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DAMPING CHARACTERISTICS OF CABLE-STAYED BRIDGES

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SUMMARY

This paper presents two analyses for aiming to study damping characteristics of a cable-stayed bridge which are significant importance for seismic design. Equivalent damping ratio caused by either friction force at movable supports or radiation of energy from foundation was studied. It was found from the analyses that damping ratio caused by friction at movable supports and radiation at foundation significantly depends on mode shape.

INTRODUCTION

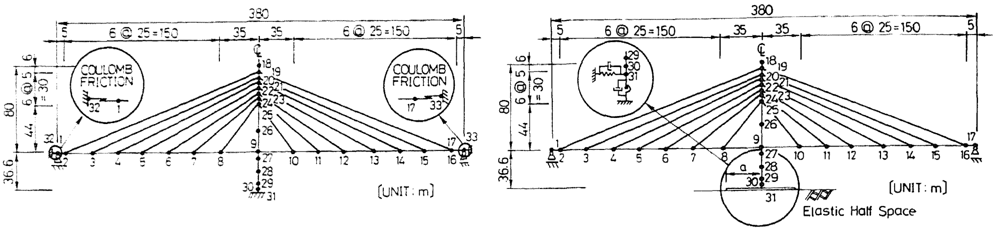
Although damping characteristics are one of the significant factors for seismic design of cable-stayed bridges, few investigations have been so far made (Ref. 1-4). It is general in seismic design to assume a damping ratio of 2 - 5 % for superstructure, while damping ratio estimated from forced vibration tests is generally much smaller. Damping of cable-stayed bridge may be developed by various factors such as hysteretic damping of materials, energy dissipation at movable supports, radiational damping at foundation, viscous damping with air, sag of cables, etc.

In this paper, damping ratio associated with energy dissipation at movable supports and radiational damping at foundation was studied. For isolating such effects, it was assumed here that energy dissipation occurs only either at movable supports or foundation. Structures including deck, tower and cables were assumed as elastic with no energy dissipation.

Fig.1 shows a cable-stayed bridge studied. Analytical idealizations which will be described later are presented. The bridge is of two span continuous girders with single tower. The length of deck is 380m. Fourteen cables are placed symmetrically in a form of "fan". The girder is rigidly connected with tower, with two ends being supported by movable supports. The weight of girder, tower and cables is 4,435tf, 734tf, and 120tf, respectively. Only oscillation in longitudinal (bridge axis) direction was considered here because this is a major interest in design.

DAMPING ASSOCIATED WITH ENERGY DISSIPATION AT MOVABLE SUPPORTS

<u>Analytical Method</u> The cable-stayed bridge was idealized as a discrete analytical model as shown in Fig.1(a), in which friction force developed at



(a) Energy Dissipation at Movable Supports (b) Energy Dissipation at Foundation

Fig. 1 Cable–Stayed Bridge Studied and Analytical Idealization

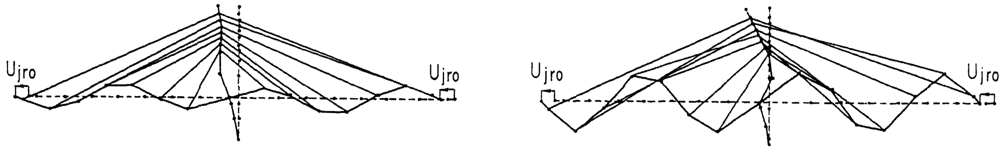
movable supports in accordance with relative displacement between girder and abutments was idealized by the Coulomb friction. Direction of the Coulomb friction force developed when the relative movement at contact plane occurs is opposite to the direction of relative velocity. The friction coefficient was assumed as 0.1 and 0.2. The equations of motion including friction force were solved in an incremental form according to the standard dynamic response analysis procedure.

The idealized cable-stayed bridge was laterally pulled in accordance with a mode shape specified, and then released smoothly to result in a free oscillation. Damping ratio was obtained from decay of the free oscillation as

$$\delta = \frac{2\pi h}{\sqrt{1-h^2}} = \log_e \frac{a_m}{a_{m+1}} \qquad (1)$$

in which δ and h represent logarithmic damping ratio and damping ratio of critical, respectively, and a_m and a_{m+1} represent amplitude of free oscillation at m-th and (m+1)th oscillation, respectively. As well as initial displacement, initial acceleration determined by multiplying the initial displacement by square of angular natural frequency of the mode specified was considered.

<u>Effect of Energy Dissipation at Movable Supports</u> Fig.2 (a) and (b) represent 5th and 6th mode shape of the bridge predicted by disregarding the friction at both movable supports. They are first and second predominant modes in longitudinal direction, respectively, with natural period of 0.52sec and 0.48sec.



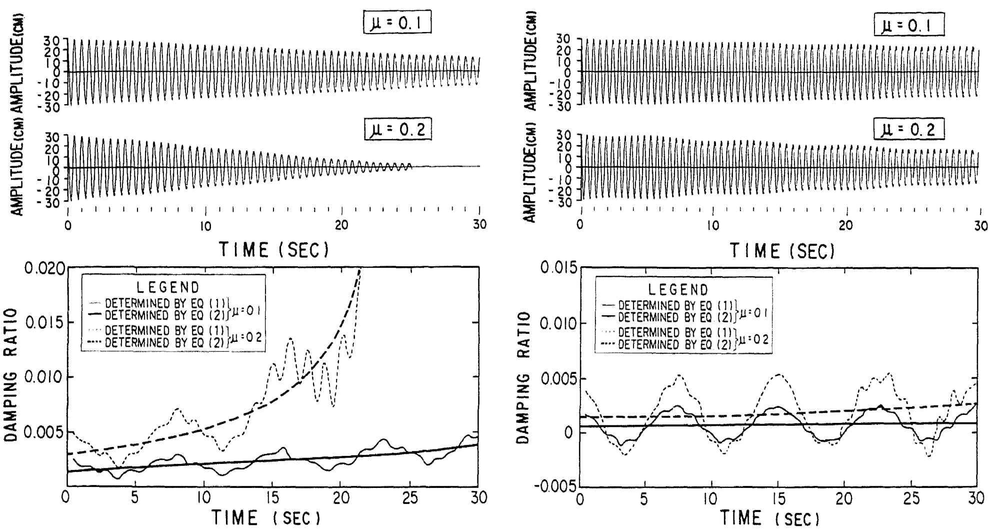
(a) 5th Mode (T_5=0.52 sec) (b) 6th Mode (T_6=0.48 sec)

Fig. 2 Predominant Modes

Fig.3 shows decay of free oscillation of the girder as well as damping ratio determined by Eq.(1). Although the free oscillation seems to decay linearly with time, there are some variations from such linear decay. Therefore, damping ratio h directly determined by Eq.(1) based on amplitude of successive peaks shows variation in time. Hence, damping ratio h was also determined from linear approximation of the successive peaks of free oscillation as

$$k = \frac{\Sigma a_m t_m - (\Sigma a_m \Sigma t_m)/n}{\Sigma t_m^2 - (\Sigma t_m)^2/n} \tag{2}$$

in which k, t_m and n represent the gradient of least square fitted line with the peak of free oscillation, time when successive peak a_m occurs, and number of peaks considered, respectively. Damping ratio h thus determined is presented in Fig.4. These results show general trend of the damping ratio determined by Eq.(1).



(a) 5th Mode (b) 6th Mode

Fig. 3 Decay of Free Oscillation and Damping Ratio

From Figs.3 and 4, it is interesting to note an amplitude dependence of the damping ratio, i.e., damping ratio increases as the amplitude of free oscillation decreases. It should be noted that such a tendency is opposite with that which has been considered for damping characteristics of cable-stayed bridges. It is also interesting to note that damping ratio h varies in accordance with mode. Damping ratio is larger in 5th mode than 6th mode. Damping ratio increase, of course, linearly with increase of friction coefficient.

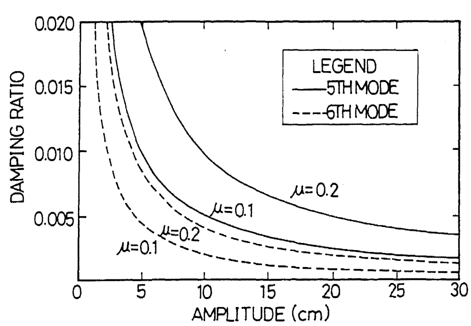


Fig. 4 Damping Ratio vs. Amplitude of Free Oscillation Relation

Table 1 summalizes the damping ratio obtained from the free oscillation. In case of friction coefficient of 0.1, damping takes a value ranging from 0.0007 (6th mode) to 0.0016 (5th mode).

Table 1. Damping Ratio Caused by Friction Force at Movable Supports

Natural Mode	Friction Coefficient	Relative Displacement at Movable Support		
		30 cm	20 cm	10 cm
5 th	0.1	0.0016	0.0024	0.0049
	0.2	0.0032	0.0049	0.0099
6 th	0.1	0.0007	0.0010	0.0020
	0.2	0.0013	0.0020	0.0039

ENERGY DISSIPATION AT FOUNDATION

Analytical Method The same bridge was idealized as shown in Fig.1(b), in which the foundation was idealized as a rigid mass-less circular plate. Subsoils supporting the foundation was assumed as an elastic half space with shear modulus of G_s and Poisson's ratio of ν. Dynamic complex stiffness of the rigid circular plate can be obtained in such an idealization. Radius of the rigid circular plate was simply assumed so that it gives the same surface area with the foundation, and was varied as a parameter to be studied. The bridge and the subsoils were assumed as elastic with no energy dissipation, which implies that the energy dissipation of the bridge takes place through only radiation of energy from the foundation to subsoils.

Dynamic complex stiffness of the foundation was assumed in a frequency independent form as (Ref.5)

$$K_x = 8Ga/(2-\nu) \quad , \quad C_x = \pi G a^2/V_s$$

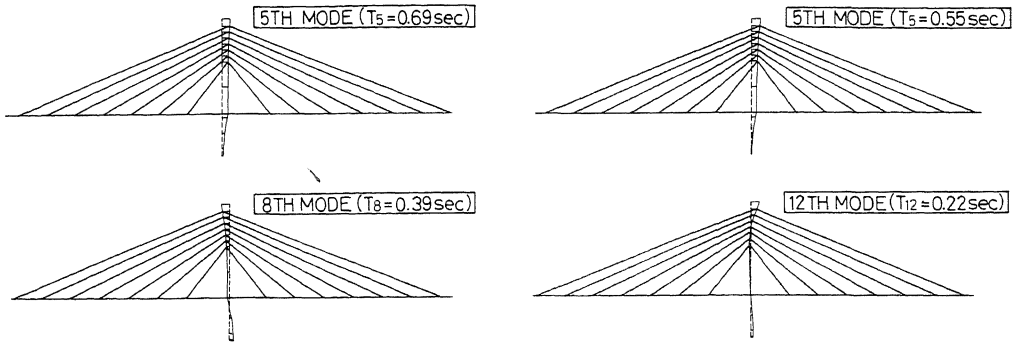
$$K_r = 8Ga^3/3(1-\nu) \quad \text{and} \quad C_r = 0.25\pi\sqrt{2(1-\nu)/(1-2\nu)}\,G a^4/V_s \tag{3}$$

in which Kx and Cx represent spring and damping coefficients for sway, and Kr and Cr represent those for rocking, respectively. Vs and a represent shear wave velocity of subsoils and radius of foundation, respectively. The damping ratio was computed by the complex eigenvalue analysis.

Effect of Energy Dissipation at Foundation Table 2 shows natural period and damping ratio computed by the complex eigenvalue analysis for the bridge placed on subsoils with shear wave velocity of 150m/sec and 300m/sec. Radius of the foundation and Poisson's ratio was assumed as 30m and 0.4, respectively. Fig.5 shows two predominant mode shapes, which include sway motion of tower and deck, and bending motion of tower. Damping ratio for the sway mode takes 0.175 (Vs = 150m/sec) and 0.034 (Vs = 300m/sec), which seems appreciably high as compared

Table 2. Damping Ratio Associated with Radiation of Energy at Foundation

Shear Wave Velocity Vs [m/sec]		1	2	3	4	5	6	7	8	9	10	11	12
150	Natural Period T [sec]	2.83	1.26	0.89	0.71	0.57	0.49	0.44	0.41	0.30	0.28	0.25	0.23
	Damping Ratio h	0.0	0.0	0.0	0.0	0.175	0.0	0.0	0.744	0.0	0.0	0.004	0.0
300	Natural Period T [sec]	2.83	1.26	0.89	0.71	0.54	0.49	0.44	0.30	0.28	0.25	0.23	0.21
	Damping Ratio h	0.0	0.0	0.0	0.0	0.034	0.0	0.0	0.0	0.0	0.002	0.0	0.830

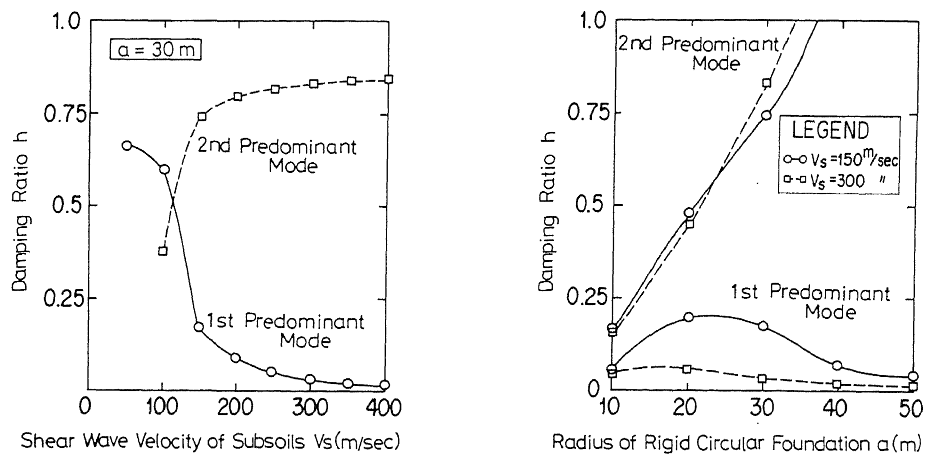


(a) Vs=150 (m/sec) (b) Vs=300 (m/sec)

Fig. 5 Predominant Modes

with the damping ratio assumed in design. Of particular interest is much higher damping ratio for the bending mode of tower, i.e., 0.744 for Vs = 150m/sec and 0.830 for Vs = 300m/sec. This clearly shows a significant effect of radiational energy dissipation associated with rocking motion of the foundation.

Fig.6 shows effects of the shear wave velocity Vs of subsoils and the radius of foundation a, in which damping ratio for two predominant modes is presented. It should be noted here that mode number selected here depends on each case; for example 8th mode and 12th mode are selected as the 2nd predominant mode for Vs = 150m/sec and 300m/sec, respectively (refer to Fig.5). However as was the case presented in Fig.5, the 1st and 2nd predominant mode can be regarded as the sway mode of tower and deck, and the bending mode of tower, respectively.



(a) Effect of Shear Wave Velocity of Subsoils (b) Effect of Radius of Foundation

Fig. 6 Variation of Damping Ratio in Terms of Vs and a

It is apparent from Fig.6 that the shear wave velocity of subsoils is a significant factor for the damping ratio. Damping ratio for the 1st predominant mode decreases as Vs increases. Damping ratio of 2 - 5 %, which is generally assumed in seismic design, is developed at about Vs = 300m/sec. On the other hand, damping ratio for the 2nd predominant mode increases as Vs increases. It is extremely high. It is also seen from Fig.6 that the radius of foundation is the other important factor for damping ratio. Effect of the radius is especially significant for the 2nd predominant mode, in which rocking motion of the foundation is predominant.

CONCLUSION

For aiming to investigate damping characteristics of cable-stayed bridge, damping ratio associated with longitudinal free oscillation subjected to friction force at movable supports and radiational energy dissipation at foundation was studied with use of nonlinear dynamic response analysis and complex eigenvalue analysis. From the analyses presented herein, the following conclusions may be deduced:

1) Energy dissipation associated with friction force at movable supports brings an increase of damping ratio of the bridge in accordance with decrease of amplitude of free oscillation.

2) Energy dissipation associated with radiation of energy from foundation to subsoils depends on stiffness of subsoils and radius of foundation. Damping ratio associated with rocking motion of foundation is significantly high.

3) Damping ratio developed by friction force and radiational damping varies significantly with mode shape. Therefore, damping characteristics of cable-stayed bridges should be determined carefully based on mode shape.

REFERENCES

1) Itoh, M. and T. Katayama : Damping of Bridge Structures, Proc. JSCE, Vol.117, pp.12-22, 1965.
2) Kuribayashi, E. and T. Iwasaki : Dynamic Properties of Highway Bridges, 5th WCEE, 1972.
3) Narita, N. : Fundamental Investigation on Wind Resistant Design of Cable-stayed Bridge with Solid Cross Section, Dr. Thesis submitted to Univ. of Tokyo, 1978.
4) Kato, M. and S. Shimada : Statistical Analysis on Measured Bridge Vibration Data, Proc. JSCE, Vol.311, pp.49-58, 1982.
5) Yamahara, H. : Investigation on Vibrational Properties of Foundations and Structures on Elastic Medium (Part 1), Proc. AIJ, Vol.115, pp.6-14, 1965.