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SIMPLIFIED SEISMIC ANALYSIS METHOD OF CYLINDRICAL TANKS ON RIGID FOUNDATION WITH SOIL-STRUCTURE INTERACTION SUBJECTED TO HORIZONTAL GROUND MOTIONS

Nobuyuki SHIMIZU

Department of Mechanical Engineering, Iwaki Meisei University,
Iwaki, Fukushima, Japan

SUMMARY

This paper deals with proposing a simplified seismic analysis method of cylindrical storage tanks on rigid foundation with soil-structure interaction subjected to horizontal ground motions. Comprehensive analyses are made of the
(1) mass-spring vibration model of tank to sway and rocking motions, and
(2) equations of motions of the tank-foundation-soil total system subjected to horizontal ground motions. Numerical verification is also presented.

1 INTRODUCTION

When we analyse the cylindrical tanks containing liquid on rigid foundation (slab) with soil-structure interaction by a simplified seismic analysis method, the followings need to be clarified,

(1) Vibration model of soil (Ref.1) and effective input (Ref.2) to the system
(2) Vibration model of the tank to both sway (Ref.3) and rocking (Ref.4) motions
(3) Equations of the tank-foundation-soil total system in a simple form.

We will study these items for the model given in Fig.1.

2 SUB-STRUCTURE METHOD FOR TANK-FOUNDATION-SOIL SYSTEM

2.1 Formulations by Sub-structure Method

By applying the sub-structure method described in the reference (2) to the finite element model, equations of motions of the tank-foundation-soil total system subjected to horizontal ground motions at the bedrock can be derived.

When rigid foundation is assumed, the equations of motions are written in the following form,

$$\begin{bmatrix} K_{ss} & k_{sd} \\ k_{ds} & k_{dd}+A_0 \end{bmatrix} \begin{Bmatrix} U_s \\ \begin{pmatrix} u \\ \phi \end{pmatrix} \end{Bmatrix} = \begin{Bmatrix} 0 \\ A_0 \begin{pmatrix} u_e \\ \phi_e \end{pmatrix} \end{Bmatrix} \quad (1)$$

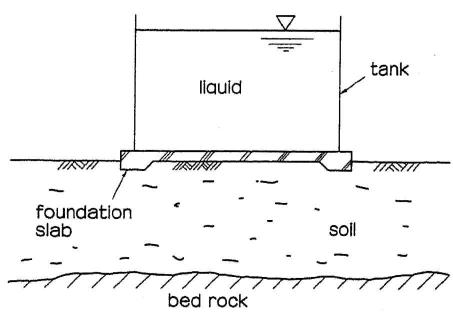

Fig. 1 Tank-Foundation-Soil Model

where K and k denote the stiffness matrices and U and $(u\ \phi)^T$ the displacement vectors. Subscript d refers the superstructure (i.e. the tank and foundation) and subscript s the soil contacting with the foundation. $[A_0]$ denotes the stiffness matrix of the soil at the contact surface of the super-structure and the soil. This $[A_0]$ is called soil spring hereafter.

Eq.(1) is the fundamental equation for the unknown displacements of the super-structure (U_S) and of the foundation at the point O $(u\ \phi)^T$ to the effective input $(u_e\ \phi_e)^T$ under the assumption that the foundation is rigid.

2.2 Soil Spring We consider the massless foundation embedded in the soil surface. As is well known horizontal force F and moment M about the horizontal axis through the center of the surface of foundation due to the horizontal displacement u_B and the rotational displacement ϕ_B are written as

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = -\begin{bmatrix} k_{xx} & k_{xr} \\ k_{rx} & k_{rr} \end{bmatrix} \begin{Bmatrix} u_B \\ \phi_B \end{Bmatrix} \tag{2}$$

$$k_{xx}=(k_{11}+ia_0c_{11})K_x,\quad k_{rr}=(k_{22}+ia_0c_{22})K_r,\quad k_{xr}=k_{rx}=(k_{21}+ia_0c_{21})K_xR_B, \tag{3}$$

$$a_0=\omega R_B/V_{s1},\quad i=\sqrt{-1},\quad K_x=8G_1R_B/(2-\nu_1),\quad K_r=8G_1R_B^3/(3(1-\nu_1)). \tag{4}$$

ω indicates angular frequency. R_B, V_{s1}, G_1, and ν_1 are the radius of the foundation, shear wave velocity, shear modulus and poisson ratio of the surface soil layer. k_{ij} and $c_{ij}(i,j=1,2)$ are generally given by the numerical values. They are given in the reference (1) for half space soil.

3 MASS-SPRING MODEL OF TANK

Let us consider a tank clamped at the rigid base which is subjected to sway and rocking motions. The tank responds to the motion of sway of the base u_x and rocking of the base ϕ. The motion is considered by separating into two independent motions, i.e. sway motion and rocking motion. The responses of the tank will be independently analyzed for the separated two motions as shown in Figs.2(a) and 3(a). The total response of the combined motion will be obtained by superposing the responses to these motions.

3.1 Mass-spring Model to Sway Motion

Fundamental equations of motions for the tank to sway motion u_x is written from the reference 3 as

$$[M_e](\ddot{d}_x) + [K_t](d_x) = -[M_e](f)_x\ddot{u}_x. \tag{5}$$

We only consider the first natural mode of vibration. Displacement vector of the tank shell is written as

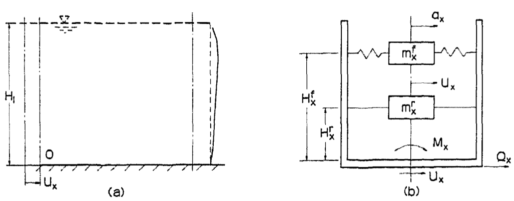

Fig. 2 Sway Motion of Tank and Vibration Model

$$(d_x) = (p)b_xq_x \tag{6}$$

in terms of the first natural mode of vibration (p), the time function q_x and the mode participation factor b_x. The time function q_x is governed by

$$\ddot{q}_x + 2h_0\omega_0\dot{q}_x + \omega_0^2q_x = -\ddot{u}_x. \tag{7}$$

Base shear Q_x, overturning moments M_{xe} (for excluding bottom pressure) and M_{xi} (for including bottom pressure) are written as

$$Q_x = m_x^f\ddot{q}_x + m_x^r\ddot{u}_x,$$
$$M_{xe} = m_x^fH_x^{fe}\ddot{q}_x + m_x^rH_x^{re}\ddot{u}_x,\quad M_{xi} = m_x^fH_x^{fi}\ddot{q}_x + m_x^rH_x^{ri}\ddot{u}_x. \tag{8}$$

Where the subscript x indicates the quantity for sway motion and the superscripts f and r for flexible shell motion and for rigid tank movement. The vibration model for Eq.(8) is shown in Fig.2(b).

3.2 Mass-spring Model to Rocking Motion

Fundamental equations of motions for the tank to rocking motion is written from the reference 4 as

$$[M_e](\ddot{d}_r) + [K_t](\dot{d}_r) = -[M_e](f)_r\ddot{u}_r. \quad (9)$$

Again first natural mode of vibration is considered. Displacement vector of the tank shell is written as

$$(d_r) = (p)b_r q_r \quad (10)$$

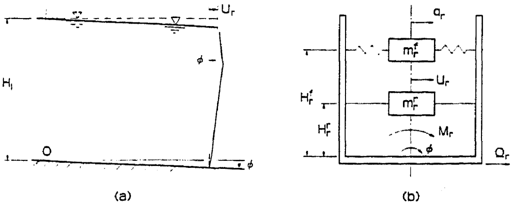

Fig. 3 Rocking Motion of Tank and Vibration Model

where q_r is the function of time, b_r mode participation factor. The function of time q_r is governed by

$$\ddot{q}_r + 2h_0\omega_0\dot{q}_r + \omega_0^2 q_r = -\ddot{u}_r. \quad (11)$$

Base shear Q_r, overturning moments M_{re} (for ebp) and M_{ri} (for ibp) are written as

$$Q_r = m_r^f\,\ddot{q}_r + m_r^r\,\ddot{u}_r,$$

$$M_{re} = m_r^f H_r^{fe}\,\ddot{q}_r + m_r^r H_r^{re}\,\ddot{u}_r, \quad M_{ri} = m_r^f H_r^{fi}\,\ddot{q}_r + m_r^r H_r^{ri}\,\ddot{u}_r \quad (12)$$

The subscript r indicates the quantity for rocking motion. The vibration model for Eq.(12) is given as in Fig.3(b).

3.3 Parameters of Mass-spring Model

The first natural period T_0 ($\omega_0=2\pi/T_0$), vibration parameters m and H defined in sections 3.1 and 3.2 are calculated for the typical tank geometry and shown in Fig.4 to 7. In these figures, m_l is the

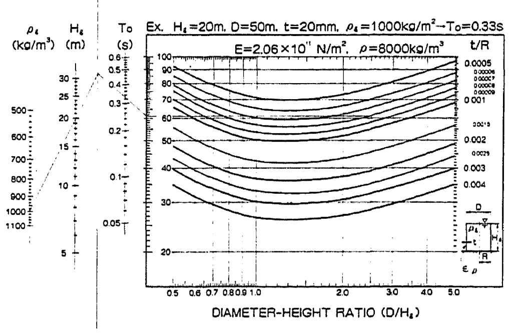

Fig. 4 Nomogram of Ist Natural Period

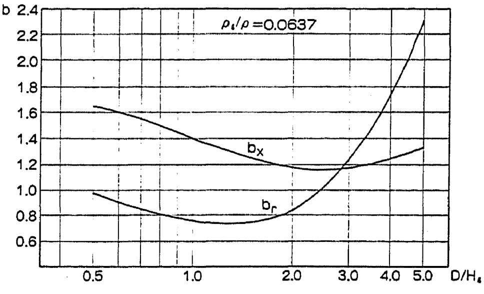

Fig. 5 Mode Participation Factor

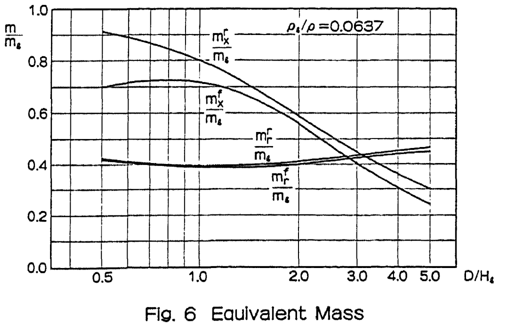

Fig. 6 Equivalent Mass

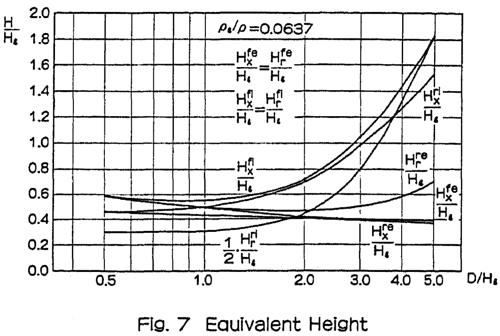

Fig. 7 Equivalent Height

total mass of liquid, i.e., $m_l=\rho_l \pi R2H_l$, where ρ_l indicates the density of liquid, R the radius of the tank, H_l the liquid height of the tank.

From the definition of the vibration parameters below, there exist the following relations;

$$m_x^f/m_r^f = m_x^f H_x^{fe}/(m_r^f H_r^{fe}) = m_x^f H_x^{fi}/(m_r^f H_r^{fi}) = b_x/b_r. \tag{13}$$

3.4 Response to Combined Motion of Sway and Rocking Response to the combined motion of sway and rocking can be obtained from the responses to independent sway motion and rocking motion described in sections 3.1 and 3.2 with the use of the vibration parameters given in section 3.3.

Base shear Q and overturning moments M_e (for ebp) and M_i (for ibp) become

$$\begin{aligned} Q &= m_x^f \ddot{q}_x + m_x^r \ddot{u}_x + m_r^f \ddot{q}_r + m_r^r \ddot{u}_r \\ M_e &= m_x^f H_x^{fe} \ddot{q}_x + m_x^r H_x^{re} \ddot{u}_x + m_r^f H_r^{fe} \ddot{q}_r + m_r^r H_r^{re} \ddot{u}_r \\ M_i &= m_x^f H_x^{fi} \ddot{q}_x + m_x^r H_x^{ri} \ddot{u}_x + m_r^f H_r^{fi} \ddot{q}_r + m_r^r H_r^{ri} \ddot{u}_r. \end{aligned} \tag{14}$$

4 EQUATIONS OF MOTIONS OF TOTAL SYSTEM

We consider the simplified vibration model of the total system shown in Fig.8. In the figure, u_e and ϕ_e indicate the effective inputs for sway and rocking motions. u_B and ϕ_B are the displacement responses of the foundation for horizontal and rotational component, u_x and ϕ are the absolute displacements for horizontal and rotational component. There exist the following relations,

$$u_x = u_B + u_e, \quad \phi = \phi_B + \phi_e. \tag{15}$$

Now, rotational displacement is transformed into the equivalent horizontal displacement at the top of the tank wall (height H_l), that is,

$$u_r = H_l\phi, \quad v_B = H_l\phi_B, \quad v_e = H_l\phi_e. \tag{16}$$

Horizontal force and overturning moment about the horizontal axis through the point O exerted from the super-structure become

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{Bmatrix} Q_T \\ M_T \end{Bmatrix} + \begin{bmatrix} m_B & -(H_B/2)m_B \\ -(H_B/2)m_B & I_B \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\phi} \end{Bmatrix} \tag{17}$$

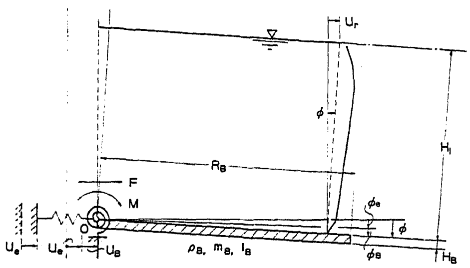

Fig. 8 Vibration Model of Total System

where Q_T and M_T indicate the horizontal force and overturning moment exerted from the upper structure (i.e. tank and liquid). They are obtained from Eq.(14) as Q_T=Q and M_T=Mi . The second term of the right hand side of Eq.(17) is the horizontal force and overturning moment about the horizontal axis through the point O of the foundation itself. The foundation mass m_B and the rotational inertia of the foundation I_B about the horizontal axis through the point O become

$$m_B = \rho_B \pi R_B^2 H_B, \quad I_B = I_{B0} + m_B(H_B/2)^2 \tag{18}$$

where I_{B0} indicates the rotational inertia of the foundation about the centroid of the foundation. Substituting Eq.(2) into (17) and using Eqs.(8) and (12) into the derived equation, one can obtain the equations of motions about the displacement vector $(y)=(u_x, u_r, u_B, v_B)^T$. Since the tank shell displacement is

$$(d) = (p)u_s, \quad u_s = b_x q_x + b_r q_r \tag{19}$$

then the equations of motions obtained above can be transformed into the equations of motions about the displacement vector $(y) = ((u_s/b_x), u_B, v_B)^T$ as

$$[M](\ddot{y}) + [C](\dot{y}) + [K](y) = -[M](f)_u\ddot{u}_e - [M](f)_v\ddot{v}_e \tag{20}$$

$$
[M]=\begin{bmatrix} m_x{}^f & m_x{}^f & m_r{}^f \\ & m_x{}^r+m_B & m_r{}^r-\dfrac{H_B}{2H_1}m_B \\ \text{SYM.} & & m_r{}^r\dfrac{H_r{}^{ri}}{H_1}+\dfrac{I_B}{H_1{}^2} \end{bmatrix},\quad [C]=\begin{bmatrix} 2h_0\omega_0 m_x{}^f & 0 & 0 \\ & \dfrac{a_0c_{11}}{\omega}K_x & \dfrac{a_0c_{21}}{\omega}\cdot\dfrac{K_xR_B}{H_1} \\ \text{SYM.} & & \dfrac{a_0c_{22}}{\omega}\cdot\dfrac{K_r}{H_1{}^2} \end{bmatrix},
$$

$$
[K]=\begin{bmatrix} \omega_0{}^2 m_x{}^f & 0 & 0 \\ & k_{11}K_x & k_{21}\dfrac{K_xR_B}{H_1} \\ \text{SYM.} & & k_{22}\dfrac{K_r}{H_1{}^2} \end{bmatrix},\qquad \begin{aligned} (f)_x &= (0\quad 1\quad 0)^T \\ (f)_r &= (0\quad 0\quad 1)^T \end{aligned} \tag{21}
$$

This is the simplified mass-spring model describing the motion of the tank containing liquid with rigid foundation resting on elastic soil.

5 NUMERICAL COMPUTATIONS

5.1 Analysis Model   Analysis model is shown in Fig.9. The soil system of an upper subsoil layer overlying the half space soil is considered. The model parameters used are listed in Table 1.

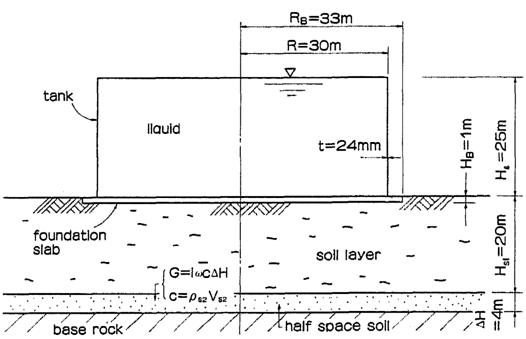

Fig. 9 Analysis Model for Numerical Computation

Table 1 Model Parameters

Object	Item		Value	
Liquid	Density	ρ_l	500.0	kg/m^3
Tank	Young's Mod's	E	2.06×10^{11}	N/m^2
	Density	ρ	7855.0	kg/m^3
	Poisson's Ratio	ν	0.3	
Rigid Foun'n	Young's Mod's	E_B	2.06×10^{10}	N/m^2
	Density	ρ_B	2500.0	kg/m^3
	Piosson's Ratio	ν_B	0.167	
Upper Subsoil Layer	Shear Wave Vel.	V_{s1}	200.0	m/s
	Density	ρ_{s1}	1800.0	kg/m^3
	Poisson's Ratio	ν_{s1}	0.4	
Half Space Soil	Shear Wave Vel.	V_{s2}	500.0	m/s
	Density	ρ_{s2}	1800.0	kg/m^3
	Poisson's Ratio	ν_{s2}	0.4	

5.2 Soil Spring and Effective Input   The k_{ij} and c_{ij} in Eq.(21) describing the soil spring are given in Fig.10. The curves were computed for the above soil model with massless foundation by the computer code DYSOL which employs the thin layer element method (a combined method of the finite element and the thin layer element which describing the energy transmission to the far field soil media).

The effective input u_e was confirmed to be almost identical to the free field surface ground motion u_0. This is easily understood because the foundation thickness employed is small enough comparing to the radius of the foundation. For this reason, u_0 is used insted of u_e as the effective input. It has also been confirmed that the effective input for rotational component v_e is sufficiently small. Therefore, v_e is neglected hereafter in the response computation.

5.3 Parameters of Super-structure   Natural frequency and other vibration parameters are obtained from Figs. 4 to 7 and listed in Table 2. The critical damping ratio h_0 of the tank itself is assumed to be 5 %.

5.4 Computational Results The first natural frequency ω_e and the corresponding critical damping ratio h_e of the total system are obtained as 16.7 rad/s (2.66 Hz) and 0.23 (23%) by the complex modal analysis method.

El Centro (1940 NS) earthquake ground motion $\ddot{u}_g$ was applied to the bedrock of free field soil system in order to obtain the effective input. The response of the free field surface ground motion $\ddot{u}_0$ was used as the effective input $\ddot{u}_e$ of the total system in the seismic response computations. The maximum value of $\ddot{u}_g$ was taken to be 81.3 gal so that the maximum value of $\ddot{u}_0$ became the normalied value of 300 gal.

Base shear Q and overturning moment M_e (for ebp) were computed and are listed in Table 3. In the table, the results computed by the code DYSOL are also presented. Two results show in good agreement.

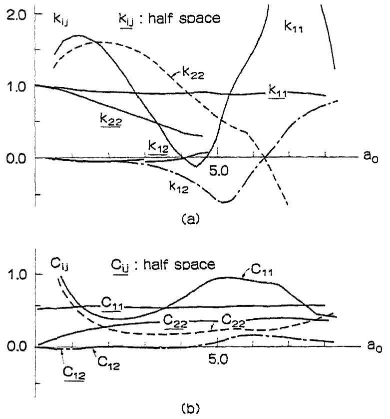

Fig. 10 Soil Spring

Table 2 Vibration Parameters

Parameter	Value
ω_o	21.8 rad/s
h_o	0.05
b_x / b_r	1.51 / 1.23
m_x^f	1.65×10^7 kg
m_x^r	1.78×10^7 kg
m_r^f	1.34×10^7 kg
m_r^r	1.46×10^7 kg
H_x^{fe} / H_x^{fi}	11.1 m / 20.4 m
H_x^{re} / H_x^{ri}	9.9 m / 20.5 m
H_r^{fe} / H_r^{fi}	11.1 m / 20.4 m
H_r^{re} / H_r^{ri}	10.8 m / 27.6 m
m_B	8.56×10^6 kg
I_B	2.33×10^9 kgm^2

Table 3 Comparisons of Results by the Proposed Method and DYSOL

Force and Moment	Proposed Method	DYSOL
Q N	8.9×10^7	7.6×10^7
$M_e(ebp)$ $N \cdot m$	9.3×10^8	8.6×10^8

6 CONCLUSIONS

The simplified equations of motions for seismic response analysis of cylindrical storage tanks on rigid foundation with soil-structure interaction subjected to horizontal ground motions were proposed. From the numerical study, the equations was shown to be practical in the computation of seismic responses of the tanks. Thus, a simplified seismic response analysis method has obtained.

REFERENCES

1. Veletsos, A. S. and Wei, Y. T., "Lateral and Rocking Vibration of Footings," J.Soil Mech. Found. Div., ASCE, 97, 1257-1281, (1971).
2. Elsabee, F. and Morray, J. P., "Dynamic Behavior of Embededded Foundations," Department of Civil Engineering Report, Publication No.R77-33, Massachusetts Institute of Technology, (1977).
3. Shimizu, N., Yamamoto, S. and Kawano, K., "Study on Seismic Design Method of Cylindrical Tanks -Part 2," Japan Soc. Mech. Eng'rs, 427(C), 328-348, (1982).
4. Shimizu, N., "Study on Seismic Design Method of Cylindrical Tanks -Part 4," Japan Soc. Mech Eng'rs, 492(C), 1676-1682, (1987).