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## SOIL-STRUCTURE INTERACTION EFFECTS FOR VERTICALLY EXCITED TANKS

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### SUMMARY

After reviewing briefly the nature of hydrodynamic effects induced in vertically excited, liquid-containing tanks that are rigidly supported at the base, the paper highlights the consequences of soil-structure interaction, and presents a simple practical procedure with which the critical design forces for both rigidly and flexibly supported tanks may be evaluated.

### INTRODUCTION

Studies of the dynamic response of liquid-storage tanks have shown that the hydrodynamic effects induced in flexible tanks by a vertical component of ground shaking may be significantly larger than those induced in rigid tanks of the same dimensions, and that the peak values of these effects may be of the order of the hydrostatic effects for a ground motion with a peak acceleration of one-third the acceleration due to gravity (e.g., Refs. 1,2,3).

These studies referred to rigidly supported tanks for which the motion experienced at the tank base is the same as the free-field ground motion. Although it has been recognized that, because of the high radiational damping capacity of vertically excited foundations, soil-structure interaction may significantly reduce the design forces for tanks supported on soft soils (Ref. 2), the consequences of such action have been examined only recently (Refs. 4-7).

The objectives of this paper are: (1) to review briefly the nature of the hydrodynamic effects induced in tanks that are rigidly supported at the base, paying special attention to the influence of the flexibility of the tank wall; (2) to highlight the principal consequences of soil-structure interaction; and (3) to provide information with which the critical design forces for both rigidly and flexibly supported steel tanks may be evaluated readily. The material is presented in physically motivated terms with minimum reference to the underlying mathematics. The mathematical details, along with complementary numerical data, are given in Refs. 4 and 6.

### SYSTEM CONSIDERED

A ground-supported, upright, circular, cylindrical, steel tank of radius,  $a$ , height,  $H$ , and constant wall thickness,  $h$ , is considered. The tank is supported through a rigid base at the surface of a homogeneous elastic halfspace

and is full with liquid of mass density,  $\rho_\ell$ . The liquid is considered to be incompressible, inviscid and free at its upper surface. The tank wall is presumed to be clamped at the base; its mass density is denoted by  $\rho$ ; the modulus of elasticity and Poisson's ratio for the tank material are denoted by  $E$  and  $\nu$ , respectively; and the corresponding quantities for the supporting soil are denoted by  $\rho_s$ ,  $E_s$  and  $\nu_s$ . The excitation is a vertical component of ground shaking with a free-field acceleration  $\ddot{x}_g(t)$ , in which  $t$  denotes time.

Points for the tank and the contained liquid are specified by the radial and axial components of a cylindrical coordinate system,  $r$  and  $z$ , the origin of which is taken at the tank base. The ground displacement and its derivatives are considered to be positive in the upward direction.

#### RIGIDLY SUPPORTED TANKS

Hydrodynamic Pressure Let  $p(z,t)$  be the hydrodynamic pressure exerted against the tank wall, positive when acting radially outward. For the conditions considered, this pressure is uniformly distributed in the circumferential direction and its axial distribution depends on the flexibility of the tank wall. For a completely rigid wall, the pressure increases linearly from top to bottom and may be expressed as

$$p(z,t) = \left[1 - \frac{z}{H}\right] \rho_\ell H \ddot{x}_g(t) \quad (1)$$

Note that the temporal variation of this pressure is the same as that of the acceleration of the input motion, and that its peak value at the junction of the wall and the base is  $\rho_\ell H \dot{x}_g$ , in which  $\dot{x}_g$  = the maximum value of  $\dot{x}_g(t)$ .

Tank flexibility affects both the magnitude, axial distribution, and temporal variation of the hydrodynamic wall pressure and of the associated tank forces. However, as demonstrated in Ref. 4, a reasonable, generally conservative, approximation to the resulting hydrodynamic effects may be obtained on the assumption that the vertical distribution of the hydrodynamic pressure is the same as that for a rigid tank.

The pressure  $p(z,t)$  under this assumption may be determined from Eq. 1 merely by replacing  $\ddot{x}_g(t)$  with  $A(t)$ . The latter quantity represents the pseudo-acceleration of a similarly excited, single-degree-of-freedom oscillator, for which the natural frequency,  $f$ , and percentage of critical damping,  $\zeta$ , are equal to those of the fundamental, axisymmetric, breathing mode of vibration of the tank-liquid system. Thus

$$p(z,t) = \left[1 - \frac{z}{H}\right] \rho_\ell H A(t) \quad (2)$$

and its maximum value at any point is obtained by replacing  $A(t)$  with its maximum or spectral value,  $A$ . The latter value may be determined from the response spectrum for the prescribed ground motion using the values of  $f$  and  $\zeta$  referred to above. Inasmuch as  $A$  may be much greater than  $\dot{x}_g$ , it should be clear that the peak values of  $p(z,t)$  for a flexible tank may be significantly greater than for the associated rigid tank.

With the magnitude and distribution of the maximum values of the hydrodynamic wall pressure established, the corresponding values of the displacements of the tank wall and of the internal forces may be determined by static analysis. Since the distributions of the hydrodynamic and hydrostatic pressures are identical for the assumption made, the maximum effects of the two pressures will be proportional, the proportionality ratio being  $A/g$ , in which  $g$  is the gravitational acceleration. Although the contribution of the radial inertia of

the flexible tank wall is not considered explicitly in this approach, it is provided for implicitly in the evaluation of  $p(z,t)$ , as indicated in Refs. 2 and 4.

Natural Frequency of Tank-Liquid System The fundamental natural frequency of axisymmetric vibration of the tank-liquid system may conveniently be expressed in the form

$$f = \frac{C_\ell}{2\pi} \frac{1}{H} \sqrt{\frac{E}{\rho}} \quad (3)$$

in which  $f$  is in cycles per unit of time, and  $C_\ell$  is a dimensionless coefficient that depends on the values of  $H/a$ ,  $h/a$ ,  $\rho_\ell/\rho$  and  $\nu$ . For steel tanks that are full with water, for which  $\nu = 0.3$ ,  $\rho_\ell/\rho = 0.127$  and  $\sqrt{E/\rho} = 16,860$  ft/sec, the fundamental natural period of the tank-liquid system,  $T = 1/f$ , may be written simply as

$$T = C_T \frac{H}{100} \quad (4)$$

in which  $H$  is expressed in feet. The values of  $C_T$  for tanks with different values of  $H/a$  and  $h/a$  are shown in Fig. 1.

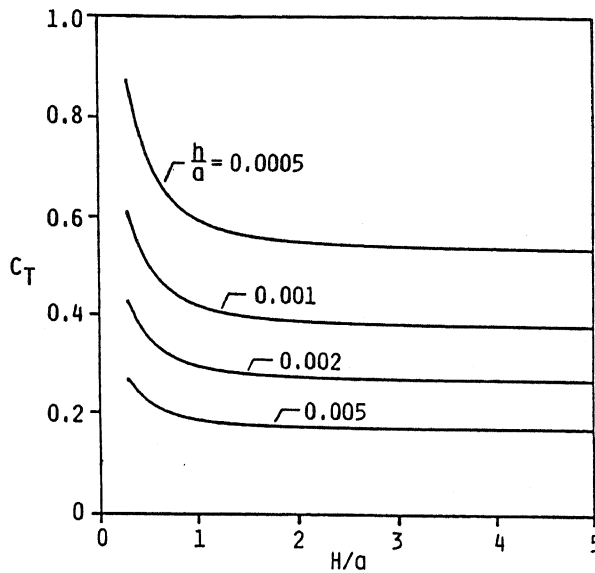


Fig. 1 Values of  $C_T$  in Expression for Fundamental Natural Period,  $T$ , of Rigidly Supported Steel Tanks Filled with Water

#### ELASTICALLY SUPPORTED TANKS

For the rigidly supported systems examined so far, the vertical motion of the tank base is the same as the free-field ground motion. By contrast, for an elastically supported tank, the two motions are different, and so are the corresponding tank responses. Two factors are responsible for these differences. First, the elastically supported system has one additional degree of freedom than the rigidly supported system, and hence different fundamental natural frequency and mode of vibration. Second, a substantial part of the vibrational energy of the elastically supported system may be dissipated into the supporting medium by radiation of waves and by hysteretic action in the soil itself. There is, of course, no counterpart of this mechanism of energy dissipation for a rigidly sup-

ported tank. The term soil-structure interaction is normally used to express the difference in the responses of the tank computed by: (a) assuming the motion of the tank base to be the same as the free-field ground motion; and (b) considering the modified or actual foundation motion, including the energy dissipating capacity of the supporting medium.

Studies of laterally excited simple structures (e.g., Ref. 8) have shown that a reasonable approximation to the interaction effects may be obtained by modifying the dynamic properties of the superstructure and evaluating the response of the modified structure to the prescribed free-field ground motion, considering it to be rigidly supported at the base. The requisite modifications involve changes in the fundamental natural frequency and damping of the fixed-base structure. This approach, which has provided the basis of the design provisions for soil-structure interaction recommended by the Applied Technology Council for buildings, has been shown in Ref. 4 also to yield satisfactory results for the vertically excited tank-liquid systems considered herein.

Effective Natural Frequency and Damping Let  $\tilde{f}$  be the modified or effective natural frequency of the interacting system, and  $\zeta$  be the associated percentage of critical damping. Further, let  $\dot{A}(t)$  be the instantaneous value of the corresponding pseudoacceleration for the postulated free-field ground motion, and  $A$  be its maximum or spectral value. The hydrodynamic wall pressure for the elastically supported tank may then be determined from Eq. 2 by replacing  $A(t)$  with  $\dot{A}(t)$ ; and the maximum values of this pressure may be determined by replacing  $\dot{A}(t)$  with  $A$ .

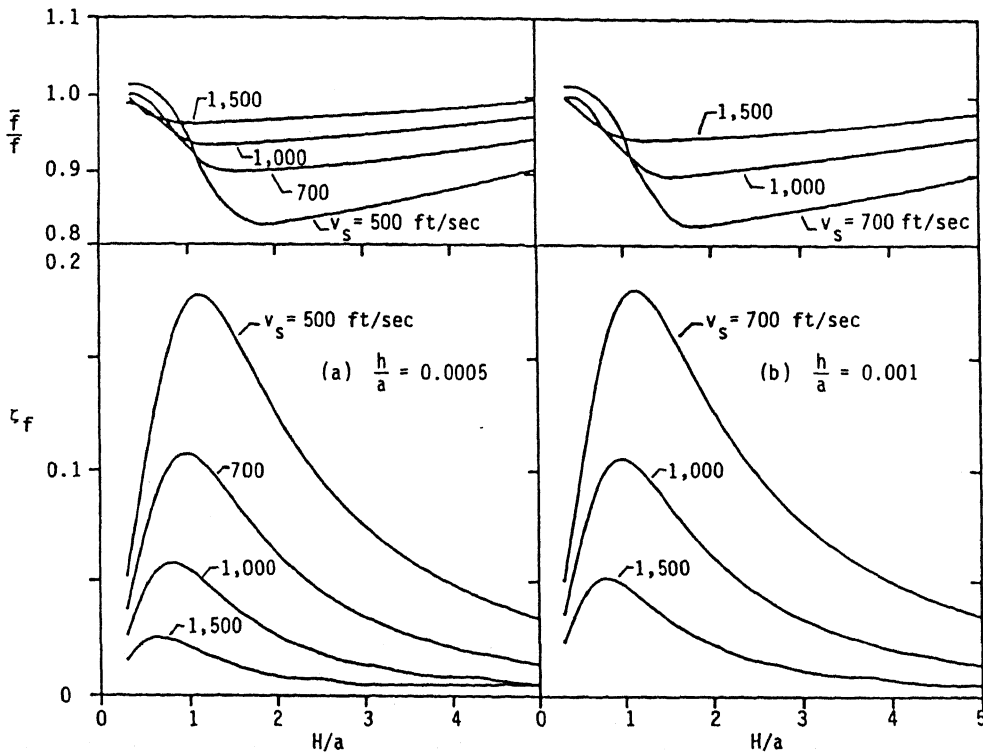


Fig. 2 Effective Natural Frequency and Foundation Damping for Elastically Supported Steel Tanks Filled with Water;  $v_s = 1/3$ ,  $\rho_s = 1.8\rho_l$

The damping factor of the flexibly supported system,  $\tilde{\zeta}$ , may be expressed in the form

$$\tilde{\zeta} = \zeta_f + (\tilde{f}/f)^3 \zeta \quad (5)$$

in which the first term on the right represents the contribution of the foundation damping, including radiation and soil material damping; and the second term represents the contribution of the structural damping. Note that  $\zeta$  is not directly additive to  $\zeta_f$ .

The quantities  $\tilde{f}/f$  and  $\zeta_f$  depend on the properties of both the structure and the supporting medium, and may be determined from Fig. 2. The results are plotted as a function of  $H/a$  for two different values of  $h/a$  and several different rigidities for the supporting medium; the latter quantity is expressed in terms of the velocity of shear wave propagation for the medium,  $v_s = \sqrt{G_s/\rho_s}$ , in which  $G_s$  = the shear modulus of elasticity for the soil. Note that the foundation damping may be quite substantial for the softer soils and the stiffer tanks (tanks with the larger values of  $h/a$ ), particularly when  $H/a$  is close to unity. A large increase in system damping, with no substantial change in natural frequency, would naturally be expected to lead to a significant reduction in response.

The mass of the foundation mat was presumed to be negligible for the solutions presented in Fig. 2, and no provision was made for the effect of soil material damping. Consideration of the first factor would reduce the overall system damping, whereas consideration of the second factor would increase it. However, the net effect is expected to be inconsequential for design purposes. Other assumptions made in the development of this information are that the radius of the foundation is the same as that of the tank, and that  $\nu = 1/3$  and  $\rho = 1.8\rho_s$ . The frequency dependence of the foundation compliance function  $\zeta_s$  was duly accounted for in the solutions.

#### ILLUSTRATIVE EXAMPLE

As an illustration of the extent to which the response of a tank may be influenced by the flexibilities of its wall and of the supporting soil, consider a steel tank with  $H/a=1$  and  $h/a=0.001$  that is full of water and is subjected to the vertical component of an earthquake-induced ground motion, for which  $\ddot{x}_g = g/3$ . The velocity of shear wave propagation for the soil is taken as  $v_s = 700$  ft/sec.

If both the tank and the supporting medium are infinitely rigid, the hydrodynamic wall pressure would increase linearly from zero at the still liquid surface to a maximum at the base, and its absolute maximum value would be

$$P_{\max} = \rho_l H \ddot{x}_g = \frac{1}{3} \rho_l H g = \frac{1}{3} \gamma_l H$$

in which  $\gamma_l = \rho_l g$  = the unit weight of the contained liquid.

The effects of tank and foundation flexibilities are evaluated on the assumption that the axial distribution of the hydrodynamic wall pressure remains linear. If, as is likely to be the case, the fixed-base fundamental natural frequency of axisymmetric vibration of the tank-liquid system falls within the amplified, constant pseudoacceleration region of the design response spectrum; and if the damping factor for that mode is taken, as is reasonable to do, as  $\zeta = 0.02$ ; then, the value of  $A$  would be of the order  $2.8 \ddot{x}_g = 0.93 g$ , and the peak value of the hydrodynamic pressure at the junction of the tank wall and the base would be

$$P_{\max} = \rho_l H A = 0.93 \rho_l H g = 0.93 \gamma_l H$$

In assessing the effect of foundation flexibility, it is further assumed that the effective natural frequency of the elastically supported tank remains within the amplified, constant pseudoacceleration region of the response spectrum. The foundation damping factor for the system is determined from Fig. 2 to be  $\zeta_f = 0.177$ , and on neglecting the contribution of structural damping, one obtains  $\zeta = \zeta_f = 0.177$ . The value of  $A$  under these conditions is approximately  $1.3 \ddot{x}_g = 0.43 \ddot{g}$ . This leads to a peak wall pressure

$$p_{\max} = \rho_l H A = 0.43 \rho_l H g = 0.43 \gamma_l H$$

which is less than one-half of that computed without regard for soil-structure interaction, and only 30 percent in excess of that computed considering both the tank wall and the foundation soil to be rigid.

#### CONCLUSION

It has been shown that soil-structure interaction may significantly reduce the hydrodynamic effects in vertically excited, circular, cylindrical, steel tanks, and that the consequences of such interaction may be evaluated readily with the information presented herein.

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