SEISMIC SHEAR FORCES
IN RC CANTILEVER SHEAR WALLS

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SUMMARY

In extensive parametric studies seismic shear forces in RC yielding shear walls are investigated by means of nonlinear time history analyses. From an attempt to explain the results of these calculations the concept of modal limit forces is developed. Using this concept an approximate relation is deduced for the evaluation of shear forces in yielding cantilever shear walls by response spectrum modal analysis. A simplified formula is also given, considering only the fundamental mode explicitly, and the second mode implicitly.

INTRODUCTION

First investigations of seismic shear forces in RC yielding shear walls in (Ref.1) have led to magnification factors for shear forces due to design seismic loads, with the number of stories as parameter, introduced as well in several national codes as in (Ref.2). However, as pointed out in (Ref.2), these magnification factors are based on a limited number of cases and could be modified as further studies became available. Investigations in (Ref.3) have shown some inadequacies of these factors: so that the number of stories, admitted as parameter, is less suitable for this purpose as would be the fundamental period of the structure, and also that shear forces do not increase proportionally with the yielding moment, as supposed, but much weaker. In the present paper the investigations of (Ref.3), concerning seismic shear forces in multistory wall structures, are extended to structures with 2 to 5 stories. Subsequently, from an attempt to interpret the results of time history calculations a concept for an approximate evaluation of shear forces in yielding shear walls by response spectrum analysis is developed.

PARAMETRIC INVESTIGATIONS

Structural model Shear wall structures are reduced for analysis, as shown in Fig. 1a, to cantilevers with \( n = 2, 3, 4 \) and 5 lumped story masses \( m \) at equal distances on the height \( h \). The yielding moment of a cantilever is given, as in (Ref.3), by the relation

\[
M_y = cM_1 g / S_0(1) 
\]

with

\[
(1) 
\]
Fig. 1  Structural characteristics. a) Structural system, b) Yielding levels, c) Design spectrum

- yielding level factor;
- design value of the acceleration response spectrum for the fundamental vibrational period $T_1$ of the structure;
- overturning moment at the base of the structure, due to the design seismic load, corresponding to the fundamental vibrational mode;
- acceleration of gravity.

For a system with $n = 1$, having the yielding force $F_y$ and the yielding moment $M_y = hF_y$, Eq. (1) turns into $F_y = cmg$. For each number of story masses $4$ different yielding levels ($3$ values of $c$ in Fig. 1b and for comparison elastic behaviour), $4$ values of $T_1$ ($0.2$ s, $0.4$ s, $0.6$ s and $1.6$ s) and $5$ values of the index $\alpha = EL/GAh^2$ ($0.00$, $0.25$, $0.50$, $0.75$ and $1.00$) with $E$, $G$ flexural respshear stiffness are considered. The yielding level $2$ corresponds to the design forces of the German Seismic Code DIN 4149 with the design spectrum shape of Fig. 1c, $T^* = 0.45$ s, $k = 0.8$ and $\text{max } S_a = 0.11 \text{ g}$ for the MSK intensity $I = 8$. For response spectrum calculations the damping ratio $D = 0.05$ is admitted, and for time history calculations Rayleigh damping with the first two damping ratios $D_1 = D_2 = 0.05$. Conservatively only flexural yielding at the base, and not also shear yielding, is considered. For the case of elastic behaviour under seismic loading the base shear, belonging to the vibrational mode $i$ with the period $T_i$, may be expressed as

$$ Q_{10} = \epsilon_i n m S_a(T_i), $$

where $\epsilon_i$ is an equivalent mass factor and $S_a(T_i)$ the elastic acceleration response spectrum value.

Earthquake excitation  The $320$ structural variants, defined, as shown before, by $n$, $c$, $T_1$ and $\alpha$, are analysed, similarly as the systems considered in (Ref.3), under the acceleration time histories of a set of $10$ strong motion records, belonging to $5$ real earthquakes: San Francisco Golden Gate Park 1957. 3.22 (GG, $I = 7$, $M = 5.3$), Helena (Montana) 1935. 10.31 (HE, $I = 8$, $M = 6.0$), Hollister (Northern California) 1949. 3.9 (NC, $I = 7$, $M = 5.2$), Ferndale (California) 1975. 6.7 (FE, $I = 7$, $M = 5.7$) and Oroville (California) 1975. 8.1 (OR, $I = 7$, $M = 5.8$). In order to facilitate the observation of the influences of higher modes, the earthquake have been chosen in the range of relative low magnitudes, with predominant higher frequencies of the ground shaking. In Fig. 2a acceleration response spectra of the stronger components of each earthquake time history, scaled for the MSK intensity $I = 8$, are represented. Fig. 2b shows mean values of the flexural moment at the base in the elastic range $M_0$, related to $M_y$, and Fig. 2c mean values of displacement ductility factors $\mu$ for SDF systems ($n = 1$).

Results  Examples of results, represented as ratio between the mean value $Q$ of the base shear, computed by nonlinear time history analysis, and the design value

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Fig. 2  Seismic excitation. a) Response spectra, b) Flexural moments at the base, c) Displacement ductility factors $\mu$

$Q_1$ of the German Seismic Code DIN 4149 (yielding level 2), corresponding to the fundamental vibrational mode of a structure with $\alpha = 0$, are represented in Fig. 3. The similarity between the shapes of the curves, belonging to inelastic and to elastic behaviour, is noticed. The peculiarities of the curves corresponding to elastic behaviour can be explained by means of the characteristics of higher modes in the modal analysis, shown in Fig. 4. So the increase of $Q/Q_1$ with $n$ (Fig. 3e, f, g, h), important especially for $\alpha = 0$ and $T_1 = 1.6$ s, is explained.

Fig. 3  Influences on $Q/Q_1$. a), b) Influence of $T_1$, c), d) Influence of $\alpha$, e), f) Influence of $n$
by the increase of $\varepsilon_2/\varepsilon_1$ (Fig. 4a). Further the increase of $T_2/T_1$ with $\alpha$ (Fig. 4b) leads for high values of $T_1$ to a decrease of $Q_{20}$, because $T_2$ reaches in the region of the descending branch of the acceleration response spectrum curve and $\varepsilon_2$ decreases also. Thus the increase of $Q/Q_1$ with $T_1$ due to $Q_{20}$, observed for $\alpha = 0$, is not found for $\alpha = 0.25$ (Fig. 3a, b). However for low values of $T_1$ the period $T_2$ remains in the region of the ascending branch of $S_a(T)$ and $Q_{20}$ increases with $\alpha$. Thus the increase of $\alpha$ leads for low values of $T_1$ to an increase and for high values of $T_1$ to a decrease of $Q/Q_1$ (Fig. 3c, d).

A comparison of the values $Q/Q_1$ in Fig. 3, obtained for elastic and for inelastic systems, shows that the influence of higher modes on the shear force of elastic systems is found qualitatively also in the case of inelastic systems. Even niceties as the influences of $n$ and of $\alpha$ on the parameters of higher modes $(T_2/T_1, \varepsilon_2/\varepsilon_1)$ can be observed in the diagrams of $Q/Q_1$ for inelastic systems. This leads to the idea to approximate the shear forces in inelastic shear walls by the response spectrum modal analysis. However, an increased influence of higher modes in the case of inelastic structures, found already in (Ref.1), is noticed. So the increase of $Q/Q_1$ between $n = 1$ and $n = 2$, due to the intervention of the second mode for $n = 2$, is more pronounced for inelastic than for elastic systems (Fig. 3e, f, g, h). Similar phenomena are stated in connection with the influence of $\alpha$ on $Q/Q_1$. As shown, $Q/Q_1$ does not increase with $T_1$ for inelastic systems with $\alpha = 0.25$, $Q_{20}$ being relative low for $\alpha > 0$, (Fig. 3b and Fig. 4b). However for inelastic systems with $\alpha = 0.25$ even the low values of $Q_{20}$ are able to produce an increase of $Q/Q_1$ (Fig. 3b). The consideration of the increased influence of higher modes in the case of inelastic structures implies some modifications in the response spectrum method.

**APPROXIMATE RELATIONS**

In order to adapt the response spectrum modal analysis to yielding structures, the concept of modal limit forces (modal forces, limited by yielding conditions, imposed for each mode separately) is developed. Modal limit forces represent the maximum limit values that modal forces in an elastoplastic structure could attain, if only one certain vibrational mode $i$ would be excited. So the modal limit shear force $Q_{iy}$ at the base of the system in Fig. 1a, belonging to mode $i$, is defined by the relations

$$Q_{iy} \leq Q_i M_y/M_i \quad \text{and} \quad Q_{iy} \leq Q_{io}.$$  \hspace{1cm} (3)
Fig. 5  Response spectrum versus time history analysis. a), b), c), d) Comparison of calculation results, e), f) Correction factors \( \gamma \)

where \( Q_i \) and \( M_i \) are design values of the shear force and the bending moment in mode \( i \), \( Q_{10} \) is the shear force of the elastic system in mode \( i \), and \( M_y \) the yielding moment of the wall structure. It is proposed to approximate the shear force at the base of the inelastic system by the SRSS relation

\[
Q = \gamma Q^* = \gamma \sqrt{\sum_{i=1}^{n} Q_{iy}^2},
\]

(4)

where \( \gamma \) is a correction factor. \( Q^* \) corresponds to a response spectrum modal analysis, in which the modal forces are considered separately until yielding.

As usually the first condition (3) is decisive for the fundamental mode, but the second condition (3) for higher modes, the form of the relation (4) explains qualitatively the increased importance of higher modes and the relative weak increase of \( Q \) with \( M_y \) (Fig. 1b and Fig. 3). Quantitative aspects are shown in Fig. 5a, b, c, d, where mean values \( Q^*/Q \) and coefficients of variation \( V \) of the ratio between shear forces \( Q^* \), calculated by Eq. (4), and shear forces \( Q \),

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calculated by nonlinear time history analysis, are represented. For elastic systems and \( \gamma = 1 \) the proposed procedure becomes identical with the usual response spectrum modal analysis in connection with the SRSS formula. For inelastic systems it leads to results of an exactitude comparable with that obtained for elastic systems. Only for yielding level 1 systems, with extremely high plastic deformations, unacceptable in practice (Fig. 2b, c), the exactitude is worse. In order to render the approximate analysis results for inelastic systems at least as conservative as those calculated for elastic systems, a correction factor

\[
\gamma = \frac{(Q^*/Q)_0}{1 + \frac{V}{Q}}
\]

(5)

is introduced, where \( Q^*/Q \) and \( V \) correspond to inelastic and \((Q^*/Q)_0\) as well as \( V_0 \) to elastic systems. In Fig. 5e, f calculated values \( \gamma \) are represented together with the approximate expression

\[
\gamma = 1 + 0.1 (M_0/M_y - 1) \equiv 1.
\]

(6)

In code design, according to usual assumptions, may be considered \( \gamma = 1 \).

For the case of simplified dynamic analysis, taking into account only the fundamental mode, a simplified relation is given, introducing the second mode implicitly. As on the one hand the simplified dynamic analysis is usually allowed to be applied only for systems with \( T_1 \leq 2 T^* \) (Fig. 1c), but on the other hand the second period \( T_2 \) amounts approximately \( T_1/6 \), for the simplified design spectrum of Fig. 1c is found always \( S_{ad}(T_2) = \max S_{ad} \). Than the seismic shear force at the base can be approximated as \( Q = \omega Q_I \), where

\[
\omega = K \sqrt{(M_y/KM_I)^2 + 0.1 (\max S_{ad}/S_{ad}(T_1))} \equiv K,
\]

(7)

\( Q_I \) and \( M_I \) are design values of the shear force and the bending moment, according to the simplified dynamic analysis, and \( K \) is the behaviour factor, introduced in (Ref.2) and (Ref.4). The fact that \( K \) appears only in the first term under the square root denotes that only the contribution of the fundamental mode is considered reduced by yielding. Hence the increased importance of the second mode (represented by the second term) at yielding systems.

CONCLUSIONS

Interpreting the results of nonlinear time history calculations, an approximate evaluation of seismic shear forces in RC yielding shear walls is developed. The proposed relations have been introduced in the draft EUROCODE 8 (Ref.4).

REFERENCES