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APPLICATION OF RESPONSE SPECTRUM METHOD TO A BRIDGE SUBJECTED TO MULTIPLE SUPPORT EXCITATION

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ABSTRACT

The earthquake behaviour of a four-span continuous girder railway bridge subjected to multiple support excitations is investigated using the response spectrum method. Small-amplitude oscillations and linear-elastic material behaviour are assumed. Soil-structure interaction effects are disregarded and only the out-of-plane response of the bridge is considered. The results of the response spectrum analysis are compared with those from a time history analysis. Different combination rules for the superposition of modal maxima as well as supports are employed, such as square-root-of-sum-squares, double sum and p-norm methods.

INTRODUCTION

In the earthquake analysis of structures it is usually assumed that the ground motion is the same at all supports. However, this assumption is not justified for long structures like bridges, because observations have shown that the earthquake ground motion can vary considerably within relatively small distances. It is also clear, that non-uniform support movements can cause quasi-static distortions and secondary forces in statically indeterminate bridges and that they may have an important influence on the dynamic response.

The earthquake analysis of structures can be carried out economically by the response spectrum method, however, for the analysis of the dynamic response due to multiple support excitation, wind loading on structures, etc., the conventional response spectrum method cannot be used. Therefore, the objective of this paper is to investigate the applicability of a modified response spectrum method for bridges subjected to non-uniform support excitations. Multiple support excitation problems can be analysed accurately in the time domain, however, a response spectrum analysis is preferred, firstly, because of its computational economy and, secondly, because the earthquake ground motion is normally defined in terms of a response spectrum rather than an accelerogram. Also, the selection of adequate support accelerograms at a given bridge site is often a rather difficult task.

METHODOLOGY

Assumptions and structural model A four-span prestressed concrete box girder bridge, as shown in Fig. 1, has been chosen for the dynamic analysis. In the present study only the out-of-plane response of the bridge is considered. The lowest eigenfrequency of long continuous girder bridges supported by slender columns is generally associated with an out-of-plane mode of vibration. In addition, bridge structures are relatively weak in transverse direction, because the lateral loads (wind, earthquake) are much smaller than the vertical ones due to dead load and traffic. Therefore, an SH-wave propagating along the bridge axis is expected to cause the most critical earthquake effect on the superstructure. The bridge is modelled by finite elements (beam elements) with the element masses concentrated in the nodal points. Soil-structure interaction effects are disregarded. Small-amplitude oscillations and linear-elastic material behaviour are assumed. Proportional damping is assumed with a damping ratio of 2% for all modes of vibration.

Equations of motion The equations of motion of a multi-degree-of-freedom oscillator with s supports, each of them subjected to a different excitation, can be taken as follows [2,3,4]:

$$\underline{M}\,\underline{\ddot{u}} + \underline{C}\,\underline{\dot{u}} + \underline{K}\,\underline{u} = -\,\underline{M}\sum_{i=1}^{s}\underline{r}_i\,\ddot{u}_{gi}(t) \qquad (1)$$

where $\underline{M}$, $\underline{C}$ and $\underline{K}$ are the mass, damping and stiffness matrices respectively; $\ddot{u}_{gi}(t)$ is the ground acceleration of the i-th support; and $\underline{u}$ is a vector of dynamic nodal point displacements relative to the pseudo-static displacements ($\underline{u}_s$) caused in the bridge due to support movements, i.e. the total displacement can be expressed as:

$$\underline{u}_t(t) = \underline{u}(t) + \underline{u}_s(t) = \underline{u}(t) + \sum_{i=1}^{s}\underline{r}_i\,u_{gi}(t) \qquad (2)$$

in which the influence vector $\underline{r}_i$ is identical to the displacement of the bridge due to a unit movement of the i-th bridge support $u_{gi} = 1$. Once the displacement time history is known, the element forces can be calculated. In accordance with eq. (2), they contain a dynamic and a pseudo-static component. In the subsequent part, however, only the dynamic effect is taken into account. For example, in the case of a uniform excitation of all supports the pseudo-static response is identical to a rigid body displacement, which does not produce any member forces.

Modelling of earthquake ground motion The peak ground acceleration, velocity and displacement at the site were estimated as 0.3 g, 17.7 cm/s and 13.9 cm respectively. Two smooth design response spectra after Newmark and Hall were constructed, using mean and mean plus one-sigma response spectrum amplification factors respectively and a damping ratio of 2% [5]. For all five supports, different response spectra were then constructed in such a way that the individual spectra lay between the mean and mean plus one-sigma spectra, as shown in Fig. 2. For the time history analysis independent spectrum-compatible accelerograms were generated artifically for each support, each of them with a peak acceleration of 0.3 g (Fig. 3). It should be pointed out that the acceleration time history affects the dynamic response, eq. (1), whereas the displacement time history governs the pseudo-static response, eq. (2).

Dynamic analysis Two types of earthquake analyses were carried out using the computer program SAPIV [1]:

(i) exact time history analysis for multiple support excitation; and
(ii) multiple support response spectrum analysis.

Only the dynamic response due to out-of-plane excitations was investigated. The finite element model of the bridge comprised 44 dynamic degrees of freedom and 6 modes were found to be appropriate for the response spectrum analysis.

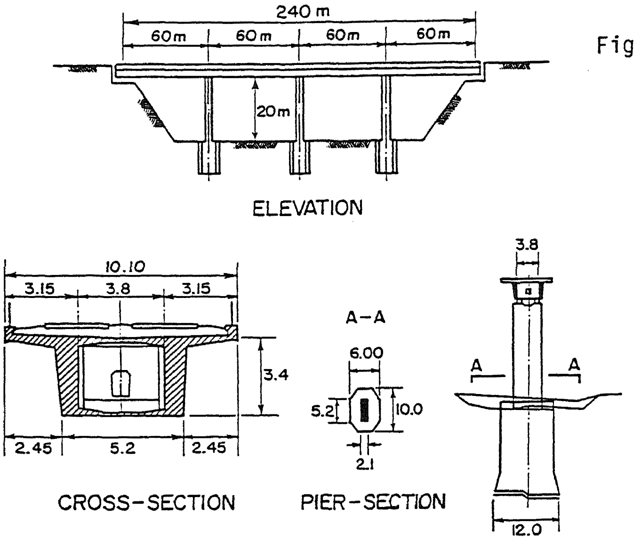

Fig. 1 Four-span continuous girder railway bridge.

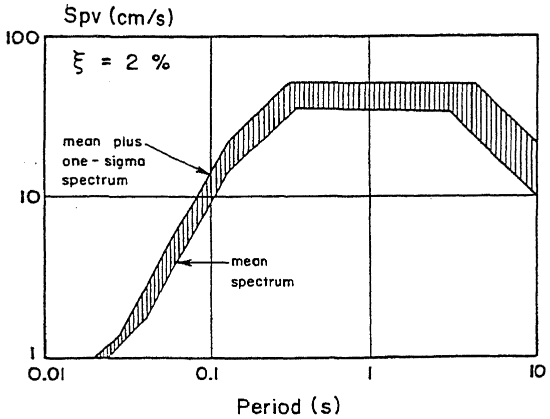

Fig. 2 Envelopes of support design response spectra for 2% damping [5].

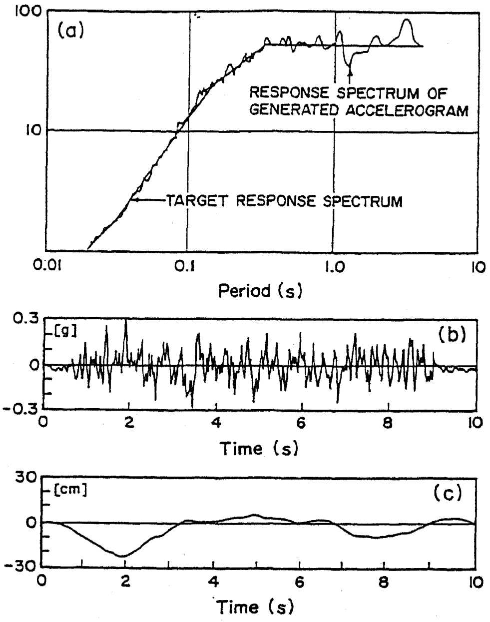

Fig. 3 Example of spectrum-compatible support movement (a: response spectra; b: accelerogram; c: ground displacement).

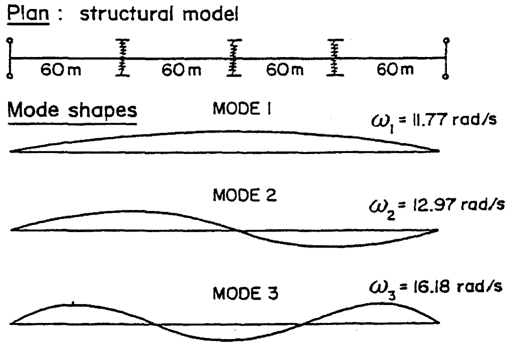

Fig. 4 Out-of-plane modes of bridge.

The eigenfrequencies of the first six out-of-plane modes obtained from an eigenvalue analysis vary between 1.87 to 7.16 Hz (Fig. 4).

Response spectrum analysis: The modal maxima obtained by the different response spectra of all bridge supports can be combined as follows:

(i) Square-root-of-sum-squares rule (SRSS)

$$R_a = [\sum_{j=1}^{M} R_j^2]^{1/2} \qquad (3)$$

where R_a and R_j are the total response and the maximum response of mode j respectively, and M is the number of modes.

(ii) P-norm rule (PN)

$$R_a = [\sum_{j=1}^{M} |R_j|^p]^{1/p} \qquad (p \geq 1) \qquad (4)$$

(iii) Double sum rule (DS)

$$R_a = [\sum_{k=1}^{M} \sum_{\ell=1}^{M} E_{k\ell} R_k R_\ell]^{1/2} \qquad (5)$$

with

$$E_{k\ell} = \{1 + [\frac{\omega'_\ell - \omega'_k}{\xi'_\ell \omega_\ell + \xi'_k \omega_k}]^2\}^{-1}$$

$$\omega'_k = \omega_k[1 - \xi_k^2]$$

$$\xi'_k = \xi_k + 2/(t_d \omega_k)$$

where ω_k and ξ_k are the circular frequency and damping ratio of the k-th mode respectively, and t_d is the duration of the earthquake.

Different rules can be used, firstly, to combine the modal maxima due to the movement of one particular support and, secondly, to combine the effects of all supports. Modal maxima and support excitations are assumed to be statistically independent in the case of the SRSS and PN methods. The DS rule, however, takes into account correlation between modes.

DISCUSSION OF RESULTS

The bridge shown in Fig. 1 was analyzed by two methods, first, by a time history analysis using independent spectrum-compatible accelerograms and, second, by the response spectrum method. The results of the exact time history analysis for the two cases of non-uniform and uniform (accelerogram according to mean plus one-sigma spectrum) support excitation are shown in Fig. 5. For the case of non-uniform support excitation, using the response spectrum method, the following combination rules were compared:

- SRSS(A): SRSS for combination of supports and modes;
- DSC(A) : SRSS for combination of supports and DS for modes;
- PN1(A) : PN (p = 1.8) for combination of supports and modes; and
- PN2(A) : PN (p = 2.2) for combination of supports and modes.

In addition, for the case of a uniform ground excitation with mean plus one-sigma spectrum (Fig. 2), the total response using the SRSS method has been calculated. The comparison of the results of these 5 different methods with the exact time history solution for non-uniform support excitation are shown in Fig. 6. We can note from this figure and Table 1 that considerable deviations occur. The exact response can either be overestimated or underestimated. For design purposes, however, a method is required which exceeds the exact response. We can also notice from Figs. 5 and 6 that the response due to a uniform excitation

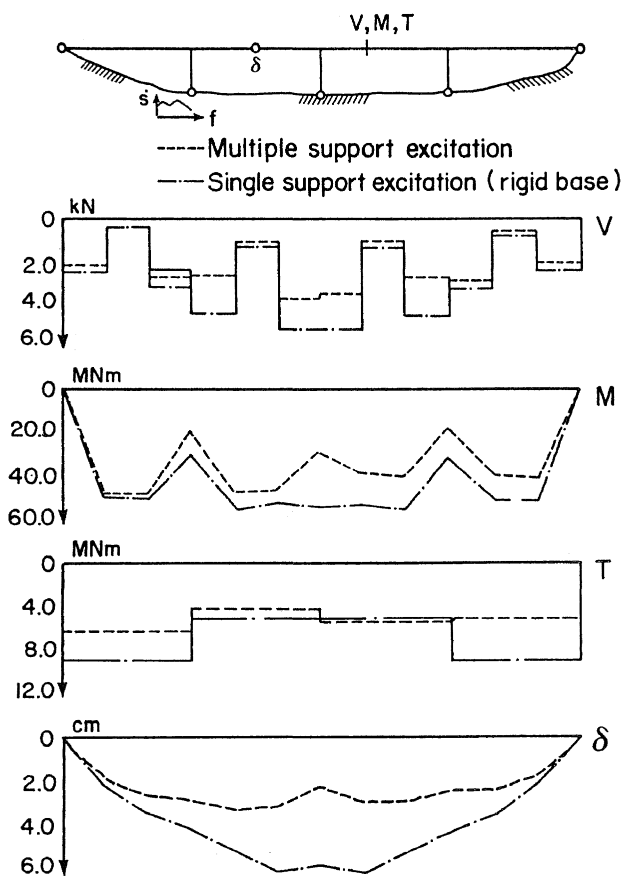

Fig. 5 Results of time history analysis with non-uniform and uniform support excitations (V: shear; M: bending moment; T: torsional moment; δ: out-of-plane displacement)

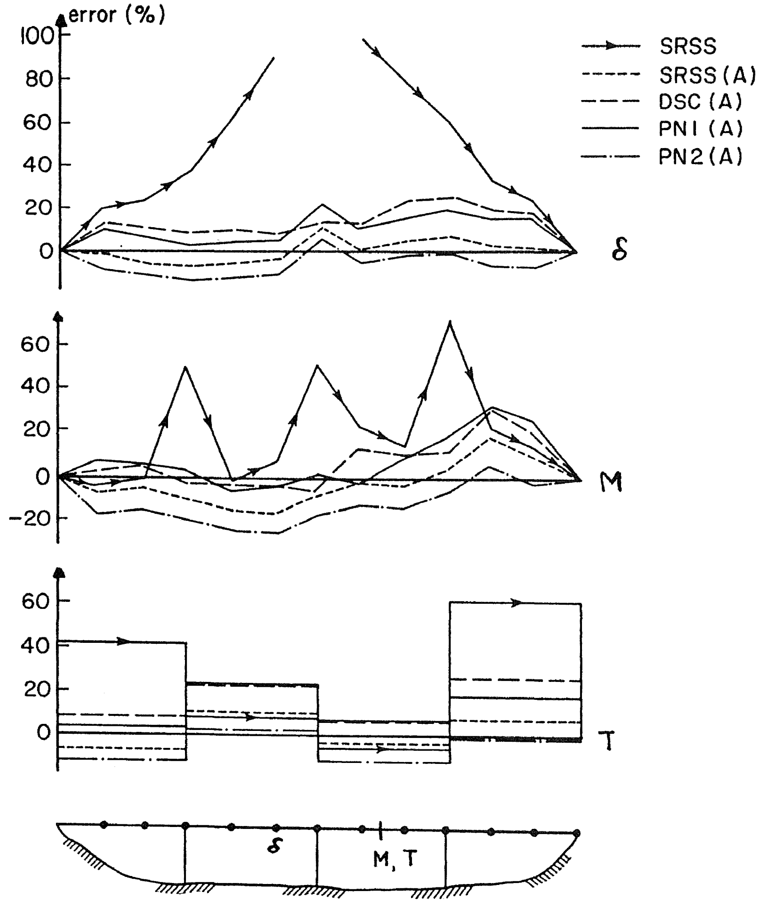

Fig. 6 Comparison of results of response spectrum analysis with exact solution (M: bending moment; T: torsional moment; δ: out-of-plane displacement).

deviates considerably from that due to multiple support excitation. It must be added, that the pseudo-static response cannot be dealt with in a response spectrum analysis. Such effects must be analyzed separately and then superimposed with the dynamic response.

Table 1: Absolute maximum relative errors of different combination rules (error in % of maximum time history response)

Description	SRSS(A)	DSC(A)	PN1(A)	PN2(A)	SRSS
Deflection	13.0	26.5	22.5	14.0	141.1
Shear force	19.1	21.3	34.2	17.1	64.2
Bending moment	18.8	30.3	34.7	24.1	73.3
Torsional moment	10.2	26.1	23.4	13.9	62.2

CONCLUSIONS

1. Considerably smaller absolute maximum dynamic responses (bending, shear, torsion, deflection) are predicted for a bridge subjected to non-uniform support excitation than for uniform support excitation.
2. The square-root-of-sum-squares, double sum and p-norm combination rules provide acceptable results for multiple support excitation, however, there are limitations with respect to accuracy.
3. For a conservative design a combination method has to be selected in which the actual response is exceeded.
4. The maximum dynamic response of a continuous girder brige with uniform excitation can be predicted with greater accuracy using the response spectrum method than in the case of multiple support excitation.

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