Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.VI)

10-4-18

SEISMIC RELIABILITY ANALYSIS METHODS FOR ELEVATED SPHERICAL TANKS

Albert T. Y. TUNG[1] and Anne S. KIREMIDJIAN[2]

1. Research Assistant, The John A. Blume Earthquake Engineering Center, Department of Civil Engineering, Stanford University, Stanford, California 94305
2. Associate Professor, Department of Civil Engineering, Stanford University, Stanford, California 94305

SUMMARY

This paper presents two alternative approaches for assessing the seismic reliability of elevated spherical tanks (EST), one on the component level and another on the system level. In order to model the dynamic effects of liquid sloshing, a discretized mass mechanical system is used. The first procedure computes the annual failure probabilities of the structural components at intact state. The second procedure obtains the most likely failure paths and the overall system reliability by utilizing the program FAILSF developed at Stanford. The two approaches are applied to analyze the seismic performance of a 20m diameter tank located in the San Francisco Bay Area.

INTRODUCTION

Elevated spherical tanks (figure 1) are commonly used in refineries and chemical plants to store liquid or gas under pressure. Since many of them are located in seismic regions and contain flammable and toxic material, their safe performance under seismic loads is of critical importance. This paper presents two methods for evaluating the seismic performance of elevated spherical tanks. A liquid containing spherical tank satisfying the current seismic design codes is analyzed using both approaches as an example.

MECHANICAL MODEL

When the content of the sphere is liquid and is not completely full, the effects of liquid sloshing must be accounted for in the dynamic analysis. A two degree-of-freedom mechanical system (figure 2) is used to model the forces and moments exerted by the fluid on the tank. The validity of the model's parameters have been substantiated by theoretical and experimental investigations (ref. 1). Neglecting sloshing effects of higher modes and assuming rigid tank, the main parameters are (1) m_c, the convective portion of the total fluid mass m_f; (2) m_i, the inertia mass which is the sum of the structural mass m_s and the stationary portion of the fluid mass m_f; (3)λ_c, the frequency parameter of the sloshing mass m_c ($\lambda_c^2 = rk_c/gm_c$ where r is the radius of the tank, k_c is the stiffness of m_c and g is the gravitational acceleration). The parameters m_c and λ_c are shown as functions of the ratio h (fill height) over r in figures 3 and 4 respectively. Due to the unique geometry of EST, h_c and h_i are taken to be zero.

Once the parameters of the mechanical model of a particular tank under consideration is found, the base shear BS can be predicted by means of modal superposition. To facilitate subsequent calculations, an effective mass may be used, i.e., $m_{ef} = BS/S_{ai}$ where S_{ai} is the spectral acceleration value associated with m_i.

COMPONENT RELIABILITY

With the knowledge of the hazard curve for the region and the response spectrum (or the dynamic amplification factor spectrum) representative of the local soil condition, the annual probability of failure for components at intact state can be computed as

$$P_c(\text{failure}) = \int_h \int_{S_{ai}} P_c(\text{failure} \mid S_{ai}, h) f(S_{ai} \mid h) f(h)\; dS_{ai}\; dh \tag{1}$$

where $P_c(\text{failure} \mid S_{ai}, h)$ is the component failure probability conditional on a given pair of spectral acceleration S_{ai} and fill height h. $f(S_{ai} \mid h)$ is the probability density function (PDF) of S_{ai} at a given fill height and $f(h)$ is the PDF of fill height which may be obtained from past operating records of the tank. Assuming lognormal distribution for resistance R (ref. 2) the conditional component failure probability is given by

$$\begin{aligned} P_c(\text{failure} \mid S_{ai}, h) &= P(R < L \mid S_{ai}, h) \\ &= P((u\,\sigma_{\ln_R} + \ln \check{m}_R) < \ln L \mid S_{ai}, h) \\ &= P(u < \frac{\ln(S_{ai}\, s(h) + b(h)) - \ln \check{m}_R}{\sigma_{\ln R}}) \\ &= \Phi(\frac{\ln(S_{ai}\, s(h) + b(h)) - \ln \check{m}_R}{\sigma_{\ln R}}) \end{aligned} \tag{2}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF). $\sigma_{\ln R} = [\ln((\sigma_R/\mu_R)^2 + 1)]^{1/2}$, $\check{m}_R = \mu_R \exp(-\sigma_{\ln R}{}^2/2)$ and μ_R and σ_R are respectively the mean and standard deviation of resistance. While resistance is an independent variable, the induced load L in a structural component is a random function of fill height such that

$$L = S_{ai}\, s(h) + b(h) \tag{3}$$

where $s(h)$ is obtained by first performing deterministic structural analysis of the tank system at different fill heights (figure 5) and then expressing the slopes as a function of h (figure 6). $b(h)$ is obtained similarly except that it is the intercept rather than the slope of figure 5.

The conditional PDF of S_{ai} is derived from its complimentary CDF written as

$$P(S_{ai} > s_a \mid h) = \int_U \int_\xi P(S_{ai} > s_a \mid \xi, h, U) f(\xi) f(U)\; d\xi\; dU \tag{4}$$

where $f(\xi)$ is the PDF of percent critical damping and $f(U)$ is the PDF of the uncertainty factor representing the inherent and mathematical modeling errors of S_{ai} . If the spectral acceleration is expressed as the product of peak ground acceleration (PGA) and dynamic amplification factor (DAF), and includes the influence of damping and uncertainty on the value of spectral accelerations such that $S_{ai} = \text{DAF}(h)\, (\xi_r/\xi)^{1/2}\, U\, \text{PGA}$, then the conditional probability in equation 4 can be obtained from the hazard curve since

$$P(S_{ai} > s_a \mid \xi, h, U) = P(\text{PGA} > \frac{s_a}{\text{DAF}(h)\, (\xi_r/\xi)^{1/2}\, U}) \tag{5}$$

where ξ_r is the average critical damping of the supporting frame.

SYSTEM RELIABILITY

The system reliability analysis is accomplished by using the reliability analysis package FAILSF developed at Stanford. FAILSF is based on FAILUR (ref. 3) and SHASYS (ref. 4) developed previously but with the added capability for considering failure modes such as plastic hinges for structural members and axial, shear and moment failure modes for foundations. The interactive program FAILSF is applicable for the analysis of general space frame structural systems which satisfy the following requirements: (1) system failure can be described as combinations of component failures, i.e., the use of minimal cutset representation to define system failure; (2) the failure equation for each component is expressible as a function of independent random variables with known distribution functions. FAILSF employs the first/second- order reliability methods and allows perfectly plastic, perfectly brittle or semi-brittle modes of failure for components. Furthermore, FAILSF uses an element replacement approach to formulate the element failure events and uses the branch and bound algorithm to generate the most likely failure sequences (ref. 3, 5).

The loading on the structure is represented by the dead load which acts vertically and the live load which acts horizontally. Both are random and are given respectively as

$$D = W\,\{d\} \tag{6}$$
$$L = S_{ai}\,S'_{ai}\,e(W)\,\{l\} \tag{7}$$

where W and S_{ai} are envelope PDF's with unit mean for the total weight $(m_c\,g + m_i\,g)$ and the spectral acceleration respectively; $S'_{ai}(W)$ is the mean spectral acceleration as a function of W; $e(W)$ is the ratio of the effective weight over the total weight of the structure, also a function of W; $\{d\}$ and $\{l\}$ are load patterns for mean vertical and horizontal loads.

At a particular damage state, i.e., after p members have failed, the limit state equation g corresponding to the failure of a member in the next damage state $p+1$ is

$$\begin{aligned} g(p+1) &= \{R(p+1) \\ &< \{[\sum_{k=1}^{p} a_x(p+1,k)R(k)\eta(k) + b_x(p+1,\{l\})\,S_{ai}\;e(W)\,S_{ai}{}'(W) \\ &\quad + c_x(p+1,\{d\})\,W]^2 + [\sum_{k=1}^{p} a_y(p+1,k)R(k)\eta(k) + b_y(p+1,\{l\}) \\ &\quad S_{ai}\;e(W)\,S_{ai}{}'(W) + c_y(p+1,\{d\})\,W]^2\}^{1/2}\} \end{aligned} \tag{8}$$

for two dimensional failure (e.g., foundation shear). For one dimensional failure (e.g., axial member), $(a_y(i,j)$, $b_y(i,j)$, and $c_y(i,j)$ in equation 9 above are all zeros. In the same equation, $a_x(i,j)$ is the stress induced in the member that is in damage state i, in the direction x of its member coordinate system by a unit force applied at the node(s) of the member or the support constraint that has failed in damage state j; $b_x(i,\{l\})$ is the stress induced in the member that is in damage state i in the x direction of its coordinate system by the mean horizontal load defined by load pattern $\{l\}$; $c_x(i,\{d\})$ is similar to $b_x(i,\{l\})$ except that it is induced by the mean vertical load defined by $\{d\}$; $R(i)$ is the resistance capacity of the member of damage state i; $\eta(j)$ is the reduction factor for resistance capacity of the member failed in damage state j ($\eta = 0$ and $\eta = 1$ imply perfectly brittle and perfectly plastic failures respectively).

The above limit state equation is evaluated repeatedly in FAILSF with different coefficients a_i, b_i, and c_i which are computed by the structural analysis subroutine for different components and damage states. Note that the loadings induced in various components are entirely functions of W and, if $f(S_{ai} \mid h) = f(S_{ai})$ is assumed, then $S'_{ai}(W)$ becomes a constant.

EXAMPLE

A typical liquid containing elevated spherical tank (figure 1) located in the San Francisco bay area and meeting the current seismic design criteria of the petroleum industry in the U. S. is analyzed for its seismic safety. The dimensions of the tank are given in table 1. The hazard curve for the region and the DAF spectrum for the firm soil of the site are shown in figures 7 and 8 respectively.

For the component analysis, the following failure modes are analyzed: (1) brace buckling, (2) brace yielding, (3) concrete pedestal shear, (4) anchor bolt yielding, (5) column buckling. The damping ratio ξ is assumed to be normally distributed with mean of 0.05 and COV of 0.12. The uncertainty factor U is assumed to be lognormally distributed with unit mean and COV of 0.27. The distribution of fill height is based on the assumption that the volume of the liquid content is uniformly distributed. Since the period (as a function of h) associated with the inertia mass of the tank is confined in a relatively narrow range of 0.09 seconds to 0.38 seconds, the distribution of S_{ai} is taken to be independent of h (see figure 8) and this simplification is accounted for in the uncertainty factor. $f(S_{ai})$ is obtained by fitting the CDF of S_{ai} with a lognormal distribution which results in a mean of 0.12g and COV of 0.498. Failure modes range from perfectly plastic to perfectly brittle as indicated by the η values in table 2. The results of the component level analysis are also shown in table 2.

On the system level, consistent with the component failure analysis, all member resistances are assumed to be lognormally distributed. Also, W is taken to be uniformly distributed and the PDF of S_{ai} is the same as the one obtained in the component analysis but with a mean of unity. The failure criteria used to determine failure of the supporting frame is the percentage loss of original stiffness and failure is assumed to have occurred when the structure has lost 70% of its original (intact state) horizontal stiffness. The first eight most likely failure paths are used to approximate the overall failure probability with the number of failed components in the most likely failure paths range from 3 to 5. The majority of the most likely failure paths are initiated by brace buckling (7 out of 8). The overall system reliability index for the tank is 4.71 ($P(\text{failure}) = .1240(10)^{-5}$).

CONCLUSION

The primary causes of collapse for elevated spherical tanks are the failures of their supporting frames. Typically, the supporting frames are highly indeterminate structural systems. However, indeterminacy does not imply effective redundancy (in the reliability sense). Due to load redistribution and progressive failure, the overall reliability of the system is largely dominated by the system's weakest component (e.g., braces). Based on this study of a typical elevated spherical tank located in seismic region, it can be concluded that: first, identifying and strenghtening the weakest component (such as braces) can very effectively increase the overall reliability of the system; second, the failure probability of the weakest component calculated using the more cost effective component level reliability procedure, which requires no specialized computer programs, can give a reasonable measure of the overall system reliability; finally, the current seismic design codes for critical industrial structures such as elevated spherical tanks seem to provide an adequate margin of safety .

Table 1. Example Elevated Spherical Tank Dimensions.

sphere diameter / min. sphere thickness	20m / 25mm
no. of columns / o. d. / thickness	12 / .915m /12.5mm
bracings	2L6x6x7/8
fill height, minimum / maximum	3m / 15.25m
anchor bolts, no. per column / diameter	24 / 38mm
concrete pedestal, l x w x h	1.5m x 1.5m x .915m

Table 2. Component Reliability Indices for Spherical Tank in Example.

	brace buckling	brace yielding	pedestal	anchor bolt	column
η	.3	1.	0.	1.	—
COV of η	.1	0.	0.	0.	—
β	4.51	5.52	4.84	7.22	$\geq$ 10.
P(failure)	.3244e-05	.1699e-07	.6501e-06	.2618e-12	$\geq$ .7770e-23

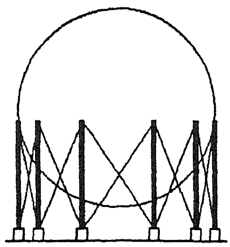

Figure 1. Elevated Spherical Tank.

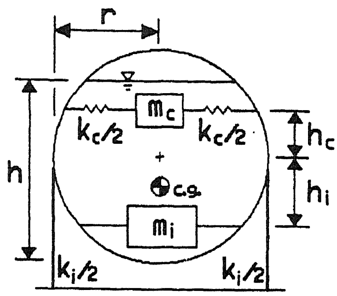

Figure 2. Equivalent Mechanical System.

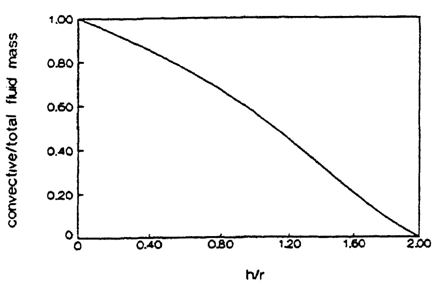

Figure 3. Values of m_c/m_f for Different h/r Ratios.

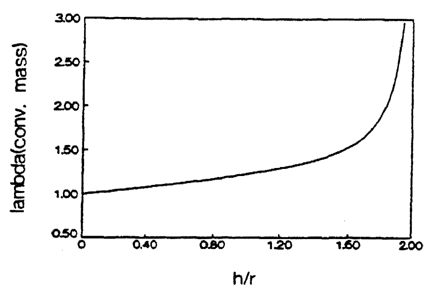

Figure 4. Frequency Parameter, λ_c, as Function of h/r.

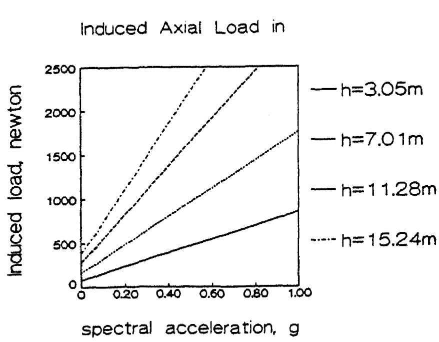

Figure 5. Example of Induced Axial Load in Member.

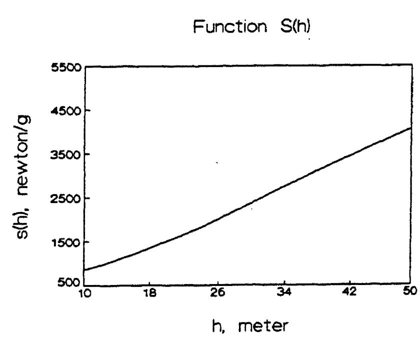

Figure 6. Variation of the Slopes, s, in Figure 5 as a Function of h.

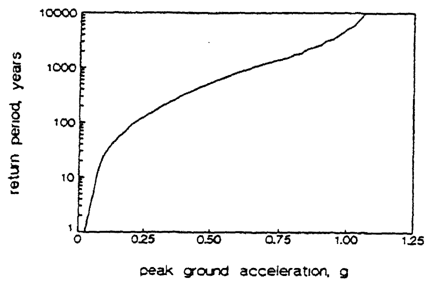

Figure 7. Example Hazard Curve for the San Francisco Bay Area

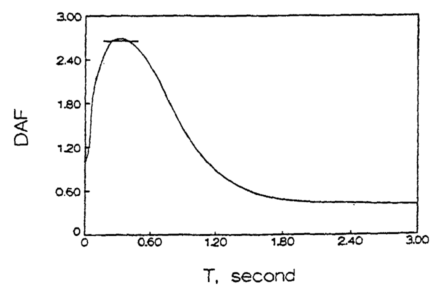

Figure 8. DAF Spectrum for Firm Soil.

REFERENCES

1. Abramson, H. N. (1966). "The Dynamic Behavior of Liquids in Moving Containers", NASA Special Report No.106, NASA, Washington D.C.
2. Ellingwood, B. (1982). "Probability Based Criteria for Structural Design", *Journal of Structural Safety*, vol. 1, pp. 15-26.
3. Guenard, Y. (1984). "Application of System Reliability Analysis to Offshore Structures", Report No. 71, The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA.
4. Karamchandani, A., Guenard, Y. and Ortiz, K. (1986). "SHASYS, A Software Package for Component and System Reliability Analysis", Report No. 78, The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA.
5. Thoft-Christensen, P. and Murotsu, Y. (1986). Application of Structural System Reliability Theory, *Springer-Verlag*, N.Y.