of RC members subjected to strong cyclic loads. In addition, a new structural damage index is herein proposed.

DAMAGE AND FAILURE OF RC MEMBERS

Any attempt of devising mathematical models to quantify damage in a rational way should set out with a clear and precise definition of damage, because "damage" is a widely used word, that describes all kinds of different phenomena and is prone to subjective interpretation. In the context of our discussion, damage of a RC member shall be defined to signify a specific degree of physical degradation with precisely defined consequences regarding the member's capacity to resist further load. A damage index is usually defined as the damage value normalized with respect to the (arbitrarily defined) failure level so that a damage index value of unity corresponds to failure.

As an illustration, Fig. 1 shows a typical response of a reinforced concrete cantilever beam to progressively increasing load cycles (Ref. 1). The stiffness of the member decreases gradually, once the yield capacity of the member has been exceeded. It takes a significant further increase in loading until the strength deteriorates as well, i.e., when the force necessary to cause a given tip deflection decreases in subsequent cycles. Fig. 1 also demonstrates the difficulty of defining failure. Thus, any failure definition such as "a strength reduction of 25% of the first yield load level" is arbitrary. Even then, such a definition is not sufficiently precise, since the apparent residual strength may increase with further displacement increase, Fig. 1.

The response of reinforced concrete to load is complicated by the complex interaction between steel and concrete. This is reflected by the numerous possible failure modes in flexure, shear or bond. Conventional reinforced concrete design philosophy calls for such member detailing that all but a few ductile failure modes are precluded. For dynamically applied cyclic loads it is difficult to predict the failure mode even for "properly" detailed members, because of the effect of the strain rate and the load reversals. Bond degradation and shear cracking typically progress more rapidly under cyclic loading than flexural strength degradation. As a result, reinforced concrete members subjected to earthquake-type loads are more likely to fail in bond or shear than in flexure, even if properly designed for monotonically applied loads.

The progressive accumulation of damage in a material up to the point of failure under repeated load application is generally known as "fatigue". Each load cycle inflicts a certain amount of irreversible damage and can be compared to the passage of some time unit of the life span of the material. If the material is subjected to a history of varying stress levels, the prediction of the fatigue life is much more difficult. In this case, it is common practice to utilize Miner's hypothesis,

$$\Sigma_i \ \frac{n_i}{N_i} \ = \ 1 \tag{1}$$

where N_i is the number of cycles with stress level S_i leading to failure, and n_i is the number of cycles with stress level S_i actually applied. This Eq. (1) assumes that the accumulation of damage is linear and independent of the load sequence. In studies of low-cycle fatigue of reinforced concrete the number of load cycles to failure is typically replaced by the cumulative dissipated energy, which is often normalized with regard to the energy stored when the member is stressed to the yield level. In reality, the amount of energy dissipated in each cycle decreases with progressive damage until failure. In analogy to an S-N curve, given the necessary experimental data, it would be possible to present a relationship between constant deformation level D_i and total energy dissipation capacity E_i. In a S-N relationship, N_i is a function of stress level S_i and can be determined experimentally. For metals, such a function, drawn on log scales, is generally approximated by a straight line. Substituting the energy dissipation capacity E_i for N_i, and deformation level D_i for stress level S_i, the corresponding $D_i - E_i$ relationship is far from linear, Fig.2. In fact, for very low deformation levels, the load-deformation relationship remains linear, and no energy is dissipated at all.

It can be observed from some laboratory experiments (Ref. 2) that the failure mode is closely related to the formation of initial cracks that eventually may become critical. Consider the two idealized load histories of Fig. 3. It is conceivable that the four low-level load cycles of history "a" introduce a cracking pattern which results in a different kind of damage due to the final strong load cycle, than if this same strong load cycle were to be applied to the undamaged member, as in history "b". Thus, not only the total energy dissipation capacity of the member is dependent on the load history, but its failure mode might be as well.

PREVIOUS DAMAGE MODELS

Numerous models have been proposed in the past to represent damage of structural members or entire structures. Some of these were derived for metal structures. Because of fundamental differences between reinforced concrete and homogeneous materials such as steel, these models are not directly applicable to reinforced concrete. Other models are based on empirical damage definitions (Ref. 3). These all but disregard the mechanics of the materials involved when subjected to cyclic load, and therefore do not lend themselves to rational predictions of the strength reserve and response characteristics of a structure with a specified degree of damage.

Several investigators have introduced energy indices which are functions of a few selected parameters (Refs. 2,4). Other notable examples are the damage ratio introduced by Lybas and Sozen (Ref. 5), and the flexural damage ratio and the normalized dissipated energy used by Banon (Ref. 6) as basic damage state variables to derive contours of equal probability of failure. Of the more recent damage models, the widely cited model of Park and Ang (Ref. 7) should be noted,

$$D_e = \frac{\delta_{max}}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \tag{3}$$

where δ_{max} = maximum deformation experienced so far, δ_u = ultimate deformation under monotonic loading, Q_y = calculated yield strength, dE = dissipated energy increment. β is a somewhat arbitrarily constructed function of several parameters, $\beta = \left(-0.357 + 0.73\frac{l}{d} + 0.24n_o + 0.314P_t\right)\ 0.7^{\rho_w}$ with $\frac{l}{d}$ = shear span ratio, n_o = normalized axial force, ρ_w = confinement ratio, P_t = longitudinal steel ratio. In Ref. 8, the authors have critically reviewed these and many other damage models.

A NEW DAMAGE MODEL

Stiffness Degradation. Under load reversals, a RC member experiences a progressive stiffness reduction due to concrete cracking and bond degradation of the steel-concrete interface primarily in the plastic hinge. The model of Roufaiel and Meyer (Ref. 9) is used to simulate this behavior. It takes into account the finite size of plastic regions. Fig. 4 illustrates the various branches of hysteretic behavior: 1) elastic loading and unloading; 2) inelastic loading; 3) inelastic unloading; 4) inelastic reloading during closing of cracks; and 5) inelastic reloading after closing of cracks. In a reversed load cycle with high shear, previously opened shear cracks tend to close, leading to an increase in stiffness and a characteristic "pinched" shape of the moment-curvature curve. Roufaiel and Meyer have modeled this effect by introducing the "crack-closing" moment M_p^+, associated with curvature ϕ_p^+. If shear stresses are negligible and the hysteresis loops are stable during cyclic loading, no pinching is likely to occur and branches 4 and 5 will form a single straight line.

Strength Degradation. In addition to stiffness degradation, RC members experience strength degradation under cyclic loading beyond the yield level. Atalay and Penzien (Ref. 10) had noticed some correlation between commencement of strength degradation and the spalling of the concrete cover. But Hwang's experiments (Ref. 2) showed that strength degradation can start at considerably lower load levels. Even for loads slightly above the yield level, damage and strength degradation can be observed, provided a sufficiently large number of load cycles is applied. It is, therefore, suggested that strength degradation is initiated as soon as the yield load level is exceeded, and the strength degradation accelerates as the critical load level is reached. For this purpose, a strength drop index, S_d, is proposed (Fig. 5),

$$S_d = \frac{\Delta M}{\Delta M_f} = \left(\frac{\phi-\phi_y}{\phi_f-\phi_y}\right)^{\omega} \tag{4}$$

where ΔM = moment capacity (strength) reduction in a single load cycle up to curvature ϕ_y, ΔM_f = fictitious moment capacity (strength) reduction in a single load cycle up to failure curvature ϕ_f. For analysis purposes, the strength drop is measured from the second branch of the primary moment-curvature curve. The actual strength reductions are indicated by the shaded area in Fig. 5. For the parameter ω, calibration studies (Ref. 8) suggest a value of 1.5. With ΔM denoting the strength drop in one load cycle to some curvature ϕ, the residual strength after this one load cycle, Fig. 4, is given by

$$m_1(\phi) = M_y + (\phi-\phi_y)p(EI)_e - [(\phi_f-\phi_y)p(EI)_e + M_y - M_f]\left(\frac{\phi-\phi_y}{\phi_f-\phi_y}\right)^{\omega} \tag{5}$$

In order to incorporate this concept of strength degradation into the hysteresis model, an imaginary point with coordinates $(\overline{\phi}_x, \overline{M}_x)$, is introduced, at which the load-deformation curve is aimed during reloading, Fig. 5. Details are given in (Ref. 8).

Definition of Failure. For RC members undergoing cyclic loading, several investigators (Refs. 2,4,7,10) have defined failure as the point where the member strength(moment) at maximum displacement(curvature) has dropped below 75% of the initial yield strength(moment). But if the member is subsequently loaded beyond this maximum displacement or curvature, its moment can be observed to increase well above the 75% level (Ref. 2), even though it has already been assumed to have "failed"(Fig. 1). For this reason it is necessary to relate the failure definition to the actual strength reserve or residual strength, which is a function of the experienced loading history.

First, the failure moment M_f and the corresponding curvature ϕ_f need to be defined for monotonic loading. The failure curvature ϕ_f is defined to be the curvature at which one of the following three critical strains is reached first: 1) the concrete's crushing strain, 2) the tensile steel's rupture strain, and 3) the compression steel's buckling strain, following spalling of the concrete cover. Given the complete stress-strain curves for steel and concrete and the cross-sectional dimensions, it is relatively straightforward to compute the monotonic moment-curvature curve, by determining the moment M_i associated with any curvature ϕ_i (Ref. 8). The failure moments for other curvature levels are assumed as

$$M_{fi} = M_f \cdot \frac{2\Phi_i}{\Phi_i + 1.0} \tag{6}$$

where M_{fi} = failure moment for given curvature level ϕ_i, M_f = failure moment for monotonic loading, $\Phi_i = \phi_i/\phi_f$(curvature ratio), and ϕ_f = failure curvature for monotonic loading. According to Fig. 6, the failure moment M_{fi} decreases with smaller curvature levels ϕ_i, i.e. larger strength drops from the monotonic loading curve are needed to lead to failure. If the total strength drop down to the failure moment M_{fi} at some curvature ϕ_i is known, the number of cycles for this curvature level needed to cause failure, can be determined.

New Damage Index. Based on the above definition of failure, a new damage index, D_e, is proposed as a measure of damage sustained by RC members undergoing inelastic cyclic loading. It combines a modified Miner's hypothesis with damage modifiers, which reflect the effect of the loading history, and it considers the fact that RC members typically respond differently to positive and negative moments:

$$D_e = \Sigma_i \left(\alpha_i^+ \frac{n_i^+}{N_i^+} + \alpha_i^- \frac{n_i^-}{N_i^-} \right) \tag{7}$$

where i = indicator of displacement or curvature level, $N_i = (M_i - M_{fi})/\Delta M_i$ = number of cycles to cause failure at curvature level i, n_i = number of cycles actually applied at curvature level i, α_i = damage modifier, and $+$ and $-$ are indicators of loading sense. The loading history effect is captured by including the damage modifier α_i, which, for positive moment loading, is defined as

$$\alpha_i^+ = \frac{\Sigma_{j=1}^{n_i^+} k_{ij}^+}{n_i^+ \times \overline{k_i}^+} \cdot \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+} = \frac{M_{i1}^+ - \frac{1}{2}(n_i^+ - 1)\Delta M_i^+}{M_{i1}^+ - \frac{1}{2}(N_i^+ - 1)\Delta M_i^+} \cdot \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+} \tag{8}$$

where $k_{ij}^+ = M_{ij}^+/\phi_i^+$ is the stiffness during the j-th cycle up to load level i, $\overline{k_i}^+ = \frac{1}{N_i^+}\Sigma_{j=1}^{N_i^+} k_{ij}^+$ is the average stiffness during N_i^+ cycles up to load level i, and $M_{ij}^+ = M_{i1}^+ - (j-1)\Delta M_i^+$ is the moment reached after j cycles up to load level i. The definition of Eq. (8) needs some explanation. The energy that is dissipated during a single cycle up to a given curvature level decreases for successive cycles. That means the damage increments also decrease. In a constant-amplitude loading sequence, the first load cycle will cause more damage than the last one. Therefore, the α_i-factor decreases as load cycling proceeds, being a function of the stiffness ratio. The factor $(\phi_i^+ + \phi_{i-1}^+)/(2\phi_i^+)$, is necessary to normalize the damage increments in the case of changing load amplitudes. For negative loading, the damage modifier is defined similarly.

To illustrate the accuracy, with which the proposed mathematical model of Eq.(7) can simulate hysteretic response of RC members, many experimental results have been reproduced numerically in (Ref. 8). Agreement between numerical and experimental results was in general excellent. Fig. 7 represents an example that is typical for the kind of agreement achieved.

Structural Damage Index. Important decisions concerning the residual strength and safety of a damaged building are most conveniently based on a single structural or global damage index, as long as the building's use and importance is accounted for. A structural index can be composed of individual story damage indices (7), each of which is a weighted average of the damage indices of all potential plastic hinges in the story under consideration,

$$D_{S_k} = \frac{\Sigma_{i=1}^{n_k} D_i^k \cdot E_i^k}{\Sigma_{i=1}^{n_k} E_i^k} \tag{9}$$

where D_{S_k} and D_i^k denote the damage index for the k-th story and for joint i in story k, respectively. n_k is the number of potential plastic hinges in the k-th story (2 ×

number of elements in story k). E_i^k is the energy dissipated in joint i of story k. Then, the structural damage index can be defined as,

$$D_g = \Sigma_{k=1}^{N} D_{S_k} I_k \tag{10}$$

where N is the total number of stories. $I_k = \frac{N+1-k}{N}$ is the weighting factor for story k, which expresses the greater importance of the lower stories of a building ($I_k = 1$ for $k = 1$). It is noteworthy that a structural damage index such as the one defined above cannot reflect the structure's increased vulnerability under further loading, if one or more critical elements have been severely damaged or failed altogether. That is, any rational evaluation of a structure's reliability can only be meaningful if the mechanical degradation process of all significant structural members are accurately accounted for.

CONCLUSIONS

A new damage model and associated damage index have been developed which are believed to be more rational than previously proposed models and take into account factors such as loading sequence which are usually ignored in others. An accurate determination of damage is essential for meaningful nonlinear dynamic analysis of concrete structures, because the damage index is closely tied to the strength reserve of a member, after it has undergone large inelastic cycles.

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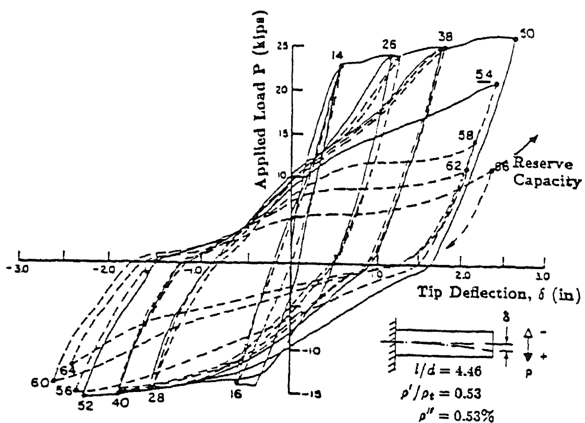

Fig. 1 — Typical Inelastic Response of RC Member (Ref. 1)

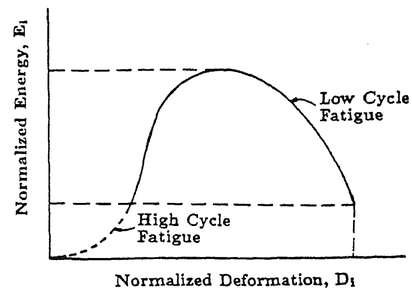

Fig. 2 — E_i–D_i Curve for RC Member

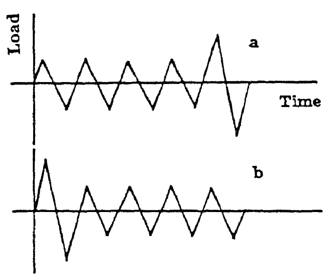

Fig. 3 — Two Different Load Histories

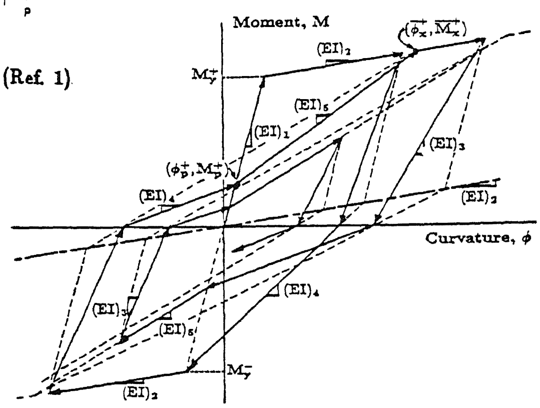

Fig. 4 — Typical Hysteretic Moment-Curvature Relationship

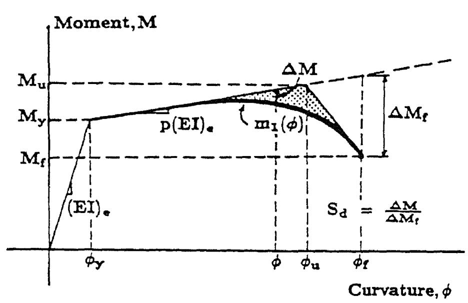

Fig. 5 — Strength Degradation Curve

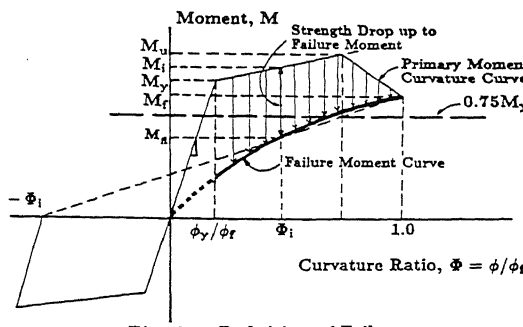

Fig. 6 — Definition of Failure

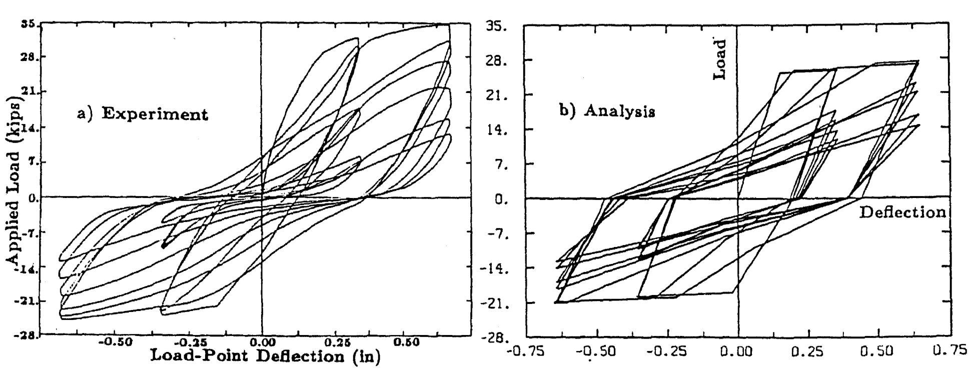

Fig. 7 — Experimental and Analytic Load-Deformation Curves for Beam S2-3 tested by Hwang (Ref. 2)