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EQUIVALENT STIFFNESS OF COMPOSITE BEAMS IN FRAMES DURING EARTHQUAKES

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SUMMARY

Two moments of inertia, positive and negative bending, develop in composite beams of a frame during earthquakes. In this study, we represented these two moments of inertia in composite beams by a single equivalent moment of inertia. In the numerical analyses, the equivalent moment of inertia was calculated by multiplying an equivalent coefficient by the moment of inertia of a steel beam in a composite beam. Analyses of 4- and 7-story frames were carried out using these equivalent coefficients and exact stiffness values of composite beams. The results were compared and we found that both results were in good agreement.

INTRODUCTION

Both positive and negative bending moment regions develop in a composite beam when it receives bending moments during earthquakes. Thus, the moment of inertia in a beam can be different in two bending moment regions of a beam. In the structural analyses of steel frames which consist of composite beams, in many cases, the elastic stiffness of the composite beams is not represented by the stiffness of a beam which has undergone changes in its cross section, but by the elastic stiffness of an equivalent steel beam. When calculating the equivalent elastic stiffness, the moments of inertia are approximated by the following four values: 1) moment of inertia of composite beams which consist of a positive bending region only, 2) average value of the positive and negative moments of inertia, 3) moment of inertia of a steel beam with a multiplication coefficient to adjust the value of the moment of inertia, and 4) moment of inertia of the steel beam without a concrete slab. In the present paper, we accurately represented the moment of inertia of a composite beam using method 3) of the above-mentioned four values.

In the actual calculation process, the exact elastic flexural stiffness of various composite beams was calculated using beams which had undergone changes in their cross-sectional areas. Then, the equivalent coefficients (the coefficient to be multiplied by the moment of inertia of steel beams) of composite beams in multilayer and multispan balanced frames were calculated for windward and leeward composite beams as well as interior composite beams. As examples of the numerical analyses, a 4-story 4-span frame and a 7-story 6-span frame were used. We compared the results obtained by the equivalent coefficients with those obtained by the exact elastic stiffness of composite beams. In addition, the results from the equivalent coefficients were compared with the results obtained by the D-method (Ref. 1), in which exact stiffness was used.

ELASTIC FLEXURAL STIFFNESS OF COMPOSITE BEAMS

As shown in Fig. 1, the bending moments applied to the two ends, A and B, are designated M_{AB} and M_{BA} ($=\alpha M_{AB}$), and the end rotation for each is defined by θ_A and θ_B, respectively. The elastic flexural stiffness (Refs. 2 and 3) can be shown by eq. (1)

$$\begin{bmatrix} M_{AB}/{}_sM_p \\ M_{BA}/{}_sM_p \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \theta_A/{}_s\theta_p \\ \theta_B/{}_s\theta_p \end{bmatrix} \tag{1}$$

where, ${}_sM_p$: full plastic moment of the steel beam in a composite beam ${}_s\theta_p$: ${}_sM_pL/6E_{s}I$, E_sI: flexural rigidity of the steel beam in a composite beam.

The elements in the stiffness matrix, $k_{ij}(i,j=1,2)$, were calculated by solving the equilibrium differential equations of the composite beam, $dN/dZ=0$ and $d^2M/dZ^2=0$ using the finite difference method. Here, N represents the axial force and M represents the bending moments. The following assumptions were used in the analysis: 1) There was no slippage between the steel beam and the concrete slab (thus, Navier's assumption has been maintained). 2) Concrete is effective in the compressive region and steel bars are effective in the negative bending region. 3) Young's modulus ratio of steel to concrete was set as 14. 4) The cross section of the composite beam was divided into 5 to 8 elements, and N and M were calculated based on the resultant force exerted on the center of each element. The ratio of the beam height to the flange width of steel beams was set between 2 and 3, thus, H-400x200x8x13, H-500x200x10x16 and H-600x200x11x7 were used. The thickness of the concrete slabs used in the analysis was 10 cm and the width was determined so that the ratio of a positive moment of inertia of composite beams, I_+, to the moment of inertia of the steel beams, ${}_sI$, ($\beta=I_+/{}_sI$) became 1 to 3. The steel bar used was D10@200 and the covering depth was 2.5 cm. In addition, the moment ratio of both ends, $\alpha=M_{BA}/M_{AB}$ was 0 to -1.0.

Using H-400x200x8x13 as an example, we presented the relationship between the elements of the stiffness matrix, k_{ij}, and the moment ratio of both ends, α, in terms of the ratio of the moment of inertia, β, in Fig. 2. As can be seen from the figure, when $-1.0 \leqq \alpha \leqq -0.4$, regardless of the moment ratio α, all elements, k_{ij}, can be considered to be constant . When the values of k_{ij} are calculated in terms of the ratio of the moment of inertia, β, they can be shown as follows:

$$\begin{bmatrix} k_{11} & K_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 0.57\beta + 0.10 & 0.16\beta + 0.17 \\ 0.16\beta + 0.17 & 0.14\beta + 0.52 \end{bmatrix} . \tag{2}$$

Similarly, stiffness matrices were obtained for H-500x200x10x16 and H-600x200x11 x7 and these elements, k_{ij}, can be represented by eq. (2) when $-1.0 \leqq \alpha \leqq -0.4$.

EQUIVALENT MOMENT OF INERTIA OF COMPOSITE BEAMS IN A BALANCED FRAME

We studied the behavior of the multilayer and multispan frame shown in Fig. 3 under the condition of an earthquake. The moment of inertia of a composite beam in a frame consists of a positive moment of inertia and a negative moment of inertia (hereafter, this type of frame will be called a "composite beam frame"). We consider the steel frame shown in Fig. 4, where the moment of inertia of the composite beam of the same frame shown in Fig. 3 was replaced by ϕ times the moment of inertia, ${}_sI$, (hereafter this type of frame will be called an "equivalent steel beam frame"). ϕ was obtained by equating the horizontal stiffness of two corresponding columns of two different types of frames; this value was designated as the equivalent coefficient. As shown in Fig. 4, the equivalent coefficients at the windward, interior and leeward sides were designated as ϕ^+, ϕ and ϕ^-, respectively.

Interior Composite Beam in the Intermediate Layer Here we calculated the equivalent coefficient ϕ for a composite beam, AB, which is the intermediate layer shown in Fig. 3(b). When we calculate the normalized horizontal stiffness of column AC in the composite beam frame of Fig. 3(b) and in the equivalent steel beam frame of Fig. 4(b) of the intermediate layer using the following assumptions, we obtain eqs. (3) and (4). The assumptions are as follows: Rotation angle: $\theta_A = \theta_B = \theta_C = \theta_D = \theta_E$, Sway deflection: $R_{AC} = R_{AD}$, Relative stiffness ratio: $k_{AC} = k_{AD}$ (The relative stiffness ratio of a steel beam in a composite beam is 1.0.), Composite beam: AB and AE are identical. Then,

$$\left(\frac{q}{r}\right)_{CF} = \frac{\Sigma k_{ij} \times 2k_{AC}}{\Sigma k_{ij} + 2k_{AC}} \quad (3), \qquad \left(\frac{q}{r}\right)_{EF} = \frac{2\phi \times 2k_{AC}}{2\phi + 2k_{AC}} \quad (4)$$

where $\Sigma k_{ij} = k_{11} + k_{12} + k_{21} + k_{22}$ and q and r are the shear force of a column and sway deflection normalized by ${}_sM_p/h$ (h: story height) and ${}_s\theta_p$, respectively. The suffixes CF and EF indicate the composite beam frame and an equivalent steel beam frame, respectively. By equating eqs. (3) and (4), the equivalent coefficient ϕ will be given by the following equation:

$$\phi = \Sigma k_{ij}/2 \quad (5)$$

Windward Composite Beam in the Intermediate Story We now obtain the equivalent coefficient ϕ^+ of a composite beam, AB, in the intermediate layer in Fig. 3(a). When the normalized horizontal stiffness of column AC at the windward side of the intermediate layer in Figs. 3(a) and 4(a) is calculated using the following assumptions, we obtain eqs.(6) and (7). The assumptions are: Rotation angle: $\theta_A = \theta_C = \theta_D$, $\theta_B = P^+\theta_A$, Sway deflection: $R_{AC} = R_{AD}$, Relative stiffness ratio: $K_{AC} = k_{AD}$. Then,

$$\left(\frac{q}{r}\right)_{CF} = \frac{(k_{11} + p^+k_{12})2k_{AC}}{k_{11} + P^+k_{12} + 2k_{AC}} \quad (6)$$

$$\left(\frac{q}{r}\right)_{EF} = \frac{\phi^+(2/3 + 1/3P^+)2k_{AC}}{\phi^+(2/3 + 1/3P^+) + 2k_{AC}} . \quad (7)$$

By equating eqs. (6) and (7), the equivalent coefficient ϕ^+ will be

$$\phi^+ = \frac{3(k_{11} + P^+k_{12})}{2 + P^+} . \quad (8)$$

The ratio P^+ of θ_B to θ_A will be calculated using moment equations at joints A and B and the assumptions listed below. Then, eq. (9) will be obtained as follows and the assumptions are: Rotation angle: $\theta_B = \theta_F = \theta_H = \theta_G$ sway deflection: $R_{AC} = R_{AD}$ Relative stiffness ratio: $k_{AC} = k_{AD} = k_{BF} = k_{BH}$, Composite beams : AB and BG are identical. Then,

$$P^+ = \frac{k_{11} - k_{21} + 2k_{AC}}{k_{11} + k_{22} + 2k_{AC}} . \quad (9)$$

Leeward Composite Beam in the Intermediate Layer The same procedure used for the calculation of the equivalent coefficient for the windward composite beam can be applied to the calculation of equivalent coefficient of a leeward composite beam, AB, at the intermediate layer in Fig. 3(c). The calculated results of ϕ^- and p^- are shown in Table 2.

A similar procedure, which was applied in the above calculation, can be applied to the calculation of equivalent coefficients ϕ^+, ϕ and ϕ^- for the composite beam, AB, in the top and ground layers. The assumptions needed to calculate the equivalent coefficients, ϕ^+ and ϕ^- are listed in Table 1 and the equivalent coefficients and equation for P^+ and P^- are shown together in Table 2.

EXAMPLES OF FRAME ANALYSIS

Analysis To study the reliability of equivalent coefficients, frame analyses on 4-story 4-span, and 7-story 6-span frames were carried out. In both frames, each span was 6.2 m and the height of the 1st floor was 3.8 m, while the height of the other stories was 3.6 m each . The ratio ,β, of the positive moment of inertia of a composite beam, I_+, to that of the steel beam, ${}_sI$, and the equivalent coefficients for each layer are shown in Tables 3 and 4. The horizontal forces applied to the floor levels of each story were determined by adopting the A_i distribution of the Japan Building Code. The vertical floor load in these calculations was 0.8 t/m^2 for each floor. The direct stiffness method and the D-method were used in the numerical analysis.

Results The story displacement and story stiffness for a 4-story frame are shown in Fig. 5. These values for the 7-story frame are shown in Fig. 6. In these figures, the analytical results of steel frames without concrete slabs (equivalent to ϕ^+, ϕ and ϕ^- =1.0) are also shown. The dot-dashed line represents the results from the direct stiffness method. Since the results obtained from the exact stiffness and from the equivalent stiffness are almost identical, the two results are represented by the dot-dashed line. In the case of the D-method, (the dotted line), the two results are also represented by the line. Figures 7 and 8 show the bending moments and shear forces for the 4- and 7-story frames.

CONCLUSIONS

In this paper, we represented the moment of inertia of a composite beam, which consists of a positive and a negative bending moments of inertia, by a single equivalent moment of inertia of a steel beam. In order to obtain the equivalent moment of inertia, an equivalent coefficient, which is a factor to be multiplied by the moment of inertia of a steel beam, was introduced. The equivalent coefficient of composite beams for the windward, interior and leeward sides in the frame are shown in Table 2. From the numerical analyses of 4-story and 7-story frames, the following conclusions have been obtained:

1) The story displacement and story stiffness calculated by equivalent coefficients agreed well with the results obtained from the exact stiffness matrix. This conclusion is valid in the direct stiffness method and the D-method.

2) Regarding the bending moment distribution, shear force distribution of columns, the results obtained from the equivalent coefficient method agreed fairly well with those obtained from the exact stiffness matrix. However, there was some minor discrepancy for the columns next to the exterior.

3) In the 7-story frame, there was basically no difference between the values obtained from the direct stiffness method and the D-method. In the 4-story frame, some difference was found between the two methods. For instance, in the case of sway deflection (not shown in the text), the largest discrepancy between the D-method and the direct stiffness method was that the D-method values was 1.2 times as large as that of the direct stiffness method values.

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Table 1 Assumption for Obtaining Equivalent Coefficient ϕ and p

Beam	Story	Assumption $(\phi, \phi^{+}, \phi^{-})$	Assumption (p^{+}, p^{-})
Interior Composite Beam	Top	$\theta_A=\theta_B=\theta_E$ $(\theta_A/\theta_C)_{CF}=(\theta_A/\theta_C)_{EF}$ Composite Beam : AB=AE	———
	Ground (Fix)	$\theta_A=\theta_B=\theta_E$ $(\theta_A/\theta_D)_{CF}=(\theta_A/\theta_D)_{EF}$ $(R_1/R_2)_{CF}=(R_1/R_2)_{EF}$ Composite Beam : AB=AE	———
Windward and Leeward Composite Beam	Top	$\left\{\begin{matrix}(\theta_A/\theta_C)_{CF}=(\theta_A/\theta_C)_{EF}\\(\theta_A/\theta_B)_{CF}=(\theta_A/\theta_B)_{EF}\end{matrix}\right\}$ $\theta_B=p^{+}\theta_A$	$M_{AC}=M_{CA}$, $M_{BH}=M_{HB}$ $k_{AC}=k_{BH}$ Composite Beam : AB=BG
	Ground (Fix)	$\left\{\begin{matrix}(\theta_A/\theta_D)_{CF}=(\theta_A/\theta_D)_{EF}\\(\theta_A/\theta_B)_{CF}=(\theta_A/\theta_B)_{EF}\end{matrix}\right\}$ $\theta_B=p^{+}\theta_A$ $(R_1/R_2)_{CF}=(R_1/R_2)_{EF}$	$M_{AC}+1/3\cdot M_{AB}=0$ $M_{BH}+1/3\cdot(M_{BA}+M_{BG})=0$ $k_{AC}=k_{BH}$ Composite Beam : AB=BG

CF: Composite Beam Frame   EF: Equivalent Steel Beam Frame

Table 2 Equivalent Coefficients ϕ and p

Beam	Equivalent Coefficient	p^{+} , p^{-}	k
Interior Composite Beam	$\phi=\dfrac{k11+k12+k21+k22}{2}$	———	———
Windward Composite Beam	$\phi^{+}=\dfrac{3(k11+p^{+}k12)}{2+p^{+}}$	$p^{+}=\dfrac{k11-k21+k}{k11+k22+k}$	$k=k_{AC}$ for top story $k=2k_{AC}$ for ground and intermediate story
Leeward Composite Beam	$\phi^{-}=\dfrac{3(k22+p^{-}k21)}{2+p^{-}}$	$p^{-}=\dfrac{k22-k12+k}{k11+k22+k}$	

Table 3 ϕ and β for 4-story Frame

Story	ϕ Windward	ϕ Interior	ϕ Leeward	β $(I+/sI)$
4 , Top(R)	1.56	1.38	1.20	1.75
Ground(2) , 3	1.83	1.57	1.31	2.11

Table 4 ϕ and β for 7-story Frame

Story		ϕ Windword	ϕ Interior	ϕ Leeward	β $(I+/sI)$
Top	R	1.46	1.30	1.15	1.60
Intermediate	6 , 7	1.45	1.30	1.16	1.60
	4 , 5	1.63	1.43	1.23	1.84
	3	1.81	1.55	1.29	2.08
Ground	2	1.80	1.55	1.29	2.08

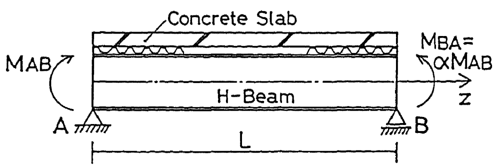


Fig. 1 Analytical Model of a Composite Beam

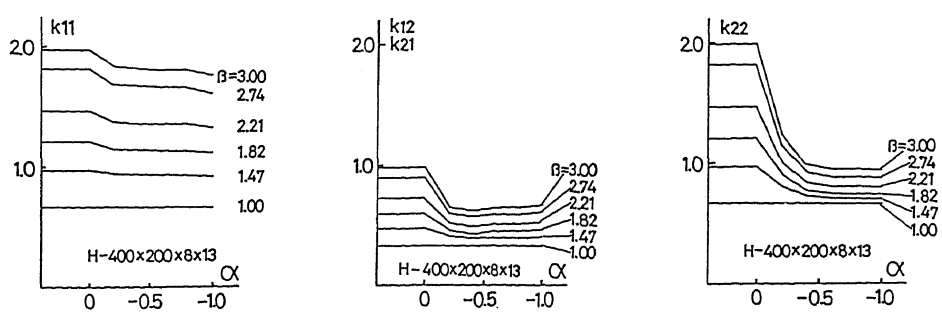


Fig. 2 Flexural Stiffness Elements of Composite Beams

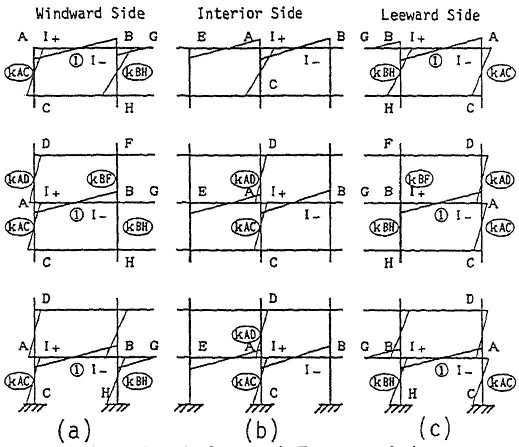


Fig. 3 Balanced Frame with Composite Beams

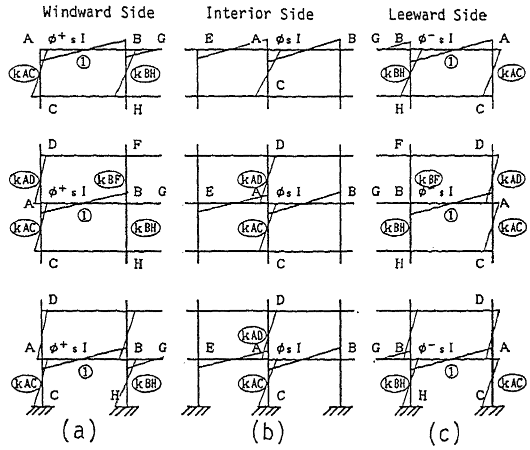
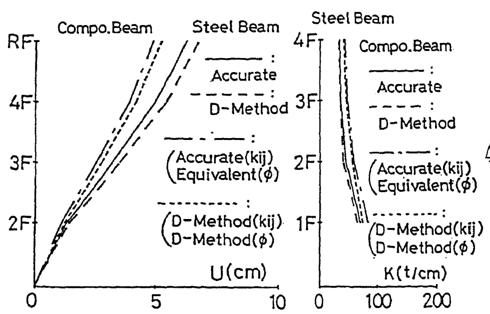
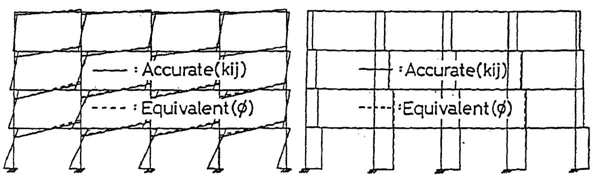


Fig. 4 Balanced Frame with Equivalent Steel Beams



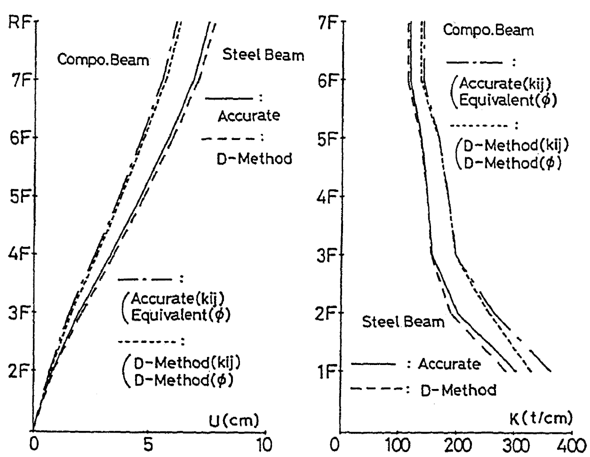
(a) Story Displacement (b) Story Stiffness

Fig. 5 Analytical Results of 4-Story Frame



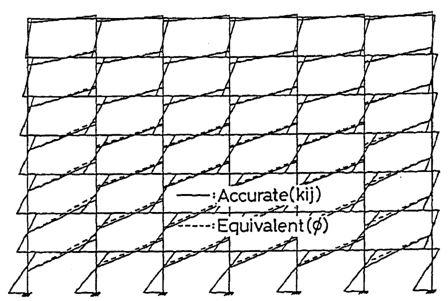
(a) Bending Moment (b) Shear Force

Fig. 7 4-Story Frame

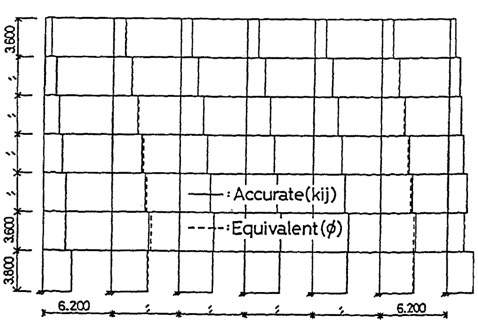


(a) Story Displacement (b) Story Stiffness

Fig. 6 Analytical Results of 7-Story Frame



(a) Bending Moment



(b) Shear Force

Fig. 8 7-Story Frame