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THEORETICAL AND NUMERICAL STUDY OF FLOOR SPECTRA

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SUMMARY

This paper describes methods used in France to generate floor spectra for spent nuclear fuel reprocessing plants.

Floor spectra play an important role in plant design. They are used at the beginning of project studies to design plant equipment and must ensure adequate precision without being overly conservative.

INTRODUCTION

Despite its apparent simplicity, calculation of floor spectra becomes highly complex when a high degree of precision is required. While many theoretical studies have been published on this subject over the past few years, few have actually made comparisons of numerical results.

This study analyzes the theoretical aspects of the TIROIR calcultation program developed in France by CEA/DEMT and the US-developed program FSG. Both of these programs, which offer different advantages, have been used widely at SGN.

The numerical comparisons described here concern a typical spent fuel reprocessing plant, assuming linear behavior of structures. The calculations also use the primary structure modal base to study the response of damped oscillators by direct transfer from the floor spectrum.

THEORETICAL ASPECTS OF PROGRAM TIROIR

It has been confirmed ref. [1] that an earthquake signal, which is a non-stationary, random signal, can generally be broken down to the product of a stationary, random function f(t) and a positive, slightly variable envelope:

$$S(t) = f(t) \,.\, E(t) \,. \qquad (1)$$

It is therefore possible to calculate an increasing factor for the average response of a simple oscillator:

$$X'' + 2.\beta.\Omega_0.X' + \Omega_0.X = f(t) \tag{2}$$

by value:

$$\left[\Omega'_o . X(t)\right]^2 = \frac{e^{-2.\beta.\Omega_o.t}\int_0^t e^{2.\beta.\Omega_o.\theta} E^2(\theta).d\theta}{\int_0^t E^2(t).dt} \int_{-\infty}^{\infty} \frac{\left[F^T(n_1)\right]^2 .n_1{}^2.2.\beta'.\Omega_o.dn_1}{4.\pi^2(n_o - n_1)^2 + \beta'^2.\Omega_o{}^2} + \frac{V^2}{\Omega_0{}^2} . \frac{\int_0^{n_1} \left[F^T(n_1)\right]^2 .n_1{}^2.dn_1}{\int_0^{\infty} \left[F^T(n_1)\right]^2 .n_1{}^2.dn_1} \tag{3}$$

where T = duration of signal

$$F^T(\Omega) = \left| \int_0^T e^{-i\Omega t'}.S(t').dt' \right| \tag{4}$$

[module of the Fourier transform of S (t)]

$$\beta'^2 = \beta^2 + \frac{1}{16.t^2.(n_0 - n_1)^2} \tag{5}$$

V = asymptotic correction term applied when Ω tends toward infinity.

The displacement of a point P on the floor of a building whose modal base is known, together with its characteristic frequencies, generalized masses (Mn), participation factors qn/Mn, modal deformations Xnp and modal damping β n is expressed as:

$$X_p(t) = X_0(t) + \sum_1^N \alpha_n(t).X_{np} \tag{6}$$

where:
X_p (t) is the absolute displacement of the point
X_o (t) is floor displacement
α_n (t) is the generalized coordinate of eigenmode n
α_n (t) is the solution of:

$$\alpha_n''(t) + 2.\beta_n.\Omega_n.\alpha_n' + \Omega_n{}^2.\alpha_n(t) = -q_n/M_n.\Gamma_n(t) \tag{7}$$

It is then possible, via the Fourier transform, to obtain the Fourier spectrum of acceleration at point P as a function of the Fourier spectrum of floor acceleration, i.e.:

$$\Gamma_p(n) = \Gamma(n).\left|1 + \sum_1^N \frac{\Omega^2}{\Omega_n^2} . \frac{q_n.X_{np}/M_n}{1 - \Omega^2/\Omega_n^2 + 2.i.\beta_n.\Omega/\Omega_n}\right| \tag{8}$$

Then again, if it is assumed that acceleration can be factorized as shown in (1), quadratic combination results in:

$$E_p{}^2(t) = E^2(t).\left|1 - \sum_1^N q_n/M_n.X_{np}\right|^2 + \sum_1^N \Omega_n{}^4.X_{np}{}^2.E_{\alpha n}(t) \tag{9}$$

where Ep(t) and E(t) are functions varying slowly and $E_{\alpha n}(t)$ represents the increasing factors of αn(t) values, which are obtained as shown in (3).

The oscillator spectrum (Ω_0, β_0) associated with point P of the building is expressed as:

$$\left[\Omega_0'.\dot{X}_E{}^{(t)}\right]max = \left[e^{-2\beta_0\Omega_0 t}.\frac{\int_0^t e^{2\beta_0\Omega_0\theta}.E_p{}^2(\theta)\,d\theta}{\int_0^{T\,max} E_p{}^2(t)\,dt}\right] \int_{-\infty}^{\infty} \frac{2\beta_0'\Omega_0 dn_1}{4\pi^2(n_0 - n_1) + \beta_0{}^{2\prime}\Omega_0{}^2} + \frac{V_0{}^2}{\Omega_0{}^2} \frac{\int_0^{n_0} \left|\Gamma_0{}_{(n_1)}\right| .n_1^2\,dn_1}{\int_0^{\infty} \left|\Gamma_0(n_1)\right| .n_1^2\,dn_1} \tag{10}$$

Data required for the calculations are therefore the floor displacement envelope, E(t), the Fourier transform of the floor signal and the modal base of the load-bearing structure.

It can be shown see ref. [1] that the pseudo-speed spectrum for damped oscillators is similar to the Fourier transform of the signal. An approximative method for obtaining the Fourier transform is to build an undamped spectrum of the earthquake signal. The next step is to check that this transform enables determination of the floor spectra defined for the site before using them for a spectrum transfer operation.

THEORETICAL ASPECTS OF PROGRAM FSG

The idea developed in this program is described in detail in ref. (4), based on the response of a system with N degrees of freedom:

The method consists of calculating a new modal base for the equipment/structure system, which allows for the various parameters below, in each vibration mode:

$$\beta_i = \frac{\Omega_i - \Omega_s}{\Omega_{ai}} \qquad \text{(tuning parameter)}$$

$$\Gamma_i = \frac{m_e}{m_i} \Phi_{ik}^2 \qquad \text{(interaction parameter)} \qquad (11)$$

$$\delta_i = (\mu_i - \frac{\Omega_i}{\Omega_e} \mu_e) \frac{\Omega_i}{\Omega_e} \qquad \text{(non-classical damping parameter)}$$

where: Ω_{ai}: average damping $= \frac{(\Omega_i + \Omega_e)}{2}$

Φ_{ik}: kth element of eigenmode $\varnothing_i$
Ω_i, $\varnothing_i$, μ_i, m_i characteristics of the load bearing structure (frequency, eigenvector, damping and modal mass).

The significant parameter, which serves to determine whether the oscillator is in resonance with an eigenmode is calculated by the following:

$$\beta_i^2 < \frac{(\mu_i + \mu_e)^2}{e} \left[1 + \frac{\Gamma_i}{4 \mu_i \mu_e} \right] \qquad (12)$$

The new modal base is then calculated for a system coupled at N + 1 degrees of freedom, for use in calculating oscillator response, via a superimposition approach which makes allowance for possible mode-to-mode coupling.

The quadratic average of the response is determined by:

$$E\left[X_o^2\right] = \sum_i \sum_j F_{o,ij} \cdot S_i \cdot S_j \qquad (13)$$

S_i and S_j are the accelerations of the floor spectrum, for the frequency and damping associated with the mode under consideration.

$$F_{o,ij} = a_i\, a_j \,.\, R_e\,(T_{o,ij}) - (a_i\, c_j - a_j\, c_i)\,.\, Im\,(T_{1,ij}) + c_j\, c_j \,.\, R_e\,(T_{2.i,j}) \qquad (14)$$

$$T_{m,ij} = G_o \int_0^{\infty} \Omega^m \,.\, H_i\,(\Omega)\,.\, H_j\,(-\Omega)\,.\, d\Omega \quad m = 0\,,1\,,2 \qquad (15)$$

$$H_i(\Omega) = (\Omega_i^{*2} - \Omega^2 + 2\,.\,i\,.\,\mu_i^*\,.\,\Omega)^{-1} \qquad (16)$$

Go is a constant which characterizes spectral density; ai and ci are the generalized participation factors of the new modal base:

$$a_i = 2\, R_e\,(b_i\, s_i^*) \qquad\qquad c_i = 2\, R_e\,(b_i)$$

$$b_i = \left| \frac{s_i^* \,.\, (q^T \,.\, Ø_i^*)\,(Ø_i^{*T} \,.\, M \,.\, r)}{s_i^{*2} \,.\, Ø_i^{*T} \,.\, M \,.\, Ø_i^* - Ø_i^{*T} \,.\, K \,.\, Ø_i^*} \right| \qquad (17)$$

A detailed description of the formulas is given in ref. (4). Note that it is possible to obtain spectra which incorporate allowance for standard deviations.

NUMERICAL COMPARISONS

The comparisons made all related to structure with the following characteristics:

Table I : Characteristics of the structure

Properties at nodes			Properties of elements		
Node	Mass 10^6 kg	Inertia 10^9kg.m2	Section m2	Inertia X m4	Inertia Y m4
1	22.3	52.0			
2	13.4	3.2	877	40 000	80 000
3	17.0	3.9	850	40 000	60 000
4	14.5	2.8	794	40 000	60 000
5	17.5	3.9	595	40 000	60 000
6	8.9	1.6	527	40 000	60 000
7	7.2	1.5	500	20 000	60 000
8	7.6	1.6	326	10 000	20 000
9	7.2	1.9	116	10 000	20 000
10	1.2	0.1	116	10 000	20 000

Equivalent ground stiffness:

Kx = 6.6.10E11 N/m
Ky = 6.7.10E11 N/m
Kox = 5.6.10E14 Nm/rad
Koy = 7.3.10E14 Nm/rad

Basic frequencies:

dir: ox = 6.2 Hz
dir: oy = 6.1 Hz
dir: oz = 10.0 Hz

Floor spectra used are shown on Figure 1. The weighted damping factors calculated for each mode are incorporated into each of the spectrum transfer methods. Responses have been plotted in both horizontal directions at node 5 of the model, on Figures 2 and 3 for program TIROIR and Figures 4 and 5 for program FSG. These curves show a good correlation between the results of the respective programs. Maximum responses are very close to one another and occur at the same frequencies.

Secondary peaks at node 5, which correspond to a significant modal participation at 12.3 Hz in direction ox and at 12.7 Hz in direction oy, are clearly depicted.

The table below shows the frequencies and values of acceleration peaks (in g) recorded at node 5 in the two directions;

Table II : Frequencies and peak accelerations

	DIRECTION X		DIRECTION Y	
	FREQUENCY (Hz)	ACCELERATION (g)	FREQUENCY (Hz)	ACCELERATION (g)
	TIROIR			
PEAK 1	6. 0	1. 79	5. 9	1. 6
PEAK 2	12. 1	0. 55	12. 2	0. 55
	FSG			
PEAK 1	5. 9	1. 70	5. 9	1. 53
PEAK 2	11. 8	0. 66	12. 4	0. 65

It is clear from this table that the first "peak" values are very close to each other (less than 5 % difference) and the second a little further apart.

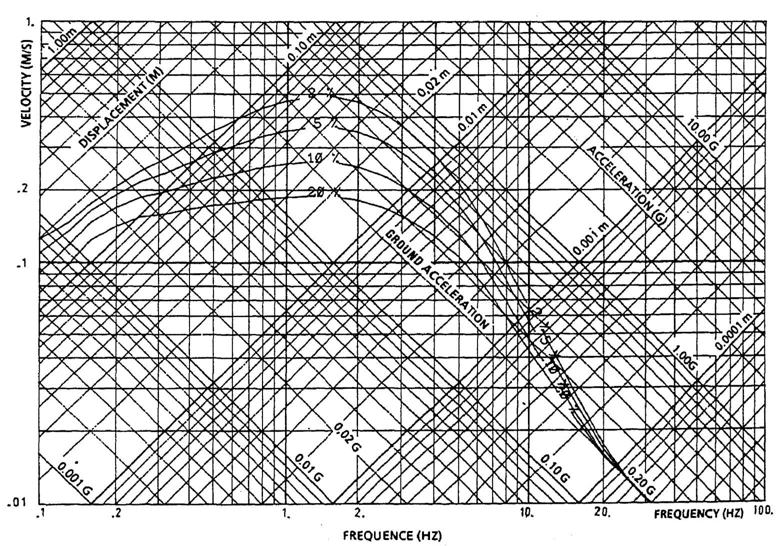

Fig. 1 Floor spectra

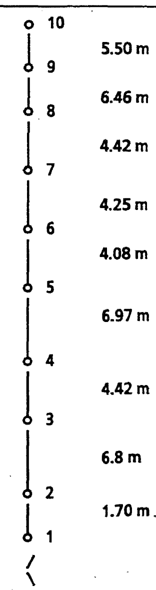

Fig. 6 Finite element model

CONCLUSION

Maximum values appearing on the graphs show good correlation, both with respect to resonance frequency position and to accelerations. Each of the two programs has certain advantages: Tiroir enables construction of synthetic accelerograms and verification of a number of earthquake signal properties. FSG makes it possible to obtain transferred spectra directly, with a mode summing rule which permits spectra to be obtained when primary-to-secondary structure interaction becomes significant.

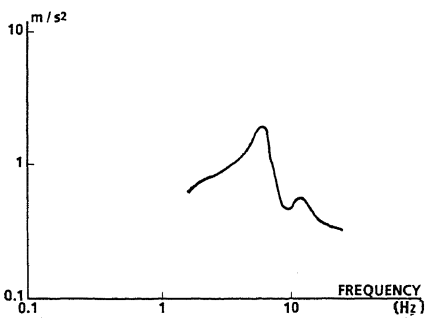

Fig. 2 Acceleration spectrum calculated by TIROIR (mode 5, directions X)

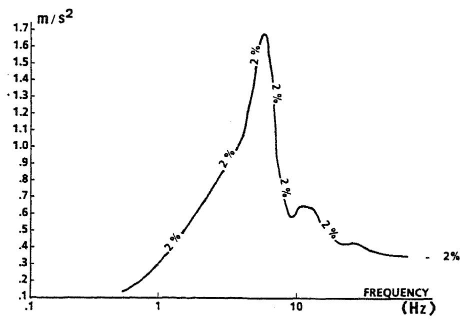

Fig. 4 Acceleration spectrum calculated by FSG (mode 5, direction X)

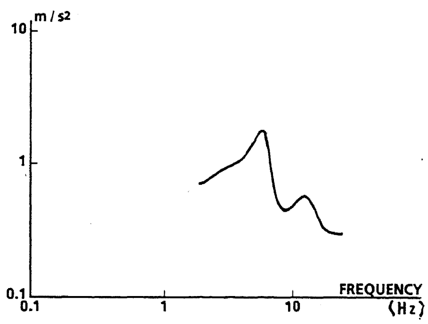

Fig. 3 Acceleration spectrum calculated by TIROIR (mode 5, direction Y)

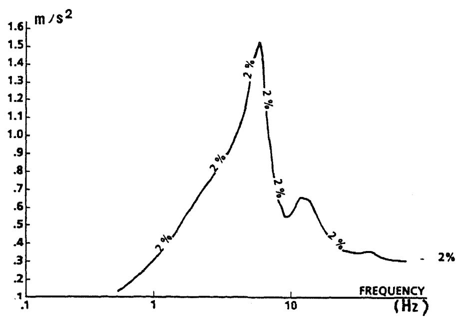

Fig. 5 Acceleration spectrum calculated by FSG (mode 5, direction Y)

REFERENCES

[1] Méthodes de calcul basées sur l'utilisation des spectres sismiques.Fondements statistiques (Livolant, Gantenbein, Gibert, CEA/DEMT)

[2] Seismic analysis of a PWR 900 MW Reactor (Gantenbein, Paper K5/1, Smirt 7, Chicago)

[3] Random Vibration (Crandall, Mark)

[4] F-S-G (Floor Spectrum Generator), (Technical Supplement by Der Kiureghian, T. Igusa).