SIMPLIFIED SEISMIC ANALYSIS METHOD OF CYLINDRICAL TANKS ON RIGID FOUNDATION WITH SOIL-STRUCTURE INTERACTION SUBJECTED TO HORIZONTAL GROUND MOTIONS

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SUMMARY

This paper deals with proposing a simplified seismic analysis method of cylindrical storage tanks on rigid foundation with soil-structure interaction subjected to horizontal ground motions. Comprehensive analyses are made of the

1 mass-spring vibration model of tank to sway and rocking motions, and
2 equations of motions of the tank-foundation-soil total system subjected to horizontal ground motions. Numerical verification is also presented.

1 INTRODUCTION

When we analyse the cylindrical tanks containing liquid on rigid foundation (slab) with soil-structure interaction by a simplified seismic analysis method, the followings need to be clarified,

1 Vibration model of soil (Ref.1) and effective input (Ref.2) to the system
2 Vibration model of the tank to both sway (Ref.3) and rocking (Ref.4) motions
3 Equations of the tank-foundation-soil total system in a simple form.

We will study these items for the model given in Fig.1.

2 SUB-STRUCTURE METHOD FOR TANK-FOUNDATION-SOIL SYSTEM

2.1 Formulations by Sub-structure Method

By applying the sub-structure method described in the reference (2) to the finite element model, equations of motions of the tank-foundation-soil total system subjected to horizontal ground motions at the bedrock can be derived.

When rigid foundation is assumed, the equations of motions are written in the following form,

\[
\begin{bmatrix}
K_{ss} & K_{sd} \\
K_{ds} & K_{dd} + K_{00}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_s \\
\ddot{u}_d
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
A_0
\end{bmatrix}
\begin{bmatrix}
\dot{u}_e \\
\dot{\phi}_e
\end{bmatrix}
\]

where \( K \) and \( k \) denote the stiffness matrices and \( U \) and \( (u \neq) \) the displacement vectors. Subscript \( d \) refers the superstructure (i.e. the tank and foundation) and subscript \( s \) the soil contacting with the foundation. \([A_0]\) denotes the stiffness matrix of the soil at the contact
surface of the super-structure and the soil. This \( [A_0] \) is called soil spring hereafter.

Eq.(1) is the fundamental equation for the unknown displacements of the super-structure \((U_e)\) and of the foundation at the point \( O \) (\( u \phi \)) to the effective input \((u_e \phi_e)\) under the assumption that the foundation is rigid.

### 2.2 Soil Spring

We consider the massless foundation embedded in the soil surface. As is well known horizontal force \( F \) and moment \( M \) about the horizontal axis through the center of the surface of foundation due to the horizontal displacement \( u_B \) and the rotational displacement \( \phi_B \) are written as

\[
\begin{pmatrix}
F \\
M
\end{pmatrix} =
\begin{bmatrix}
k_{XX} & k_{XR} \\
k_{TR} & k_{RR}
\end{bmatrix}
\begin{pmatrix}
u_B \\
\phi_B
\end{pmatrix}
\]  

(2)

\[
k_{XX} = (k_{11} + i\alpha_c G_{11}) K_x, \quad k_{TR} = (k_{22} + i\alpha_c G_{22}) K_x, \quad k_{XR} = k_{TR} = (k_{21} + i\alpha_c G_{21}) K_x R_B, \]

(3)

\[
a_0 = \nu R_B / V_{s1}, \quad i = \sqrt{-1}, \quad K_x = 8G_1 R_B / (2-\nu_1), \quad K_r = 8G_1 R_B^3 / (3(1-\nu_1)).
\]

(4)

\( \omega \) indicates angular frequency, \( R_B, V_{s1}, G_1 \), and \( \nu_1 \) are the radius of the foundation, shear wave velocity, shear modulus and poisson ratio of the surface soil layer. \( k_{ij} \) and \( c_{ij} \) are generally given by the numerical values. They are given in the reference (1) for half space soil.

### 3  MASS–SPRING MODEL OF TANK

Let us consider a tank clamped at the rigid base which is subjected to sway and rocking motions. The tank responds to the motion of sway of the base \( u_x \) and rocking of the base \( \phi \). The motion is considered by separating into two independent motions, i.e. sway motion and rocking motion. The responses of the tank will be independently analyzed for the separated two motions as shown in Figs. 2(a) and 3(a). The total response of the combined motion will be obtained by superposing the responses to these motions.

#### 3.1  Mass–Spring Model to Sway Motion

Fundamental equations of motions for the tank to sway motion \( u_x \) is written from the reference 3 as

\[
[M_x](\ddot{d}_x) + [K_x](\dot{d}_x) = -[M_u](f)\ddot{u}_x.
\]

(5)

We only consider the first natural mode of vibration. Displacement vector of the tank shell is written as

\[
(\dot{d}_x) = (p) b_x q_x
\]

(6)

in terms of the first natural mode of vibration \( p \), the time function \( q_x \) and the mode participation factor \( b_x \). The time function \( q_x \) is governed by

\[
\dot{q}_x + 2\omega_0 \nu_0 q_x + \omega_0^2 q_x = -\ddot{u}_x.
\]

(7)

Base shear \( Q_x \), overturning moments \( M_{xe} \) (for excluding bottom pressure) and \( M_{xi} \) (for including bottom pressure) are written as

\[
Q_x = m_x f\dot{u}_x + n_x r \dot{u}_x, \quad M_{xe} = m_x f e_x \dot{u}_x + n_x r h_x \dot{u}_x, \quad M_{xi} = m_x f h_x \dot{u}_x + m_x r h_x \ddot{u}_x.
\]

(8)

Where the subscript \( x \) indicates the quantity for sway motion and the superscripts \( f \) and \( r \) for flexible shell motion and for rigid tank movement. The vibration model for Eq.(8) is shown in Fig.2(b).
3.2 Mass-spring Model to Rocking Motion

Fundamental equations of motions for the tank to rocking motion is written from the reference 4 as

\[ [M_c](\ddot{d}_r) + [K_c](\dot{d}_r) = -[M_c](f_c)\ddot{u}_r. \]  
\[ (9) \]

Again first natural mode of vibration is considered. Displacement vector of the tank shell is written as

\[ (\ddot{d}_r) = (p) b_1 q_r \]  
\[ (10) \]

where \( q_r \) is the function of time, \( b_1 \) mode participation factor. The function of time \( q_r \) is governed by

\[ \ddot{q}_r + 2\zeta_0 \omega_0 q_r + \omega_0^2 q_r = -\ddot{u}_r. \]  
\[ (11) \]

Base shear \( Q_r \), overturning moments \( M_{re} \) (for ebp) and \( M_{ri} \) (for ibp) are written as

\[ Q_r = m_1 \dot{q}_r + m_2 \ddot{u}_r, \]
\[ M_{re} = m_1 H_r \ddot{q}_r + m_2 H_r \dot{u}_r, \]
\[ M_{ri} = m_1 f_i \ddot{q}_r + n_2 H_r \dot{u}_r \]  
\[ (12) \]

The subscript \( r \) indicates the quantity for rocking motion. The vibration model for Eq. (12) is given as in Fig.3(b).

3.3 Parameters of Mass-spring Model

The first natural period \( T_1 (\omega_1 = 2\pi / T_1) \), vibration parameters \( m \) and \( H \) defined in sections 3.1 and 3.2 are calculated for the typical tank geometry and shown in Fig.4 to 7. In these figures, \( m_1 \) is the
total mass of liquid, i.e., \( m_1 = \rho R^2 H_1 \), where \( \rho \) indicates the density of liquid, \( R \) the radius of the tank, \( H_1 \) the liquid height of the tank.

From the definition of the vibration parameters below, there exist the following relations;

\[
m_x^f / m_r^f = m_x^f H_x / (m_r^f H_r) = m_x^f f_x / (m_r^f H_r) = b_x / b_r. \tag{13}
\]

3.4 Response to Combined Motion of Sway and Rocking

Response to the combined motion of sway and rocking can be obtained from the responses to independent sway motion and rocking motion described in sections 3.1 and 3.2 with the use of the vibration parameters given in section 3.3.

Base shear \( Q \) and overturning moments \( M_e \) (for ebp) and \( M_I \) (for ibp) become

\[
Q = m_x^f \ddot{u}_x + m_x^f \dddot{u}_x + m_r^f \dddot{u}_r + m_r^f \dddot{u}_r
\]

\[
M_e = m_x^f H_x \ddot{u}_x + m_x^f H_x \dddot{u}_x + m_r^f H_x \dddot{u}_x + m_r^f H_x \dddot{u}_r
\]

\[
M_I = m_x^f H_x \ddot{u}_x + m_x^f H_x \dddot{u}_x + m_r^f H_x \dddot{u}_x + m_r^f H_x \dddot{u}_r. \tag{14}
\]

4 EQUATIONS OF MOTIONS OF TOTAL SYSTEM

We consider the simplified vibration model of the total system shown in Fig.8. In the figure, \( u_e \) and \( \phi \) indicate the effective inputs for sway and rocking motions. \( u_B \) and \( \phi_B \) are the displacement responses of the foundation for horizontal and rotational component, \( u_x \) and \( \phi \) are the absolute displacements for horizontal and rotational component. There exist the following relations,

\[
u_x = u_B + u_e, \quad \phi = \phi_B + \phi_e. \tag{15}\]

Now, rotational displacement is transformed into the equivalent horizontal displacement at the top of the tank wall (height \( H_1 \)), that is,

\[
u_r = H_1 \phi, \quad v_B = H_1 \phi_B, \quad v_e = H_1 \phi_e. \tag{16}\]

Horizontal force and overturning moment about the horizontal axis through the point \( O \) exerted from the super-structure become

\[
\begin{bmatrix}
F \\
M
\end{bmatrix} = \begin{bmatrix}
Q_T \\
M_T
\end{bmatrix} + \begin{bmatrix}
m_B \\
H_B
\end{bmatrix} \begin{bmatrix}
-(H_B / 2) m_B \\
-(H_B / 2) m_B
\end{bmatrix} \begin{bmatrix}
\ddot{u}_B \\
\dddot{u}_B
\end{bmatrix} \tag{17}
\]

where \( Q_T \) and \( M_T \) indicate the horizontal force and overturning moment exerted from the upper structure (i.e. tank and liquid). They are obtained from Eq.(14) as \( Q_T = Q \) and \( M_T = M_e + M_I \). The second term of the right hand side of Eq.(17) is the horizontal force and overturning moment about the horizontal axis through the point \( O \) of the foundation itself. The foundation mass \( m_B \) and the rotational inertia of the foundation \( I_B \) about the horizontal axis through the point \( O \) become

\[
m_B = \rho_B \pi R^2 H_B, \quad I_B = I_{BO} + m_B (H_B / 2)^2 \tag{18}\]

where \( I_{BO} \) indicates the rotational inertia of the foundation about the centroid of the foundation. Substituting Eq.(2) into (17) and using Eqs.(8) and (12) into the derived equation, one can obtain the equations of motions about the displacement vector \( (y) = (u_x, u_r, u_B, v_B)^T \). Since the tank shell displacement is

\[
(d) = (p)u_s, \quad u_s = b_x \ddot{a}_x + b_r \ddot{a}_r \tag{19}\]

then the equations of motions obtained above can be transformed into the equations of motions about the displacement vector \( (y) = (u_x / b_x, u_r, u_B, v_B)^T \) as

\[
[M] \ddot{y} + [C] \dot{y} + [K] (y) = -[M] (f) \ddot{u}_e - [M] (f) \dddot{v}_e \tag{20}
\]

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\[
[M] = \begin{bmatrix}
  m_x^f & m_x^f & m_x^f \\
  m_x^{r+B} & m_x^r - \frac{H_B}{2H_1}m_x^B \\
  \text{SYM.} & m_x^r H_1 & + \frac{H_1}{H_2} \\
\end{bmatrix}, \quad [C] = \begin{bmatrix}
  2h_0\omega_0 m_x^f & 0 & 0 \\
  \frac{a_0 c_{11}}{\omega} K_x & \frac{a_0 c_{21}}{\omega} K_{xR_B} \\
  \text{SYM.} & \frac{a_0 c_{22}}{\omega} K_x & \frac{a_0 c_{22}}{\omega} \frac{K_x}{H_2^2} \\
\end{bmatrix}
\]

\[
[K] = \begin{bmatrix}
  \omega_0^2 m_x^f & 0 & 0 \\
  k_{11} K_x & k_{21} \frac{K_{xR_B}}{H_1} \\
  \text{SYM.} & k_{22} \frac{K_x}{H_1} \\
\end{bmatrix}
\]

This is the simplified mass-spring model describing the motion of the tank containing liquid with rigid foundation resting on elastic soil.

5 NUMERICAL COMPUTATIONS

5.1 Analysis Model Analysis model is shown in Fig.9. The soil system of an upper subsoil layer overlying the half space soil is considered. The model parameters used are listed in Table 1.

![Fig. 9 Analysis Model for Numerical Computation](image)

<table>
<thead>
<tr>
<th>Object</th>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid</td>
<td>Density</td>
<td>$\rho_l$</td>
</tr>
<tr>
<td>Tank</td>
<td>Young’s Mod’s $E$</td>
<td>$E_B$</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>$\rho$</td>
</tr>
<tr>
<td></td>
<td>Poisson’s Ratio $\nu$</td>
<td>$\nu_B$</td>
</tr>
<tr>
<td>Rigid Foun’n</td>
<td>Young’s Mod’s $E$</td>
<td>$E_B$</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>$\rho_g$</td>
</tr>
<tr>
<td></td>
<td>Poisson’s Ratio $\nu_g$</td>
<td>$\nu_g$</td>
</tr>
<tr>
<td>Upper</td>
<td>Shear Wave Vel. $V_{ SUS}$</td>
<td>$V_{ SUS}$</td>
</tr>
<tr>
<td>Subsoil</td>
<td>Density</td>
<td>$\rho_{SU}$</td>
</tr>
<tr>
<td>Layer</td>
<td>Poisson’s Ratio $\nu_{SU}$</td>
<td>$\nu_{SU}$</td>
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<tr>
<td>Half</td>
<td>Shear Wave Vel. $V_{H}$</td>
<td>$V_{H}$</td>
</tr>
<tr>
<td>Space</td>
<td>Density</td>
<td>$\rho_s$</td>
</tr>
<tr>
<td>Soil</td>
<td>Poisson’s Ratio $\nu_s$</td>
<td>$\nu_s$</td>
</tr>
</tbody>
</table>

5.2 Soil Spring and Effective Input The $k_{11}$ and $c_{11}$ in Eq.(21) describing the soil spring are given in Fig.10. The curves were computed for the above soil model with massless foundation by the computer code DYSOL which employs the thin layer element method (a combined method of the finite element and the thin layer element which describing the energy transmission to the far field soil media).

The effective input $u_e$ was confirmed to be almost identical to the free field surface ground motion $u_0$. This is easily understood because the foundation thickness employed is small enough comparing to the radius of the foundation. For this reason, $u_0$ is used instead of $u_e$ as the effective input. It has also been confirmed that the effective input for rotational component $v_e$ is sufficiently small. Therefore, $v_e$ is neglected hereafter in the response computation.

5.3 Parameters of Super-structure Natural frequency and other vibration parameters are obtained from Figs.4 to 7 and listed in Table 2. The critical damping ratio $h_0$ of the tank itself is assumed to be 5%.
5.4 Computational Results  The first natural frequency $\omega_e$ and the corresponding critical damping ratio $h_e$ of the total system are obtained as 16.7 rad/s (2.66 Hz) and 0.23 (23%) by the complex modal analysis method.

El Centro (1940 NS) earthquake ground motion $u_e$ was applied to the bedrock of
free field soil system in order to obtain the effective input. The response of
the free field surface ground motion $u_0$ was used as the effective input $u_e$ of the
total system in the seismic response computations. The maximum value of $u_e$ was
taken to be 81.3 gal so that the maximum value of $u_0$ became the normalized value of
300 gal.

Base shear $Q$ and overturning moment $M_e$ (for ebp) were computed and are listed
in Table 3. In the table, the results computed by the code DYSOL are also
presented. Two results show in good agreement.

![Diagrams](image)

<table>
<thead>
<tr>
<th>Table 2 Vibration Parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\omega_e$</td>
</tr>
<tr>
<td>$h_e$</td>
</tr>
<tr>
<td>$b_s/b_e$</td>
</tr>
<tr>
<td>$m_f$</td>
</tr>
<tr>
<td>$m_e$</td>
</tr>
<tr>
<td>$m_r$</td>
</tr>
<tr>
<td>$m_r'$</td>
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<tr>
<td>$H_f/H_i$</td>
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<td>$H_f'/H_i'$</td>
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<td>$H_r/H_i$</td>
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<tr>
<td>$H_r'/H_i'$</td>
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<tr>
<td>$m_b$</td>
</tr>
<tr>
<td>$I_g$</td>
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</table>

<table>
<thead>
<tr>
<th>Table 3 Comparisons of Results by the Proposed Method and DYSOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force and Moment</td>
</tr>
<tr>
<td>$Q$</td>
</tr>
<tr>
<td>$M_e$ (ebp)</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS

The simplified equations of motions for seismic response analysis of
cylindrical storage tanks on rigid foundation with soil-structure interaction
subjected to horizontal ground motions were proposed. From the numerical study,
the equations was shown to be practical in the computation of seismic responses of
the tanks. Thus, a simplified seismic response analysis method has obtained.

REFERENCES


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