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### RELIABILITY IMPORTANCE MEASURES OF LIFELINE NETWORK COMPONENTS AND THEIR APPLICATION

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#### SUMMARY

A new importance measure of lifeline network components based on the reliability concepts is proposed to evaluate the ranks of system components. The proposed importance measure is compared with existing importance measures and is applied to a simple network model. Based on the application result, it is shown that the proposed importance measure is effective to evaluate the ranks of lifeline system components. This new measure of lifeline network components can be used effectively to improve the system reliability of lifelines under earthquakes, and to restore lifeline systems damaged by earthquakes.

#### INTRODUCTION

Recently, the methodologies to evaluate the reliability of lifeline network systems under earthquakes have been developed (Refs.1,2,3,4). Using these methods, we can estimate the characteristics of lifeline network systems and the connectivity or serviceability of lifeline network systems.

After evaluating reliabilities of lifeline network systems, it is sometimes necessary to improve the reliabilities of some nodes or to make a refined restoration strategy for damaged lifeline network system after earthquake (Refs.5,6). To achieve these purposes effectively, we need to introduce a importance measure of lifeline network components (Refs.7,8).

In this paper, some existing importance measures are examined. And a new importance measure is proposed. Then these importance measures are compared each other. At the end, this new importance measures is applied to a simple network model.

#### PROBABILISTIC IMPORTANCE AND CRITICALITY IMPORTANCE

In this paper, for the purpose of simplicity, we deal with reliability of connectivity of lifeline network, in which only failure of links is considered.

Reliability of Connectivity As shown in Fig.1, a complex network may be transformed into a system of series in parallel (SSP). Therefore, the reliability of connectivity of node  $i$  is

$$R_i = P[E_1 U E_2 U \dots U E_k U \dots U E_M] \dots (1)$$

in which  $E_k$  is the event which the  $k$ -th tie set is connected with a supply node.

To simplify the formulation, let us assume that the tie sets 1 to  $m$  include rink  $j$  respectively and the tie sets  $m+1$  to  $M$  don't include rink  $j$  (see Fig.1). Then Eq.(1) becomes

$$R_i = P[(Q_j \cap F_1) \cup (Q_j \cap F_2) \cup \dots \cup (Q_j \cap F_k) \cup \dots \cup (Q_j \cap F_m) \cup E_{m+1} \cup \dots \cup E_M], \quad (1 \leq k \leq m \leq M) \dots (2)$$

in which  $Q_j$ ; the event which rink  $j$  does not fail, and

$F_k$ ; the event which all rinks except rink  $j$  in  $k$ -th tie set don't fail. If we introduce following notations,

$$F_s = F_1 \cup F_2 \cup \dots \cup F_m \dots (3)$$

$$E_s = E_{m+1} \cup E_{m+2} \cup \dots \cup E_M \dots (4)$$

Eq.(2) becomes

$$\begin{aligned} R_i &= P[(Q_j \cap F_s) \cup E_s] = P[Q_j \cap F_s] + P[E_s] - P[Q_j \cap F_s \cap E_s] \\ &= P[Q_j] \{P[F_s] - P[F_s \cap E_s]\} + P[E_s] = q_j \cdot P[F_s \cap \bar{E}_s] + P[E_s] \dots (5) \end{aligned}$$

in which  $q_j$  is the reliability of rink  $j$ . Eq.(5) implies that the reliability of connectivity of demand node  $i$  is expressed as a linear function of the reliability of rink  $j$ ,  $q_j$ .

Formulation Probabilistic importance and criticality importance are importance measures of lifeline network components based on the reliability concepts (Ref.9). For a demand node  $i$ , these importance measures of rink  $j$  are defined respectively as follows:

$$I_{ij}^P = \frac{\partial R_i}{\partial q_j} = P[F_s \cap \bar{E}_s] = P[F_s \cup E_s] - P[E_s] = P_{cj}^* - P_{cj} \dots (6)$$

$$I_{ij}^C = \frac{\partial \ln R_i}{\partial \ln q_j} = \frac{q_j}{R_i} I_{ij}^P = \frac{q_j}{R_i} (P_{cj}^* - P_{cj}) \dots (7)$$

$$I_{ij}^{C'} = \frac{\partial \ln(1 - R_i)}{\partial \ln(1 - q_j)} = \frac{1 - q_j}{1 - R_i} I_{ij}^P = \frac{1 - q_j}{1 - R_i} (P_{cj}^* - P_{cj}) \dots (8)$$

in which  $P_{cj}$ ; the reliability of connectivity of node  $i$  given that rink  $j$  fails, and

$P_{cj}^*$ ; the reliability of connectivity of node  $i$  given that rink  $j$  does not fail.

Characteristics The probabilistic importance  $I_{ij}^P$  implies the increase rate of the reliability of node  $i$  with respect to the reliability of rink  $j$ , and it does not depend on the value of reliability of rink  $j$ . On the other hand, the criticality importance measures,  $I_{ij}^C$  and  $I_{ij}^{C'}$ , depend on the value of reliability of rink  $j$ ; therefore, we can consider the influence of the value of reliability of rink  $j$  on the reliability of node  $i$ .

Fig.2(a) shows the relationship between  $P_{cj}$ ,  $P_{cj}^*$  and  $R_i$ . As known from definition,  $P_{cj}$  equals to the value of  $R_i$  given that  $q_j$  is 0, and  $P_{cj}^*$  equals to the value of  $R_i$  given that  $q_j$  is 1. Fig.2(b) shows the behavior of importance measures when only  $q_j$  changes from 0 to 1.  $I_{ij}^P$  is constant against  $q_j$ . But  $I_{ij}^C$  increases non-linearly as  $q_j$  increases, and  $I_{ij}^{C'}$  decreases non-linearly as  $q_j$  increases.

Up to now, the influence of reliability of rink  $j$  on the importance of rink  $j$  is discussed; from now on, the influence of reliabilities of the rinks except rink  $j$  on the importance of rink  $j$  is discussed.

For a series system and a parallel system which consist of two rinks  $x$  and  $y$  as shown in Fig.3, the relationship between importance measures of rink  $x$  and the reliabilities of rink  $y$ ,  $q_y$ , is shown in Figs.4,5, and 6.

(1) As known from Fig.4, in the series system, the probabilistic importance of rink  $x$ ,  $I_x^P$ , increases linearly as the reliability of rink  $y$ ,  $q_y$ , increases. In the parallel system,  $I_x^P$  decreases linearly as  $q_y$  increases.

(2) As known from Fig.5, in the series system, the criticality importance of rink  $x$ ,  $I_x^C$ , has constant value 1. In the parallel system,  $I_x^C$  decreases non-linearly as the reliability of rink  $y$ ,  $q_y$ , increases.

As known from Fig.6, in the parallel system, the criticality importance of rink  $x$ ,  $I_x^C$ , has constant value 1. In the series system,  $I_x^C$  increases non-linearly as the reliability of rink  $y$ ,  $q_y$ , increases.

By the examination of the probabilistic importance and the criticality importance in this chapter, it is shown that these importance measures have some shortcomings. The probabilistic importance measure of rink  $j$ ,  $I_j^P$ , can't take the reliability of rink  $j$  into account. In series systems, the criticality importance,  $I_j^C$ , can't take the reliabilities of the rinks except rink  $j$  into account. In parallel systems, the criticality importance,  $I_j^C$ , can't take the reliabilities of the rinks except rink  $j$  into account.

#### PROPOSED LINEAR IMPORTANCE

We pointed out in previous chapter that the probabilistic importance and the criticality importance have shortcomings. We propose new importance measures defined by following equations:

$$I_{ij}^L = q_j \frac{\partial Ri}{\partial q_j} = q_j (P_{cj}^* - P_{cj}) \dots (9)$$

$$I_{ij}^{L'} = (1 - q_j) \frac{\partial Ri}{\partial q_j} = (1 - q_j) (P_{cj}^* - P_{cj}) \dots (10)$$

As shown in Fig.2(b), these importance measures,  $I_{ij}^L$  and  $I_{ij}^{L'}$ , of rink  $j$  change linearly against the reliability of rink  $j$ ,  $q_j$ . So we call these importance measures Linear Importance.

For a series system and a parallel system which consist of two rinks  $x$  and  $y$  as shown in Fig.3, the relationship between the linear importance measures of rink  $x$  and the reliability of rink  $y$  is shown in Figs.7 and 8. As known from Figs.7 and 8, in the series system, the linear importance measures of rink  $x$ ,  $I_x^L$  and  $I_x^{L'}$ , increase linearly as the reliability of rink  $y$ ,  $q_y$ , increases. In the parallel system,  $I_x^L$  and  $I_x^{L'}$  decrease linearly as the reliability of rink  $y$ ,  $q_y$ , increases.

Therefore, the linear importance measures of rink  $j$ ,  $I_j^L$  and  $I_j^{L'}$ , can take the reliability of rink  $j$  into account. Furthermore, the linear importance measures of rink  $j$ ,  $I_j^L$  and  $I_j^{L'}$ , can take the reliabilities of the rinks except rink  $j$  into account in series systems and parallel systems.

If we apply the linear importance to a large network system, we can estimate the importance of components using  $P_{cj}$  and  $P_{cj}^*$  by Monte Carlo simulation.

#### ANALYTICAL EXAMPLE

Each importance measure is applied to a simple network system as shown in Fig.9. Table 1 shows the reliability of connectivity of each demand node. Fig.10 shows the criticality importance  $IC^1$  of rink 1 to 8 for each demand node. Fig.11 shows the linear importance  $IL^1$  of rink 1 to 8 for each demand node. These

importance measures can consider the fact that it is difficult to improve the more reliable components than to improve the less reliable components. Figs.10 and 11 show that the order of the linear importance is same as the order of the criticality importance for each node. But, in comparison with the criticality importance, the linear importance is apt to evaluate that the importance of rinks for less reliable node is larger.

We can estimate the importance measures of rink  $j$  for whole network system by using following equation:

$$T_j = \sum_{i=1}^n \alpha_i I_{ij} \dots (12)$$

in which  $T_j$ ; total importance of rink  $j$   
 $\alpha_i$ ; coefficient of demand node  $i$  ( $=1.0$ )  
 $I_{ij}$ ; importance measure of rink  $j$  for demand node  $i$ , and  
 $n$ ; number of nodes

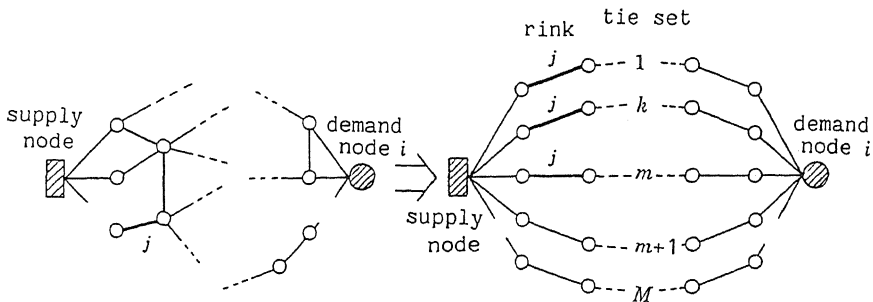
Fig.12 shows the total importance measures normalized by the maximum value respectively. As known from Fig.12, the linear importance  $IL'$  is the most suitable importance measure. According to  $IL'$ , the importance of rink 7, is fairly high because of low reliability of rink 7 and the importance of rink 8 is fairly high because of low reliability of node 5.

#### ACKNOWLEDGMENT

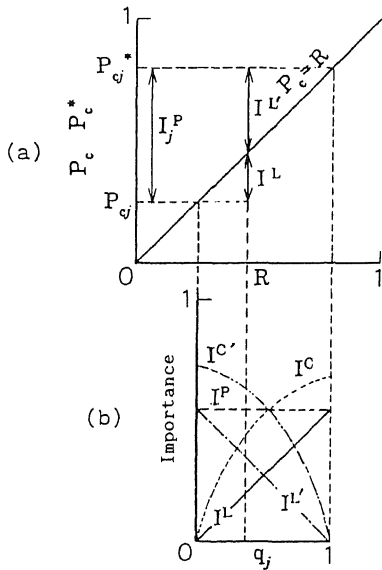
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(a) Network System (b) The Corresponding SSP  
 Fig.1 Network System and the Corresponding SSP



(a) Relationship between  $P_{cij}$ ,  $P_{cij}^*$  and  $R_i$   
 (b) Relationship between Importance Measures and  $q_j$

Fig.2 Basic Characteristics of Importance Measures

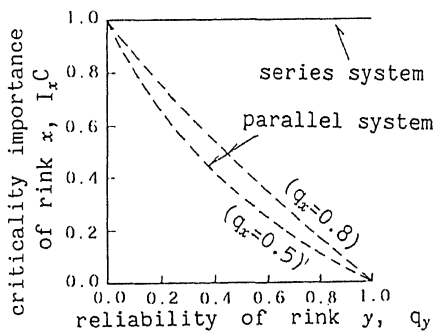
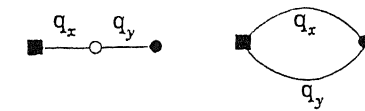


Fig.5 Characteristics of Criticality Importance  $I_x^C$



(a) Series System (b) Parallel System  
 Fig.3 Series System and Parallel System

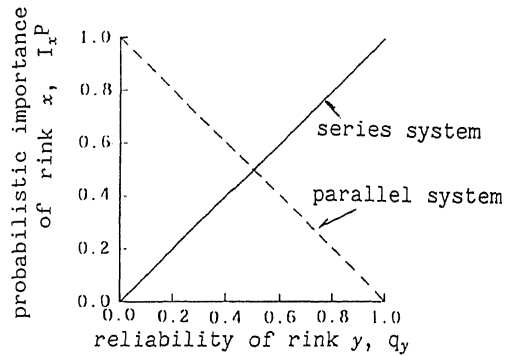


Fig.4 Characteristics of Probabilistic Importance  $I_x^P$

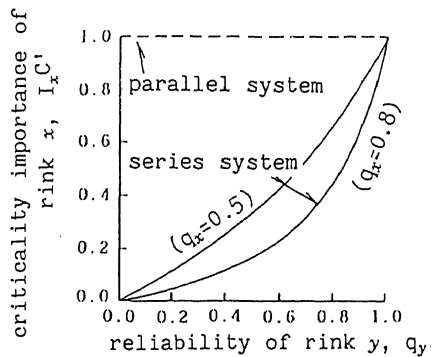


Fig.6 Characteristics of Criticality Importance  $I_x^{C'}$

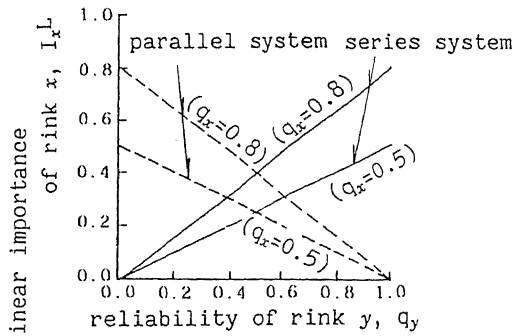


Fig.7 Characteristics of Linear Importance  $I_x L$

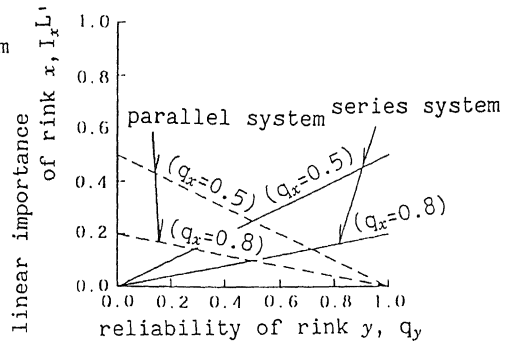


Fig.8 Characteristics of Linear Importance  $I_x L'$

Table 1 Reliability of Connectivity of Demand Node

Node	1	2	3	4	5
Reliability	0.925	0.963	0.836	0.928	0.743

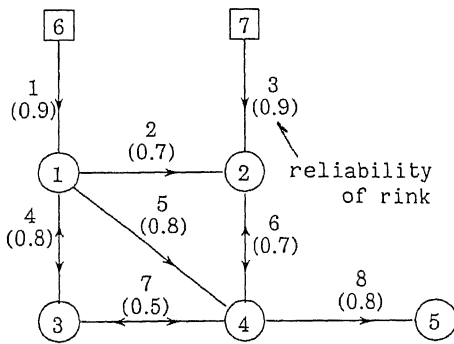


Fig.9 Model Network System

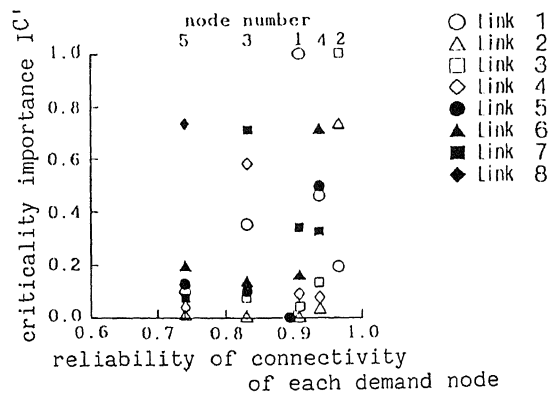


Fig.10 Criticality Importance  $I_x C'$

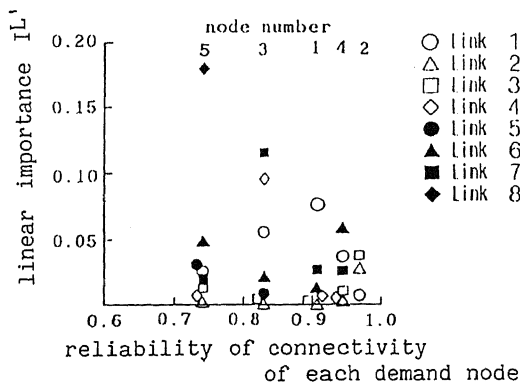


Fig.11 Linear Importance  $I_x L'$

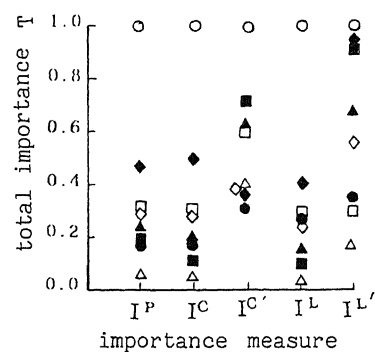


Fig.12 Total Importance