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## A PROPOSAL FOR THE EARTHQUAKE RESISTANT DESIGN OF TANKS -RESULTS FROM THE AUSTRIAN RESEARCH PROJECT

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### SUMMARY

An engineering approach is presented for calculating the maximum dynamic pressure distributions caused by horizontal and vertical earthquake excitation of tanks. Three different possibilities for superposing the dynamic pressure components due to the horizontal and the vertical earthquake components on the static pressure and the different modes of wall instabilities are discussed. The results show that the dynamic pressure component caused by vertical excitation must not be neglected especially for tall tanks. An essential aim of this project has been the development of simple formulas and diagrams for engineers dealing with the construction of liquid storage tanks made of steel.

### INTRODUCTION

Tanks damaged by earthquakes show different modes of buckling of the tank wall: diamond shaped buckles near the base, a bulge near the base or at a change of the wall thickness ('elephant footing') and, rather rarely, buckles near the top (Refs. 1 and 2). In addition to these modes of quasi-static buckling dynamic instabilities may appear (Ref. 3) which can be investigated by numerical methods as shown in (Ref. 4).

In this context the vertical component of the earthquake excitation in combination with the well studied effects caused by the horizontal component plays an important role. Furthermore, the nonlinearities due to plasticity and large deformations as well as the deformability of the foundation and the nonlinear effects of unanchored tanks must not be ignored.

### ANCHORED TANKS

The engineering approach is based on parametric studies performed for 14 typical steel tanks used to store light oil products and having different height vs. radius ratios.

Horizontal Excitation In (Ref. 5) the fundamental concept of an iterative procedure is described for calculating the maximum dynamic pressure resulting from the common vibration of the elastic shell and the liquid. Based on this concept a numerical algorithm has been developed using the 'added mass concept'. Further details are given in (Ref. 6).

From these parametric studies the diagrams shown in Fig.1 are obtained by integration of the calculated pressure distributions. These diagrams are used in the engineering approach.  $M_{SL}$  and  $M_D$  are the effective masses with respect to sloshing and to the common vibration of the shell and the fluid (interaction vibration), respectively.  $M_B$  is the effective mass with respect to rigid tank motion under the assumption of a rigid ground. The influence of a deformable soil on the dynamic behaviour of a horizontally activated tank will be treated in a future paper and will be of some concern. Multiplication of the effective masses with the corresponding maximum accelerations  $A_{SL}$ ,  $A_D$ ,  $A_B$ , leads to the maximum resultant horizontal earthquake loads. Regarding  $A_{SL}$ ,  $A_D$ ,  $A_B$  see Fig.1.  $A_H$  is the maximum horizontal free field acceleration. Multiplication with the related heights  $H_{SL}$ ,  $H_D$  and  $H_B$  gives the individual contributions to the resultant overturning moment from which the maximum axial compressive membrane force used for buckling analysis is obtainable (Ref. 5).

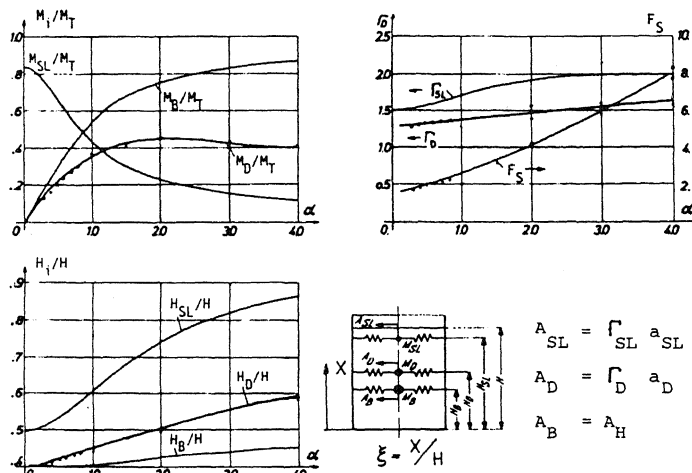


Fig.1 Relevant Parameters for the Calculation of the Tank Wall Overturning Moment vs.  $\alpha = H/R$  (H...Tank-Height, R...Tank-Radius)

In addition to the design response spectra the calculation of the relevant accelerations requires the knowledge of the natural frequencies for sloshing and interaction vibrations as well as the relevant mode participation factors  $\Gamma$ . The fundamental sloshing frequency is given by:

$$f_{SL} = \sqrt{1.84 g \tanh(1.84 \alpha) / R} / (2 \pi) \quad [\text{cps}] \quad (1),$$

and the fundamental natural frequency for the interaction vibration of the elastic tank wall and the liquid content can be approximated by

$$f_D = \sqrt{Et_{1/3} / (\rho_l H)} / (2F_s(\alpha)R), \quad t_{1/3} = t(\xi=1/3) \quad t \dots \text{thickness of tank wall} \quad (2),$$

with  $F_s(\alpha)$  taken from Fig.1. The mode participation factors  $\Gamma_{SL}$  and  $\Gamma_D$  can be obtained from the same diagram. A SRSS superposition (Ref. 5) should be applied for calculating the maximum resultant tank wall overturning moment:

$$MM = \sqrt{(M_{SL} A_{SL} H_{SL})^2 + (M_B A_B H_B)^2 + (M_D A_D H_D)^2} \quad (3)$$

It is not recommendable to use pseudo-accelerations from deformation response spectra because the pseudo-accelerations underestimate the actual spectral accelerations especially in the case of the lower sloshing frequencies (Rev. 7).

Vertical Excitation Results of parametric studies based on the theory outlined in (Ref. 8 and 9) allow the conclusion that the vertical earthquake excitation can, with the exception of the damping behaviour, be approximated by a mathematical model which does not take into account the deformability of the foundation. This means that the maximum pressure due to vertical excitation can be calculated by the following formula derived from the relations given for rigidly based tanks in (Ref. 8 and 9):

$$P_v(\xi) = \sqrt{P_{Bv}^2 + P_{Dv}^2} = \sqrt{(H \rho_L (1 - \xi) A_v)^2 + (0.815 \beta H \rho_L \cos(\pi \xi / 2) a_{Dv})^2} \quad (4).$$

$P_{Bv}$  results from the vertical excitation of the rigid tank and  $P_{Dv}$  is the contribution due to the deformability of the elastic tank wall.  $\beta$  is a correction factor (Eq.(6)) with respect to the boundary conditions.  $A_v$  is the maximum vertical acceleration and  $a_{Dv}$  is the spectral acceleration taken from the response spectrum for the vertical earthquake component at the fundamental natural frequency for axisymmetric interaction vibrations which can be estimated by

$$f_{Dv} = \sqrt{2 E I_1(\lambda_1) t_1 / 3} / (\pi \rho_L H (1 - \mu^2) I_0(\lambda_1)) / (4R) \quad [\text{cps}] \quad \lambda_1 = \pi / (2\alpha) \quad (5)$$

$E$  ...Young's modulus,  $\rho_L$ ...mass density (liquid),  $\mu$ ...Poisson's ratio  
 $I_1$ ...modified Bessel function of first kind and order 1  
 $I_0$ ...modified Bessel function of first kind and order 0

and with a damping ratio  $\xi$  as shown in Fig.2 (Ref. 9) for different shear wave velocities  $v_s$  of the soil and different radius-to-thickness ratios of the tank wall.

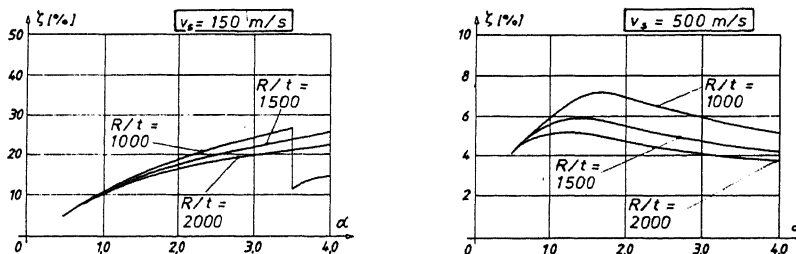


Fig.2 Damping Ratio of the Interaction Vibration; Vertical Excitation (with 5 % material damping of the soil)

The contribution  $P_{Dv}$  in Eq.(4) is estimated according to (Ref. 8), where the clamping condition of the tank wall at the bottom is neglected. Taking into account the correct boundary conditions a procedure described in (Ref. 6) leads to more accurate results with the correction factor  $\beta$ :

$$\begin{aligned} 0.0 < \alpha \leq 0.8 : & \quad \beta = 1.0 \\ 0.8 < \alpha \leq 4.0 : & \quad \beta = 1.078 + 0.274 \ln(\alpha) \end{aligned} \quad (6)$$

A more sophisticated approach to the full coupling between liquid-shell-foundation-soil in which rocking vibration is included is presented in (Ref. 10).

Combined Action of the Horizontal and the Vertical Earthquake Components Fig.3(a) shows three different possibilities of superposition of the contributions of the static pressure  $P_{stat}$ , the pressure due to the horizontal earthquake component  $P_H$  and the pressure due to the vertical earthquake component  $P_v$ , which are most critical with respect to the individual buckling modes.

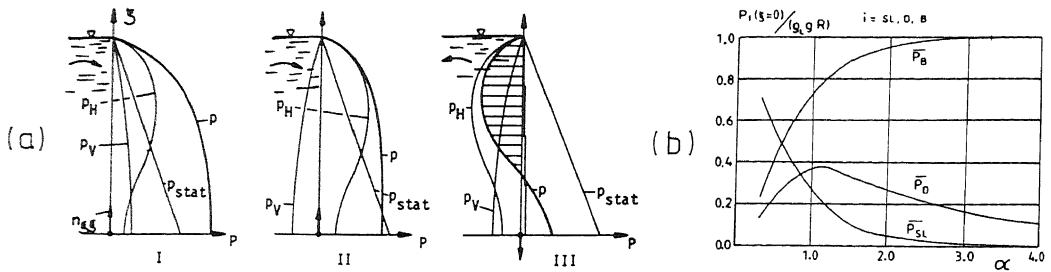


Fig.3 (a) Schematic Presentation of Different Possible Superpositions of the Maximum Pressure Contributions  
 (b) Parts of Max. Dyn. Activated Pressure Due to  $A_H=1g$  for  $\xi=0$

Case I ( $p = p_{stat} + p_H + p_V$ ) causes the highest circumferential tensile stresses and leads, especially in the case of broad tanks, to plastic buckling as described in (Ref. 11). This plastic collapse problem ('elephant footing') can be analyzed, for example by the FE-method using an axisymmetric model.

Case II ( $p = p_{stat} + p_H - p_V$ ) is most critical with respect to 'shell crippling', i.e. the elastic buckling due to axial compression forces. The stabilizing effect of the internal pressure is reduced by subtracting the dynamic pressure caused by the vertical earthquake component. This mode of buckling can be treated in analogy to axially loaded cylinders as outlined in (Ref. 5).

Case III ( $p = p_{stat} - p_H - p_V$ ) may lead to regions of low pressure mainly near the top of the tank where the wall is rather thin, and buckling due to external pressure (Ref. 2) as well as cavitation may occur.

Currently (Ref. 7) real tanks have been investigated with regard to tank wall instability under the assumption  $A_V/A_H=0.5$ , the excitations being based on the 1976 Friuli earthquake (Ref. 12). Fig.4(a) taken from (Ref. 7) shows the critical earthquake intensity measured by ( $A_H^{crit}/g$ ) for different modes of instability. It can be seen that, for the assumptions described above, elastic-plastic buckling ('elephant footing') is a potential mode of failure for all tanks. In Fig.4(a) a curve depicting results obtained by using an empirical relationship for critical axial loads of pressurized cylinders (Ref. 13) is added. The relationship of Rotter & Seide has been obtained from the results of numerous numerical examinations and it has the following form:

$$n_x^{crit} = 0.605 Et^2/R (1 - [pR/(t \sigma_Y)]^2) (1 - \frac{1}{1.12 + s^{1.15}}) (\frac{\sigma_Y/250 + s}{1 + s}) \quad (7),$$

with  $s = R/(400t)$

$\sigma_Y$  ..... yield stress [N/mm<sup>2</sup>]  
 $n_x^{crit}$  ... critical axial membrane force  
 $p$  ..... internal pressure

Fig.4(a) shows, that for an engineering approach Eq.(7) gives a useful estimate for elastic-plastic buckling. It should be noted that all parameters to be used in Eq.(7) must be evaluated at  $\xi=0$ .

The axial membrane force at  $\xi=0$  can be computed from the wall overturning moment, Eq.(3), and the fluid pressure at a tank's bottom rim ( $\xi=0$ ) can be calculated using the diagrams shown in Fig.3(b) together with the relations shown above. The pressures  $p_{stat}$ ,  $p_{SL}$ ,  $p_B$ ,  $p_D$ ,  $p_{DV}$  and  $p_{DV}$  must be superposed as follows:

$$p(\xi=0) = [ p_{stat} \pm \sqrt{p_{SL}^2 + p_B^2 + p_D^2} \pm \sqrt{p_{DV}^2 + p_{DV}^2} ] \quad (8),$$

$$p_K(\xi=0) = P_K \rho_L g R A_K / g \quad K = SL, B, D \quad (9),$$

$$p_{DV}(\xi=0) = P_{DV} \rho_L g H a_{DV} / g \quad (10),$$

$$p_{BV}(\xi=0) = -H \rho_L A_V \quad (11).$$

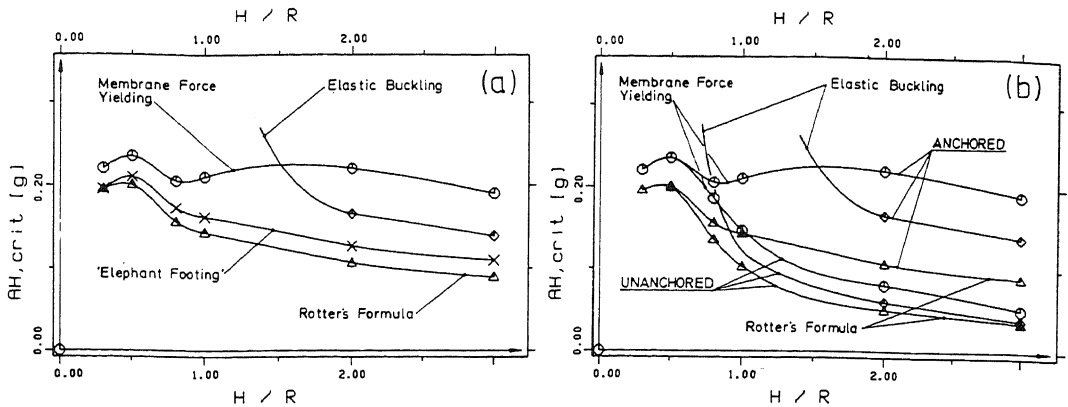


Fig.4 Critical Horizontal Free Field Acceleration for Different Modes of Tank Wall Instability.  
 (a) Anchored Tanks  
 (b) Comparison Anchored-Unanchored Tanks

UNANCHORED TANKS

The dynamic behaviour of unanchored tanks is rather different from that of anchored tanks. Partial lift off of the bottom of the tank wall is possible, caused by the overturning moment. This leads to increased axial membrane forces at the tank wall and tank wall instabilities occur at lower overturning moments. To study this phenomenon many analytical models have been developed (Refs. 14 and 15). Numerical investigations, described below, show that these models don't provide fully satisfactory procedures.

To calculate the increase of the axial membrane force a finite element shell-spring model was used (Refs. 15 and 7). The spring characteristics (axial force vs. uplift height) were calculated for real tanks and used in finite element models. The results show that the axial membrane force distribution is strongly dependent on the roof stiffness and the load factor.

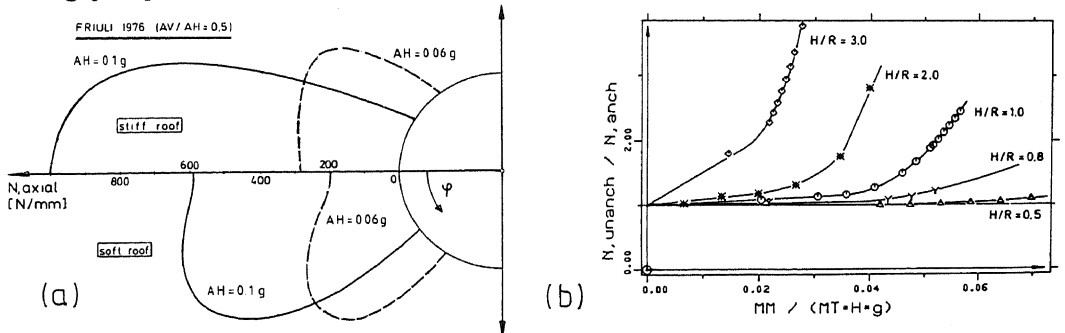


Fig.5 (a) Compressive Axial Membrane Force at  $\xi=0$  ( $H=24m, R=12m$ )  
 (b) Max. Compressive Axial Membrane Force of Unanchored Tanks Related to Anchored Tanks vs. Dimensionless Overturning Moment (with  $MT=q_1 R^2 \pi H$ )

Fig.5(a) shows the results for a tank, with a low and a high top stiffness (roof or edge ring), respectively, at different load factors. In Fig.5(b) the results of a parametric study are drawn. The influence of the increased axial membrane force on different kinds of tank wall instabilities is shown in Fig.4(b).

For these investigations the pressure distributions calculated for anchored tanks were used for the unanchored tanks. In the case of an unanchored tank a strong rocking motion with respect to the nonlinear uplift motion can be observed. That is why an iterative procedure is necessary, showing a dependance of the natural frequencies of the tank on the uplifting height. Furthermore, the mode shapes change and an additional pressure component caused by bottom lift-off occurs. Current projects (Ref. 7) try to solve these problems of the nonlinear behaviour of unanchored tanks.

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#### REFERENCES

1. Manos, G.C., Clough R.W., Tank Damage During the May 1983 Coalinga Earthquake, *J. Earthquake Engng. Struct. Dyn.* 13: 449-466, (1985)
2. Natsiavas, S., Babcock, C.D., Buckling at the Top of a Fluid-Filled Tank During Base Excitation, *Transactions of the ASME*, Vol. 109: 374-380, (1987)
3. Chiba, M., Tani, J., Yamaki N., Dynamic Stability of Liquid-Filled Cylindrical Shells Under Vertical Excitation, Part I & II, *Earthquake Engng. Struct. Dyn.* 15: 23-51, (1987).
4. Rammerstorfer, F.G., Auli W., Computation of the Stability of Dynamically Loaded Non-Linear Structures, *Numerical Methods for Non-Linear Problems*, Vol. 3: 615-637, Swansea, Pineridge Press, (1986)
5. Fischer, F.D., Rammerstorfer F.G., The Stability of Liquid-Filled Cylindrical Shells Under Dynamic Loading, E.Ramm (ed.), *Buckling of Shells*, 569-597 Springer, (1982)
6. Rammerstorfer, F.G., Scharf, K., Fischer, F.D., Seeber, R., Collapse of Earthquake Excited Tanks, *Res Mechanica*, (1988), (in press)
7. Scharf, K., Doctoral Thesis, Institute of Light Weight Structures, Technical University of Vienna, (1988), (current project)
8. Luft, R.W., Vertical Accelerations in Prestressed Concrete Tanks, *J. Struct. Engng.*, ASCE 110: 706-714, (1984)
9. Fischer, F.D., Seeber, R., Dynamic Analysis of Vertically Excited Liquid Storage Tanks Considering Liquid-Soil-Interaction, *Earthquake Engng. and Structural Dynamics*, Vol 16, pp. 329-342, (1988)
10. Fischer, F.D., Seeber, R., Rammerstorfer, F.G., Analysis of a Three-Dimensional Tank-Liquid-Soil Interaction Problem, *PVP*, (ASME), (1988), (to appear)
11. Dorninger, K., Fischer F.D., Rammerstorfer F.G., Seeber, R., Progress in the Analysis of Earthquake Loaded Tanks, *Proc. 8th ECEE* 3: 73-80, (1986)
12. Fritze, R., Grossmayer, R., Antwortspektren der Friauler Beben 1976. OGE-Bericht 3, Technical University of Vienna, (1981)
13. Rotter, J.M., Seide, P., On the Design of Unstiffened Shells Subject to a Axial Load and Internal Pressure, *ECCS Colloquium on Stability of Plate and Shell Structures*, Ghent University, 6-8 April 87, (1987)
14. Wozniak, R.S., Mitchell, W.W., Basis of Seismic Design Provisions for Welded Steel Oil Storage Tanks, *Proc. of the Session of Advances in Storage Tank Design*, API Refining Dept., pp. 485-493, (1978)
15. Auli, W., Fischer, F.D., Rammerstorfer, F.G., Uplifting of Earthquake-Loaded Liquid-Filled Tanks, *Proc. of the 1985 Pressure Vessels and Piping Conf.*, *PVP-Vol. 98-7*, pp. 71-85, New Orleans, Louisiana, (1985)