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NONLINEAR SLOSHING ANALYSIS OF LIQUID STORAGE TANKS SUBJECTED TO RELATIVELY LONG-PERIOD MOTIONS

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SUMMARY

The effect of nonlinear boundary conditions at the liquid surface on the sloshing heights in cylindrical tanks under horizontal and vertical ground motion is discussed and compared with data obtained by linear analysis. A simple formula for estimating this effect is proposed. On the basis of the large sloshing heights recorded during the 1983 Nihonkai-chubu earthquake, the sloshing heights under nonlinear conditions are estimated to be about 10-25% larger than those calculated under linear conditions. Taking this into account, the response spectra of relatively long-period ground motions deduced from the recorded sloshing heights are neary equal to the two-dimensional response spectra calculated from strong-motion seismograms.

INTRODUCTION

There is still insufficient information available on the relatively long-period ground motions that occur during earthquakes even though this phenomenon is now recognized as being of great importance. The reasons for our lack of knowledge are that the reliability of acceleration-type (SMAC-type) strong motion seismographs is not particularly good for that range of period. Deduction of long-period ground motion characteristics from low-magnification displacement seismographs must also be made with care because of their significant off-scaled extent being more than ±3 cm. In contrast, the generating mechanism of sloshing is comparatively clear. Accordingly, the investigation of sloshing behavior and the subsequent deduction of the response spectra of ground motions from recorded sloshing heights offer valuable information about relatively long-period ground motions.

Previously (Ref.1), we calculated the velocity response spectra Sv from seismograms and recorded oil sloshing data for the 1983 Nihonkai-chubu earthquake by a linear analysis based on the potential theory. The Sv values from the sloshing heights are often larger than those from the seismograms, particularly for large sloshing heights. This difference in Sv values is believed to stem from differences in the locations of the tank and the seismograph (several kilometers at most) or various errors contained in the seismograms. Moreover, a variety of factors that are not considered in the simplified analytical model, in particular the nonlinearity of boundary conditions or the effect of vertical ground motion, may affect the results. Also, when considering ground motion on the horizontal plane, maximum response spectra from the seismograms for every direction should be adopted.

We first develop equations of sloshing based on the nonlinearity of boundary conditions under horizontal and vertical ground motions. Results of the nonlinear analysis of the seismograms are compared with results of a linear analysis, from which a simple formula that estimates the effects of nonlinear boundary conditions is proposed. The response spectra deduced from the observed sloshing heights are discussed, taking into account the effects of nonlinear boundary conditions, and compared with the spectra calculated from seismograms. (The nonlinearity of the boundary conditions in this paper means the change in boundary at the free surface with the growth of sloshing computed from the finite amplitude theory.)

FUNDAMENTAL EQUATIONS FOR NONLINEAR SLOSHING UNDER HORIZONTAL AND VERTICAL EXCITATIONS

Very few studies of the sloshing response under horizontal and vertical excitations have been done based on actual seismogram data compared to observed sloshing heights. In our study of nonlinear sloshing in rigid cylindrical tank, we have introduced a term that accounts for vertical excitation to the solution of Sakata et al. (Ref.2). Throughout the analysis, we assumed the irrotational flow of incompressible and non-viscous fluid.

Use of the velocity potential, ϕ, of the relative motion of liquid in a tank gives the Laplace equation

$$\nabla^2\phi=\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad \text{..... (1)}$$

The boundary conditions at the tank wall and bottom are

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=a_0}=0 \quad , \quad \left.\frac{\partial\phi}{\partial z}\right|_{z=-h}=0 \qquad \text{..... (2), (3)}$$

The nonlinear boundary conditions at the free surface are

$$\int_0^{2\pi}\int_0^{a_0}\Big[\left.\frac{\partial\phi}{\partial t}\right|_{z=\eta}+\{g+\ddot{f}_z(t)\}\eta+r\cos\theta\ddot{f}_x(t)+\frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial\phi}{\partial\theta}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right\}\Big|_{z=\eta}$$
$$-\frac{1}{2}\{\dot{f}_z^2(t)+\dot{f}_x^2(t)\}\Big]\delta\eta\, r\,dr\,d\theta=0 \qquad \text{..... (4)}$$

$$\int_0^{2\pi}\int_0^{a_0}\Big[-\frac{\partial\eta}{\partial t}+\left.\frac{\partial\phi}{\partial z}\right|_{z=\eta}-\left.\frac{\partial\phi}{\partial r}\right|_{z=\eta}\cdot\frac{\partial\eta}{\partial r}-\frac{1}{r^2}\left.\frac{\partial\phi}{\partial\theta}\right|_{z=\eta}\cdot\frac{\partial\eta}{\partial\theta}\Big]\delta\phi\Big|_{z=\eta} r\,dr\,d\theta=0 \qquad \text{..... (5)}$$

in which r, θ and z are the cylindrical co-ordinates shown in Fig.1, a_0 the radius of the tank and h the depth of the liquid. η denotes the liquid elevation, defined as the surface displacement in the z-direction with respect to motion co-ordinates fixed at the tank. $\ddot{f}z(t)$ and $\ddot{f}x(t)$ denote the vertically and horizontally excitation accelerations. Eq. (4) expresses the pressure conditions and Eq. (5) the condition of wave continuity. Both are shown in integral form for convenience of calculation by Galerkin's method.

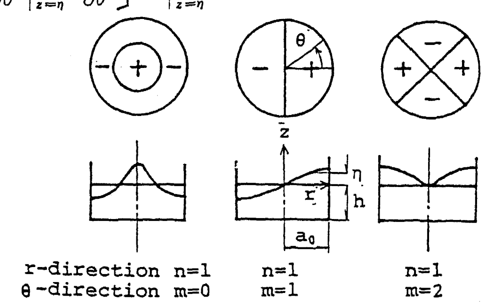

Fig.1 Cylindrical co-ordinate system and the three modes of sloshing considered.

In the approximate solution to be obtained by Galerkin's method, the following functions of linear free vibration were adopted as satisfactory ϕ and η, which take into account only th sloshing components in the direction of excitation.

$$\phi(r,\theta,z,t)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}A_{mn}(t)\cos m\theta\times J_m(\lambda_{mn}r)\times\cos h\{\lambda_{mn}(z+h)\}/\cos h(\lambda_{mn}h) \qquad \text{... (6)}$$

$$\eta(r, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} C_{mn}(t) \cos m\theta \times J_m(\lambda_{mn} r) \qquad \ldots\ldots (7)$$

in which Amn(t) and Cmn(t) are the generalized time functions. Eigen values, λmn, corresponding to the wave number, m, of the circular direction and the mode number, n, of the radial direction are the n-th positive roots that satisfy $dJm(\lambda r)/dr|_{\gamma=a_o}=0$, in which, Jm is the Bessel function of the first kind and m-th order. The modes dominant for a large tank with a floating roof were considered. The wave number, m, of the circular direction is limited to 0, 1 or 2, and the mode number, n, of the radial direction to 1 only. The three modes used are shown in Fig.1. In the following, the i-th mode means the wave number, i, of circular direction, the subscript n being omitted. Consequently, i=1 corresponds to the fundamental mode, i=0 and 2 being nonlinear terms. The eigen values are given by

$$\omega_m^2 = g\,\lambda_m \tan h(\lambda_m h) \qquad \ldots\ldots (8)$$

in which $\lambda_0 a_0=3.8317$, $\lambda_1 a_0=1.8412$ and $\lambda_2 a_0=3.0542$.

Eqs.(4)-(7) are dimensionless for the radius of a tank of a_0 and an eigen value of ω_1. The ordinary differential equations, in terms of the generalized co-ordinate, C_i, are derived by Galerkin's method. The methods for making the variables and constants dimensionless and for the derivation of the equations are given in Ref.2. The equations of motion, expressed in terms of generalized co-ordinates of surface displacement, are

$$\ddot{C}_1+2\zeta_1\dot{C}_1+\{1+\ddot{f}_z(t)/g\}\,C_1+C_1^3P^{(1)}+C_1\dot{C}_1^2P^{(2)}+C_1C_0Q_0^{(1)}+\dot{C}_1\dot{C}_0Q_0^{(2)}+C_1C_2Q_2^{(1)}+\dot{C}_1\dot{C}_2Q_2^{(2)}=-\mu S_1\ddot{f}_x(t) \qquad \ldots\ldots (9)$$

$$\ddot{C}_0+2\zeta_0\,\omega_0\,\dot{C}_0+\omega_0^2\{1+\ddot{f}_z(t)/g\}C_0 = C_1^2\,R_0^{(1)}+\dot{C}_1^2\,R_0^{(2)} \qquad \ldots\ldots (10)$$

$$\ddot{C}_2+2\zeta_2\,\omega_2\,\dot{C}_2+\omega_2^2\,\{1+\ddot{f}_z(t)/g\}\,C_2 = C_1^2\,R_2^{(1)}+\dot{C}_1^2\,R_2^{(2)} \qquad \ldots\ldots (11)$$

in which ζ_i is the i-th equivalent damping constant. S_1 is a definite integral value, and μ, $P^{(1)}$, $P^{(2)}$, $Qm^{(1)}$, $Qm^{(2)}$, $Rm^{(1)}$ and $Rm^{(2)}$ (m=0,2) are nonlinear parameters dependent on the dimensionless liquid depth $\bar{h}(=h/a_o)$.

Comparison of the results of the vibration experimental study by Yamagata et al.(Ref.3) and the present analytical ones is shown in Fig.2. The inner diameter of model tank is 2.0 m, and the liquid depth 1.263 m. The sinusoidal wave with the fundamental sloshing period of 1.49 sec is added horizontally. Input displacement is 1.9cm, and input wave number is 3. In fig.2, the results of the model test, the nonlinear analysis and the linear analysis are indicated by the symbol o, the solid line and the broken line, respectively. This figure shows that the results of nonlinear analysis match the experimental results well, with respect to the maximum wave height, the change of vibration period and the asymmetrical wave form on the plus side (rise side) and the minus side (fall side).

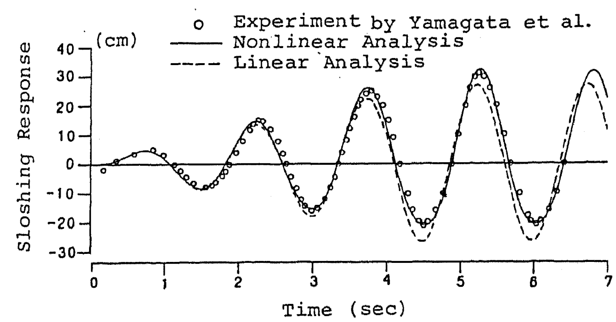

Fig. 2 Comparison of experimental (Ref. 3) and analytical results (h=0.1%).

SLOSHING RESPONSE TO EARTHQUAKE MOTIONS

The SMAC records at Akita and the low-magnification seismograms at Niigata (obtained by correcting off-scaled records) and Tomakomai for the 1983 Nihonkai-chubu earthquake were used. The dimensions of the tanks in which sloshing actually was observed in Akita, Niigata and Tomakomai also were used. Those tanks are relatively large, having capacities of 10,000-130,000 kl.

The ratios, β_1, of the maximum wave height obtained by nonlinear analysis to that obtained by linear analysis for horizontal motion alone (NS component) are given in Fig.3. Taking into account the nonlinearity, the wave heights increase about 15% for Niigata, about 20% for Akita and about 10% for Tamakomai as compared with the values from the linear analysis. The ratios, β_3, of the maximum wave height produced by simultaneous excitations to the height produced by horizontal (NS) motion alone are shown in Fig.4. Clearly, the effects of vertical excitation for maximum wave height are about ±2%, negligible from an engineering stand point.

As seen in Eqs.(10) and (11), C_o and C_2 are basically proportional to C_1^2; therefore, the rates of the effect of nonlinearity that are the ratios of C_o and C_2 to C_1 are proportional to C_1. Then, the effect of nonlinearity including the effect of vertical excitation is described for the dimensionless wave height $\bar{\eta}$ ($=\eta max/a_o$) as a form of $\beta_2=1+\gamma\bar{\eta}$, in which β_2 is the ratio of the maximum wave height obtained by nonlinear analysis to height (ηmax) obtained by linear analysis. The values of γ are calculated backward and plotted against the dimensionless liquid depth $\bar{h}$ in Fig. 5. In contrast, regression relation that fits a parabolic curve between $\bar{h}$ and γ is shown by the broken line in Fig.5 and is

$$\gamma = 0.952/\bar{h} - 0.397 \qquad \cdots\cdots (12)$$

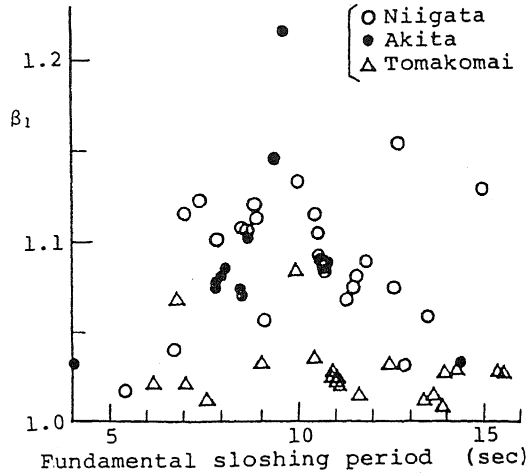

Fig.3 Ratios, β_1, of the maximum wave height obtained by nonlinear analysis to that obtained linearly for the horizontal motion only recorded in several cities during the 1983 Nihonkai-chubu earthquake.

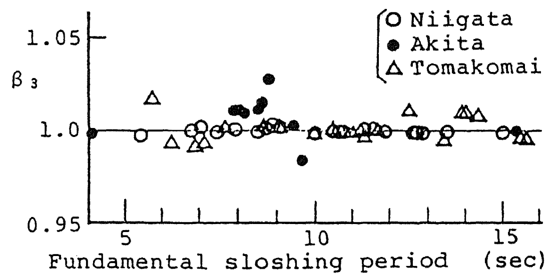

Fig.4 Ratio β_3, for the maximum wave height produced by simultaneous excitations to the height produced by horizontal motion only.

The nonlinear sloshing height that includes the effect of vertical excitation can be simply estimated by multiplying the linear value by the coefficient β_2;

$$\beta_2 = 1+(0.952/\bar{h} - 0.397)\,\bar{\eta} \qquad \cdots\cdots (13)$$

COMPARISON OF RESPONSE SPECTRA

The analytically computed sloshing heights do not always coincide with the observed heights because of various errors. Fig.3 was obtained from analytically computed heights (not the observed heights necessary to this study). Using Eq.(13), we estimated β_2' for the levels of the observed sloshing heights (Fig.6). The values of β_2' are less than 1.2 at Niigata, less than 1.1-1.28 at Akita and less than 1.15 at Tomakomai.

Dividing the Sv calculated by linear analysis from sloshing heights for each tank by the β_2' from Fig.6 and taking into account the nonlinearity, we deduced the Sv values (symbol o) and compared them with the two-dimensional velocity response spectra (solid line) from the seismograms (Fig.7(a)-(c)). Although corrected off-scaled seismograms at Niigata are used, there is good agreement. In

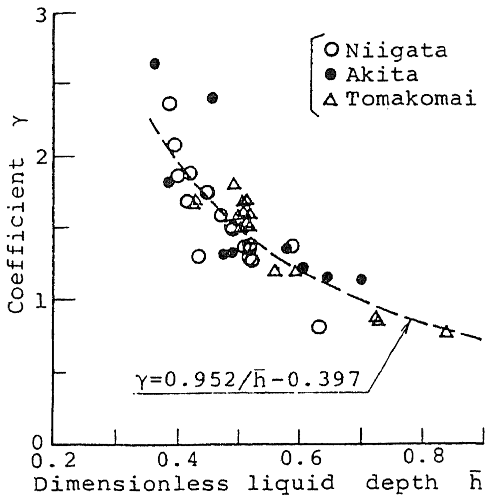

Fig.5
Relation between coefficient γ that represents the effect of nonlinearity for maxumum wave height to actual earthquake motions and the dimensionless liquid depth h.

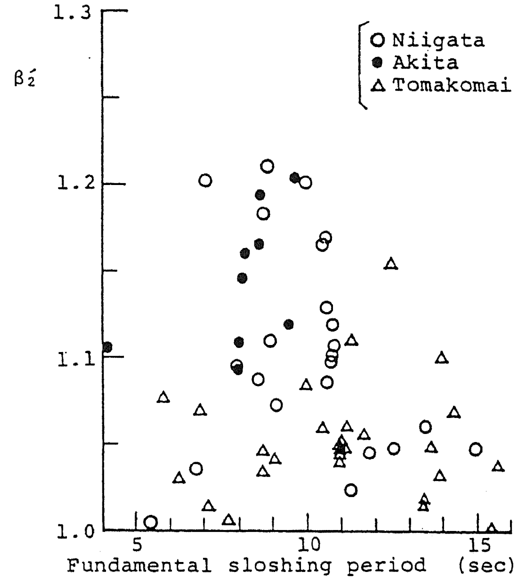

Fig.6
Ratio β_2 estimated from the relation in Eq. (13) including the effect of vertical excitation for recorded sloshing height levels.

the spectral peaks for the period 8-11 sec, the Sv values calculated backward from the sloshing heights are equal to 230 kine (for periods of 8.7, 8.9 and 10.0 sec), and the value from the seismograms is 190 kine (for a period of 9.8 sec). Thus, there is a difference between these Sv; but, the relation between them is remarkably improved in comparison with that from the linear analysis. For Akita, Fig.7(b), two-dimensional spectra were calculated from SMAC records that may contain various errors. Nevertheless, the correspondence of the Sv shown by these two methods is again fairly, improved in comparison with that from linear analysis, by the correction that accounts for the effects of nonlinearity. There is some difference between both Sv near the periods of 4 and 8.5 sec. These

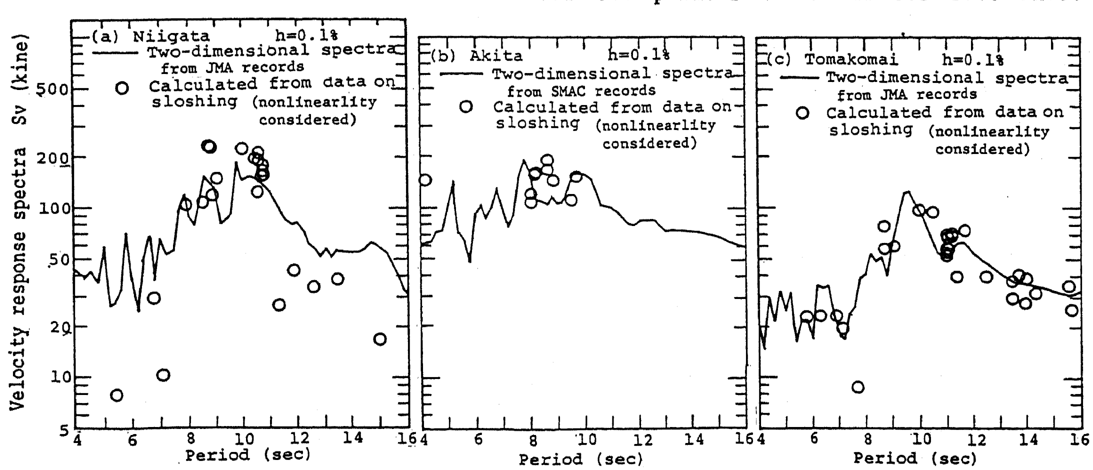

Fig.7 Comparison of the velocity response spectra estimated from the observed sloshing heights (nonlinear boundary conditions) to two-dimensional velocity response spectra (h=0.1%) obtained from seismograms.

period regions, however, are situated in the trough portion of the spectra from the seismograms and the peak values around these periods fit well. Whereas the Sv computed backward from the sloshing heights is equal to 140 kine, for the period of 4.1 sec, the Sv from the seismograms is 140 kine at 5.2 sec. Moreover, the former is 200 kine at 8.7 sec and the latter 200 kine at 7.8 sec. At Tomakomai in Fig.7(c), unlike Niigata, better records that were not off scale were obtained from a low-magnification seismograph. Therefore, both spectra match very well over a wide range of periods, even without considering the effect of nonlinearity, because the sloshing heights are not very large.

CONCLUSIONS

In order to improve the accuracy of response spectra estimated from observed sloshing heights, we have taken into account the effects of nonlinear boundary conditions and vertical excitation on the observed sloshing heights in large tanks. Our results are summarized as follows;

1) Equations for sloshing that take into account the nonlinearity of boundary conditions for horizontal and vertical excitations are Eqs.(9)-(11).
2) In the range of sloshing heights in large tanks, which are of engineering concern, the nonlinearity of sloshing is expressed in terms of a dimensionless liquid depth and a dimensionless sloshing height. The ratio of the nonlinear to the linear sloshing height is expressed in Eq.(13).
3) The effects of vertical excitation are less than ±2% and negligible in engineering terms for the seismograms used.
4) For several cities, the observed sloshing heights produced by the 1983 Nihonkai-chubu earthquake were large because of the effects of nonlinearity (as well as vertical excitation) as follows; maximum of 20% at Niigata, 28% at Akita and 15% at Tomakomai.
5) When the nonlinearity of the boundary conditions is considered, the velocity response spectra estimated from the recorded sloshing heights can be brought closer (vs. linear analysis) to the two-dimensional velocity response spectra obtained from seismograms.

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