



10-2-8

## RANDOM SEISMIC RESPONSE ANALYSIS OF SOIL CABLE-STAYED BRIDGE INTERACTION

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### SUMMARY

This paper presents a random seismic response analysis of a soil-foundation cable-stayed bridge, which has complicated vibrational characteristics because of connections of a main girder, cables and towers. The dynamic characteristics of a soil-foundation system are represented by an impedance model which can be obtained through the finite element method. The governing equation of the cable-stayed bridge system is derived using the substructure method. The dynamic response characteristics, which are caused by dynamic soil-structure interaction and random variable effects on soil, are examined by the random vibration approach. It is shown that these effects have significant roles on dynamic response evaluations of a cable-stayed bridge system.

### INTRODUCTION

Since the cable-stayed bridge has complicated vibrational systems due to different structural properties, it is important to examine the dynamic response characteristics which include the dynamic soil-structure interaction (Refs.1,4,5). It seems that the dynamic soil-structure interaction has significant roles on the dynamic response evaluation for the cable-stayed bridge with a main girder of a few hundred meters in length. It is also necessary to examine dynamic soil effects with a random variable because the dynamic soil has generally very complicated ones.

The direct application of the finite element to the total system including a soil-foundation system seems to be difficult because of the extreme increase in the degrees-of-freedom of the governing equation. The substructure method is very effective for the dynamic soil-structure interaction analysis (Refs.2,3). The substructure method is based on an independent formulation of the superstructure and the soil-foundation system, and on combining them at their interface continuity and equilibrium conditions. Then, soil-foundation system can be represented by the simplified vibrational model due to impedance functions. Since the superstructure has generally dominating vibrational modes for the response, the governing equation of the total system can be represented with the degrees-of-freedom equation of very small scale.

In this study, in order to clarify the dynamic soil-structure interaction effects on the responses of a cable-stayed bridge, a seismic response analysis is carried out using the random vibration approach by the application of the substructure method. Further, the random variable effects on the response

evaluations are examined with a perturbation method and a spectral approach.

#### FORMULATION

The Governing Equation Motion For the cable-stayed bridge as shown in Fig.1, the superstructure can be represented with an assemblage of beam elements by the application of the finite element method(Ref.4). While the displacement of each node of an element represents 6-components, the dynamic response in plane including the main girder is given by 3-components of these displacements. The dynamic response is represented with the relative displacement due to an inertia force, and with the pseudo-static displacement which is caused by the motion of the soil-foundation system. Neglecting the damping terms in the external forces, the equation of motion for the relative displacement is expressed with

$$[M_{a,a}] \{\ddot{u}_a\} + [C_{a,a}] \{\dot{u}_a\} + [K_{a,a}] \{u_a\} = -([M_{a,a}] [L] + [M_{a,b}]) \{\ddot{u}_b\} \quad (1)$$

in which  $[L] = -[K_{a,b}] [K_{b,b}]^{-1}$  and the subscript 'b' denotes the restrained displacement components and corresponds to the connecting components between the superstructure and a soil-foundation system. The subscript 'a' also denotes the unrestrained displacement, which consists of the dynamic displacement and pseudo-static displacement. Using the eigenvalue analysis for the undamped vibrational situation of Eq.(1), and applying the transformation due to the modal matrix which consists of dominating vibrational modes, Eq.(1) can be expressed by

$$\{\ddot{q}\} + [2 \beta_j \omega_j] \{\dot{q}\} + [\omega_j^2] \{q\} = -[\Phi]^T ([M_{a,a}] [L] + [M_{a,b}]) \{\ddot{u}_b\} \quad (2)$$

in which  $\{u\} = [\Phi] \{q\}$

and  $\beta_j$  and  $\omega_j$  denote the critical damping ratio and natural frequency of the jth mode, respectively.

On the other hand, the dynamic characteristics of soil-foundation system depend upon the soil condition and aspect ratio of the foundation. Using the simplified vibrational model due to the impedance functions, the equation of motion of a soil-foundation is expressed by

$$[M_b] \{\ddot{u}_b\} + [C_b] \{\dot{u}_b\} + [K_b] \{u_b\} = - [M_b] \{\ddot{x}_o\} - \{R\} \quad (3)$$

in which  $\{u_b\}$  is the swaying and rocking components of the gravitational center of each foundation, and  $\{\ddot{x}_o\}$  denotes the input seismic motion. The interaction force working on the foundation from the superstructure is represented by  $\{R\}$  which is obtained by the equation motion of the superstructure. Using the equilibrium conditions of the interaction force and the continuous conditions of the displacements are satisfied at the connecting node, the governing equation of motion of the total system is expressed by

$$[M_s] \{\ddot{y}_s\} + [C_s] \{\dot{y}_s\} + [K_s] \{y_s\} = \{F_s\} \ddot{x}_o \quad (4)$$

in which

$$[M_s] = \begin{bmatrix} [I] & [M_{s,p}] \\ [M_{p,s}] & [\tilde{M}_p] \end{bmatrix} \quad [C_s] = \begin{bmatrix} [2 \beta_j \omega_j] & [0] \\ [0] & [\tilde{C}_p] \end{bmatrix} \quad [K_s] = \begin{bmatrix} [\omega_j^2] & [0] \\ [0] & [\tilde{K}_p] \end{bmatrix}$$

$$\{F_s\} = - \begin{bmatrix} [M_{s,p}] \{1\} \\ [M_p] \{1\} \end{bmatrix} \quad \begin{aligned} [\tilde{K}_p] &= [K_p] + [G]^T ([K_{b,b}] + [K_{b,a}] [L]) [G] \\ [\tilde{C}_p] &= [C_p] + [G]^T ([C_{b,b}] + [C_{b,a}] [L]) [G] \end{aligned}$$

$$[\tilde{M}_p] = [M_p] + [G]^T ([M_{bb}] + [L]^T [M_{ab}] + [M_{ba}] [L] + [L]^T [M_{aa}] [L]) [G]$$

$$[M_{ap}] = ([M_{ab}] + [M_{ba}] [L]) [G]$$

and  $[G]$  denotes the connecting matrix of the displacements between the restrained nodes of the superstructure and the gravity center of a foundation.

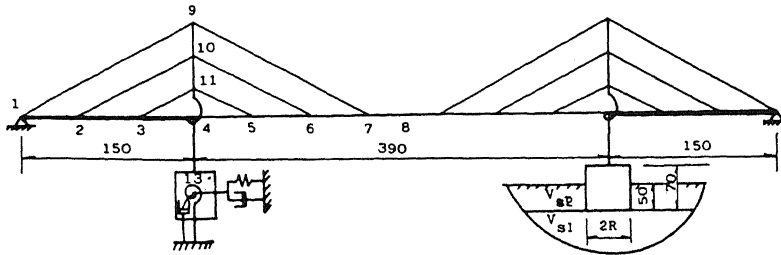


Fig.1 Analytical Model of Cable-Stayed Bridge

Random Variable Effects on the Governing Equation of Motion Since the dynamic characteristics of soil have complicated ones, it is suggested that these effects can be evaluated with the random variables for the dynamic soil-structure interaction(Ref.6). Assuming the random variables have comparatively small variations, a perturbation method provides an effective tool for the dynamic response analysis. Then, each term of Eq.(4) can be expressed by

$$[C_s] = [C^0_s] + \varepsilon_1 [C^1_s] + \varepsilon_2 [C^2_s]$$

$$[K_s] = [K^0_s] + \varepsilon_1 [K^1_s] + \varepsilon_2 [K^2_s] \quad (5)$$

$$\{y\} = \{y^0\} + \varepsilon_1 \{y^1\} + \varepsilon_2 \{y^2\}$$

in which parameter  $\varepsilon_1, \varepsilon_2$  denote random variables with a zero mean. Substituting Eq.(5) into Eq.(4), and rearranging the equation for each parameter, the governing equation can be expressed by

$$[M^0_s] \{\ddot{y}^0\} + [C^0_s] \{\dot{y}^0\} + [K^0_s] \{y^0\} = \{F^0_s\} \ddot{x}_0$$

$$[M^0_s] \{\ddot{y}^1\} + [C^0_s] \{\dot{y}^1\} + [K^0_s] \{y^1\} = -[C^1_s] \{\dot{y}^0\} - [K^1_s] \{y^0\} \quad (6)$$

$$[M^0_s] \{\ddot{y}^2\} + [C^0_s] \{\dot{y}^2\} + [K^0_s] \{y^2\} = -[C^2_s] \{\dot{y}^0\} - [K^2_s] \{y^0\}$$

The Random Seismic Response Analysis Since the governing equation of motion of the total system is given by degrees-of-freedom equation of small scale, Eq.(4) can be solved using the complex eigenvalue analysis exactly. Moreover, applying the eigenvalue analysis in the situations of the undamped system, Eq.(4) can be transformed into the more convenient form. While Eq.(4) has generally a nonproportional damping matrix, a more available equation can be obtained through a diagonalization of the damping matrix. Now giving the power spectral density function of input seismic motion, the dynamic responses for a stationary situation can be determined with the spectral approach. Using the power spectral density functions of Tajimi type, the power spectral density of the response can be obtained by

$$[S_{yy}(\omega)] = [\Phi] [H(\omega)] \{f_s\} \{f_s\}^T [H(\omega)]^T [\Phi]^T S_0(\omega) \quad (7)$$

in which

$$[H(\omega)] = [\omega_j^2 - \omega^2 + 2i\omega \beta_j \omega_j]$$

$$\{f_s\} = [\Psi] \{F_s\}$$

$$S_s(\omega) = (1 + 4h_s^2(\omega/\omega_s)^2)S_0 / ((1 - (\omega/\omega_s)^2)^2 + 4h_s^2(\omega/\omega_s)^2)$$

and  $[\Psi]$  is the modal matrix of Eq.(4), and  $S_0$  the intensity of input motion.  $h_s$  and  $\omega_s$  are characteristic values of surface soil subjected to the input seismic motion. Then, autocovariance function can be determined through the inverse Fourier transformation. Further, noticing the same form of equation as expressed in Eq.(6), the power spectral density function for  $\{y^1\}$  and  $\{y^2\}$  can be obtained by the same procedure. Further, random variables effects on the dynamic responses can be evaluated with these results.

## RESULTS AND DISCUSSIONS

Fig.1 shows a cable-stayed bridge model with the center span girder of a length 390m, which is made of steel members, and with side span girders made of a prestressed concrete. The height of the tower is 80m and the caisson foundation is rested upon a lower stratum of soil and the depth of its side stratum is 50m. This soil-foundation system is represented with an impedance model as shown in Fig.1. The ratio of the shear wave velocity for the surface and lower stratum is used as parameters for the dynamic soil-structure interaction analysis. The first 10 vibrational modes of the superstructure are used for the dynamic response analysis.

Fig.2 shows the natural frequencies on the variations of the shear wave velocity of the lower stratum from 200m/s to 600m/s. This result shows that the closely spaced natural frequencies between sixth and eighth are caused in the situations of several different ratios of shear wave velocity. This means that the dynamic soil-structure interaction has important contributions on the responses which are dominated by these frequencies. Then, determining the vibrational modes, which provide significant roles on the dynamic response, is important for the dynamic soil-structure interaction analysis.

Fig.3 shows the rms displacement response of a girder for several different foundations subjected to a white noise excitation. The displacements of each nodal point are normalized by the displacements with respect to rigidly supported base conditions. The dynamic soil-structure interaction effects on the responses due to the changes of its foundation radius gives small contributions on the various soil conditions. While the increase of the shear wave velocity of the lower stratum provides a decreasing effects of dynamic soil-structure interaction, the modal coupling effects by the closely spaced natural frequencies give a bit of increase of the response. The dynamic soil-structure interaction seems to give a bit of contributions on the dynamic response evaluation.

Fig.4 shows the dynamic response of the bending moments of each nodal point to a white noise excitation. The responses with respect to the dynamic soil-structure interaction are normalized with the results which correspond to rigidly supported base conditions. The change of foundation radius gives a bit of contributions for the dynamic response evaluation. The dynamic soil-structure interaction has a bit of effects at the center span girder. While the dynamic soil-structure interaction effects on the responses have a decrease for the increase of the shear wave velocity of the lower stratum, those provide significant contributions on the bending moment response of the section which connects a pier and a top of a foundation. It should be noticed that there is a section at which the dynamic soil-structure interaction gives important roles on the bending moment evaluation.

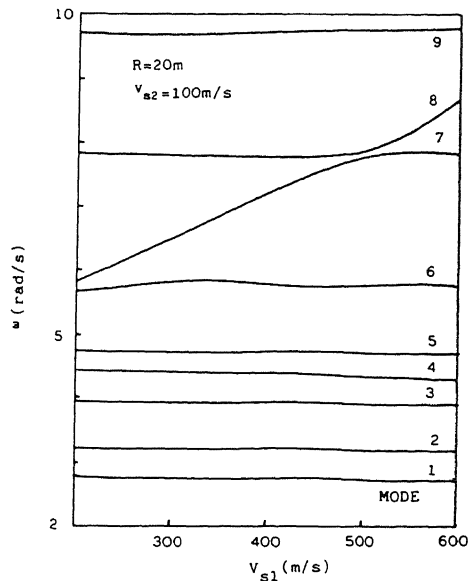


Fig.2 Natural Frequencies vs Shear Wave Velocity

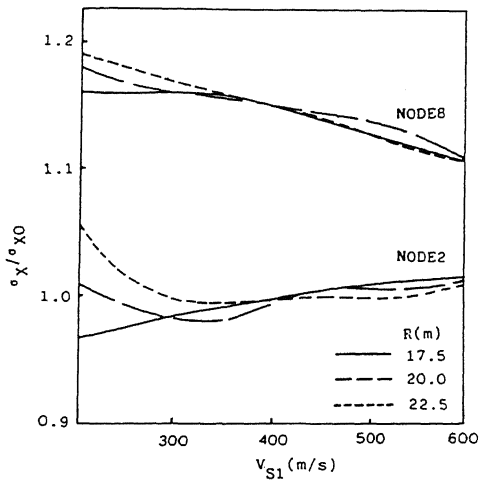


Fig.3 rms Displacements vs Shear Wave Velocity

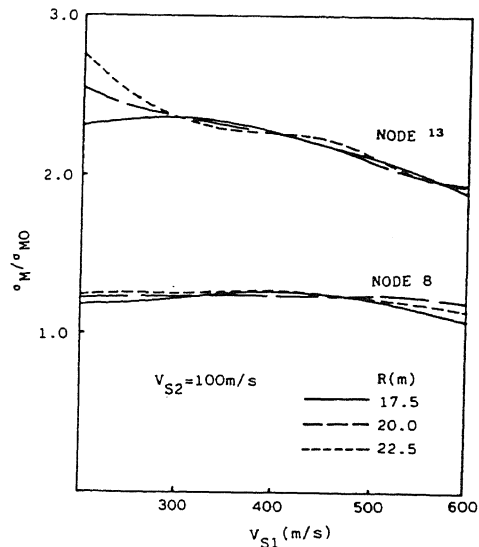


Fig.4 rms Bending Moments vs Shear Wave Velocity

Fig.5 shows the random variable effects on the displacement response due to variations of the shear wave velocity of soil. The mean value of the shear wave velocity of surface stratum is 100m/s and the one lower stratum take the values between 200m/s and 600m/s. These results are obtained for small variations of these values around each mean value. The characteristic values of an input power spectral density function are 0.5 for  $h_0$  and 10 rad/s for  $\omega_0$ , respectively. The rms acceleration of input motion also is 80 gal. While the increase of variations of shear wave velocities of soil gives something of contributions on the displacement responses, these give same effects for the increase of the shear wave velocity of lower stratum. It is shown that the random variable effects provide on the whole same results for a main girder and tower.

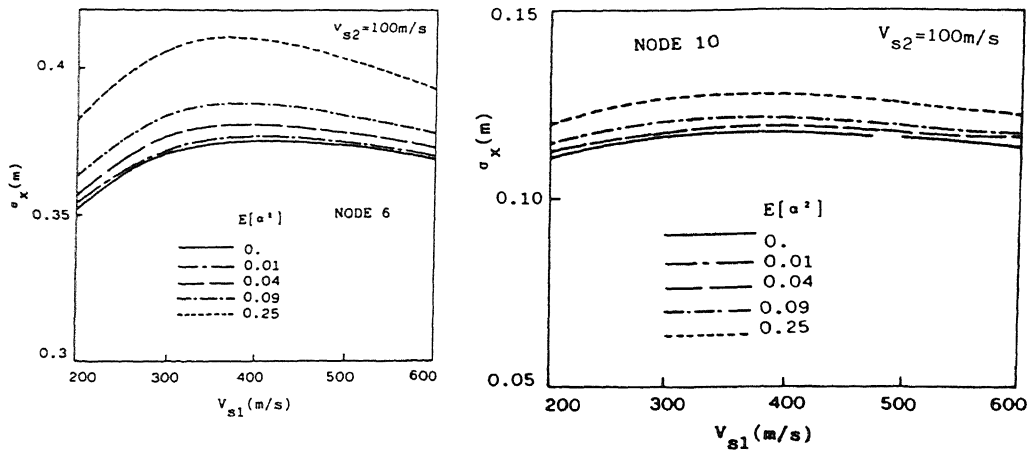


Fig.5 Effects on Variations of Shear Wave Velocity of Soil

#### CONCLUSION

From the dynamic response analysis of a cable-stayed bridge, the following results are obtained as :

- (1) While the dynamic soil- structure interaction effects on the dynamic responses generally depend upon the dynamic characteristics of soil- foundation system and superstructure, it is important to examine these effects in order to perform more exact evaluations for the dynamic response of a cable-stayed bridge. It is shown that the dynamic soil-structure interaction effects demonstrate some different response characteristics on each nodal point of a girder, pier, and tower.
- (2) The variations of the shear wave velocity of soil provide something of contributions on the dynamic response. Considering the random properties of the shear wave velocity evaluation of soil, it seems that the more exact evaluation of the dynamic response can be given through the random variable analysis, in spite of including difficulty on appropriate evaluations of these random variables.

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