THEORETICAL AND NUMERICAL STUDY OF FLOOR SPECTRA

Philippe Maurel*

*SGN, Scientific Calculation Department, St Quentin en Yvelines, France

SUMMARY

This paper describes methods used in France to generate floor spectra for spent nuclear fuel reprocessing plants.

Floor spectra play an important role in plant design. They are used at the beginning of project studies to design plant equipment and must ensure adequate precision without being overly conservative.

INTRODUCTION

Despite its apparent simplicity, calculation of floor spectra becomes highly complex when a high degree of precision is required. While many theoretical studies have been published on this subject over the past few years, few have actually made comparisons of numerical results.

This study analyzes the theoretical aspects of the TIROIR calculation program developed in France by CEA/DEMT and the US-developed program FSG. Both of these programs, which offer different advantages, have been used widely at SGN.

The numerical comparisons described here concern a typical spent fuel reprocessing plant, assuming linear behavior of structures. The calculations also use the primary structure modal base to study the response of damped oscillators by direct transfer from the floor spectrum.

THEORETICAL ASPECTS OF PROGRAM TIROIR

It has been confirmed ref. [1] that an earthquake signal, which is a non-stationary, random signal, can generally be broken down to the product of a stationary, random function $f(t)$ and a positive, slightly variable envelope:

$$ S(t) = f(t).E(t). $$  \hspace{1cm} (1)
It is therefore possible to calculate an increasing factor for the average response of a simple oscillator:

\[ X'' + 2\beta_0 \Omega_0 X' + \Omega_0^2 X = f(t) \]  

by value:

\[ [Q_0'.X_0(t)] = e^{-2\beta_0 \Omega_0 t} \int_0^T e^{2\beta_0 \Omega_0 \theta} E_0^2(\theta) d\theta \]

\[ = \left[ \frac{T}{n_1^2 + \frac{2.5.5 \cdot 2.5.5}{4 \cdot n_1^2 n_2^2 + \beta_0^2 \Omega_0^2}} \right] \int_0^T \left[ \frac{T(n_1)}{n_1^2} \right] \frac{n_1^2}{n_1^2} \cdot d\theta \]

\[ \text{where } T = \text{duration of signal} \]

\[ F^T(\Omega) = \left| \int_0^T e^{-i\Omega t} S(t') dt' \right| \]

[module of the Fourier transform of \( S(t) \)]

\[ \beta^2 = \frac{\beta^2 + \frac{1}{16 \cdot t^2 (n_0 - n_1)^2}} \]

\( V = \text{asymptotic correction term applied when } \Omega \text{ tends toward infinity.} \)

The displacement of a point P on the floor of a building whose modal base is known, together with its characteristic frequencies, generalized masses \( M_n \), participation factors \( q_n/M_n \), modal deformations \( X_n \), and modal damping \( \beta_n \) is expressed as:

\[ X_p(t) = X_0(t) + \sum_{n=1}^{N} q_n(t) \cdot X_n \]

where:

\( X_p(t) \) is the absolute displacement of the point \\
\( X_0(t) \) is the floor displacement \\
\( q_n(t) \) is the generalized coordinate of eigenmode \( n \) \\
\( a_n(t) \) is the solution of:

\[ a_n''(t) + 2\beta_n \Omega_n a_n' + \Omega_n^2 a_n(t) = -q_n/M_n \cdot \gamma_n(t) \]

It is then possible, via the Fourier transform, to obtain the Fourier spectrum of acceleration at point P as a function of the Fourier spectrum of floor acceleration, i.e.:

\[ \gamma_p(n) = \gamma(n) \cdot \left| 1 + \sum_{n=1}^{N} \frac{\Omega_n^2}{\Omega_n^2 - \Omega_0^2 + 2i \beta_n \Omega_n} \cdot \frac{q_n \cdot \gamma_n / M_n}{1 - \Omega_n^2 / \Omega_0^2 - 2i \beta_n \Omega_n} \right| \]

Then again, if it is assumed that acceleration can be factorized as shown in (1), quadratic combination results in:

\[ E_p^2(t) = E_0^2(t) \cdot \left( 1 - \sum_{n=1}^{N} \frac{q_n / M_n \cdot X_n \cdot E_n^2(t)}{1 - \Omega_n^2 / \Omega_0^2 + 2i \beta_n \Omega_n} \right) \]

where \( E_p(t) \) and \( E_0(t) \) are functions varying slowly and \( E_n(t) \) represents the increasing factors of \( q_n(t) \) values, which are obtained as shown in (3).

The oscillator spectrum \( (\Omega_0, \beta_0) \) associated with point P of the building is expressed as:

\[ \left[ Q_0'.X_0'(t) \right]_{\text{max}} = \left[ e^{-2\beta_0 \Omega_0 t} \left( \int_0^T e^{2\beta_0 \Omega_0 \theta} E_0^2(\theta) d\theta \right) \right] \left| \int_0^T \left( \frac{2\beta_0 \Omega_0 \cdot \frac{v_0^2}{T_n^2} \cdot n_1^2 \cdot d\theta}{4n_1^2(n_0 - n_1) + \beta_0^2 \Omega_0^2} \right) \right| \]
Data required for the calculations are therefore the floor displacement envelope, $E(t)$, the Fourier transform of the floor signal and the modal base of the load-bearing structure.

It can be shown see ref. [1] that the pseudo-speed spectrum for damped oscillators is similar to the Fourier transform of the signal. An approximative method for obtaining the Fourier transform is to build an undamped spectrum of the earthquake signal. The next step is to check that this transform enables determination of the floor spectra defined for the site before using them for a spectrum transfer operation.

THEORETICAL ASPECTS OF PROGRAM FSG

The idea developed in this program is described in detail in ref. (4), based on the response of a system with $N$ degrees of freedom:

The method consists of calculating a new modal base for the equipment/structure system, which allows for the various parameters below, in each vibration mode:

$$\beta_i = \frac{\Omega_i - \Omega_e}{\Omega_e}$$ (tuning parameter)

$$\gamma_i = \frac{m_i}{m_e} \phi_{ik}^2$$ (interaction parameter) \hspace{1cm} (11)

$$\delta_i = (\mu_i - \Omega_i \frac{\Omega_i}{\Omega_e})\frac{\Omega_i}{\Omega_e}$$ (non-classical damping parameter)

where: \(\Omega_{ai}\): average damping \[\frac{(\Omega_i + \Omega_e)}{2}\]

\(\phi_{ik}\): $k$th element of eigenmode $\phi_i$

\(\Omega_i, \phi_i, \mu_i, m_i\) characteristics of the load bearing structure (frequency, eigenvector, damping and modal mass).

The significant parameter, which serves to determine whether the oscillator is in resonance with an eigenmode is calculated by the following:

$$\beta_i^2 = \frac{(\mu_i + \mu_e)^2}{e} \left[1 + \frac{\gamma_i}{4 \mu_i \mu_e}\right]$$ \hspace{1cm} (12)

The new modal base is then calculated for a system coupled at $N + 1$ degrees of freedom, for use in calculating oscillator response, via a superimposition approach which makes allowance for possible mode-to-mode coupling.

The quadratic average of the response is determined by:

$$E\left[X^2\right] = \sum_i \sum_j p_{o,ij} \cdot S_i \cdot S_j$$ \hspace{1cm} (13)
$S_1$ and $S_j$ are the accelerations of the floor spectrum, for the frequency and damping associated with the mode under consideration.

$$F_{o,i,j} = a_i a_j . R_e (T_{o,i,j}) - (a_i c_j - a_j c_i) . Im (T_{1,i,j}) + c_j c_j . R_e (T_{2,i,j})$$ (14)

$$T_{m,i,j} = G_o \int_0^\alpha \Omega_m . H_i (\Omega) . H_j (-\Omega) . d\Omega \quad m = 0, 1, 2$$ (15)

$$H_i (\Omega) = (\Omega_i^2 - \Omega^2 + 2 i \mu_i \Omega)^{-1}$$ (16)

$\alpha$ is a constant which characterizes spectral density; $a_i$ and $c_i$ are the generalized participation factors of the new modal base:

$$a_i = 2 R_e (b_i s_i^0) \quad c_i = 2 R_e (b_i)$$

$$b_i = \left[ \frac{s_i^0 (q_i^0 - \Phi_i^0)}{s_i^2 \Phi_i^1 . M . \Phi_i^1 - \Phi_i^1 T . K . \Phi_i^1} \right]$$ (17)

A detailed description of the formulas is given in ref. (4). Note that it is possible to obtain spectra which incorporate allowance for standard deviations.

**NUMERICAL COMPARISONS**

The comparisons made all related to structure with the following characteristics:

**Table I : Characteristics of the structure**

<table>
<thead>
<tr>
<th>Properties at nodes</th>
<th>Properties of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>Mass $10^6$ kg</td>
</tr>
<tr>
<td>1</td>
<td>22.3</td>
</tr>
<tr>
<td>2</td>
<td>13.4</td>
</tr>
<tr>
<td>3</td>
<td>17.0</td>
</tr>
<tr>
<td>4</td>
<td>14.5</td>
</tr>
<tr>
<td>5</td>
<td>17.5</td>
</tr>
<tr>
<td>6</td>
<td>8.9</td>
</tr>
<tr>
<td>7</td>
<td>7.2</td>
</tr>
<tr>
<td>8</td>
<td>7.2</td>
</tr>
<tr>
<td>9</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Equivalent ground stiffness: Basic frequencies:

$K_x = 6.6.10^9$ N/m  
$K_y = 6.7.10^9$ N/m  
$K_{ox} = 5.6.10^9$ N.m/ rad  
$K_{oy} = 7.3.10^9$ N.m/ rad

$\text{dir: } ox = 6.2$ Hz  
$\text{dir: } oy = 6.1$ Hz  
$\text{dir: } oz = 10.0$ Hz

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Floor spectra used are shown on Figure 1. The weighted damping factors calculated for each mode are incorporated into each of the spectrum transfer methods. Responses have been plotted in both horizontal directions at node 5 of the model, on Figures 2 and 3 for program TIROIR and Figures 4 and 5 for program FSG. These curves show a good correlation between the results of the respective programs. Maximum responses are very close to one another and occur at the same frequencies.

Secondary peaks at node 5, which correspond to a significant modal participation at 12.3 Hz in direction ox and at 12.7 Hz in direction oy, are clearly depicted.

The table below shows the frequencies and values of acceleration peaks (in g) recorded at node 5 in the two directions;

<table>
<thead>
<tr>
<th></th>
<th>DIRECTION X</th>
<th></th>
<th>DIRECTION Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FREQUENCY (Hz)</td>
<td>ACCELERATION (g)</td>
<td>FREQUENCY (Hz)</td>
</tr>
<tr>
<td>TIROIR</td>
<td>6.0</td>
<td>1.79</td>
<td>5.9</td>
</tr>
<tr>
<td>PEAK 1</td>
<td>12.1</td>
<td>0.55</td>
<td>12.2</td>
</tr>
<tr>
<td>PEAK 2</td>
<td>5.9</td>
<td>1.70</td>
<td>5.9</td>
</tr>
<tr>
<td>PEAK 1</td>
<td>11.8</td>
<td>0.66</td>
<td>12.4</td>
</tr>
</tbody>
</table>

It is clear from this table that the first "peak" values are very close to each other (less than 5 % difference) and the second a little further apart.

Fig. 1 Floor spectra

Fig. 6 Finite element model
CONCLUSION

Maximum values appearing on the graphs show good correlation, both with respect to resonance frequency position and to accelerations. Each of the two programs has certain advantages: TIROIR enables construction of synthetic accelerograms and verification of a number of earthquake signal properties. FSG makes it possible to obtain transferred spectra directly, with a mode summing rule which permits spectra to be obtained when primary-to-secondary structure interaction becomes significant.

Fig. 2 Acceleration spectrum calculated by TIROIR (mode 5, direction X)

Fig. 4 Acceleration spectrum calculated by FSG (mode 5, direction X)

Fig. 3 Acceleration spectrum calculated by TIROIR (mode 5, direction Y)

Fig. 5 Acceleration spectrum calculated by FSG (mode 5, direction Y)

REFERENCES

[1] Méthodes de calcul basées sur l'utilisation des spectres sismiques. Fondements statistiques (Livolant, Gantenbein, Gibert, CEA/DEMT)
[2] Seismic analysis of a PWR 900 MW Reactor (Gantenbein, Paper K5/1, Smirt 7, Chicago)
[3] Random Vibration (Crandall, Mark)