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NONLINEAR EARTHQUAKE RESPONSE ANALYSIS OF REINFORCED CONCRETE BRIDGES USING THE FINITE STRIP METHOD

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SUMMARY

A layered finite strip method is developed for the nonlinear elasto-plastic analysis of the static and dynamic behavior of slabs. The material nonlinearity, crack opening and closing, yielding and crushing of concrete as well as yielding of the reinforcement steel, is taken into account. The Simpson's integration rule and Newmark's methods are used. The method is programmed and used to analyze to several numerical examples. The results are compared with experimental test data and other sources are also presented.

INTRODUCTION

The finite strip method is a suitable approach and has been successfully used in static and dynamic analysis of floors, roofs, bridge decks and box-girder bridges by Cheung and Cheung et al (Refs. 1,2,3). The layered finite element method has been used by Hand et al (Ref. 4). So far as the authors are aware, no study has yet been reported of a layered finite strip method. Simpson's integration is used along the span of the strip element to calculate stiffness matrices and pseudo forces. The points selected for the Simpson's integration can be arbitrarily arranged in each layer because there is no restraint of connectivity relationship among the integration points. Step by step iteration method is used to solve static equations of equilibrium or dynamic equations of motions. The strains and stresses are updated at Simpson's integration points. More closely spaced integration points can be selected in the plastic zones than in the elastic zones to make the analysis efficient and accurate.

LAYERED FINITE STRIP ELEMENT FORMULATION

The layered finite strip element developed in this paper is formed by combining the LO2 rectangular bending strip with the LO2 plane strip element (Ref. 5) as shown in Fig. 1. The assumed displacement field in layered strip element can be expressed as:

$$u = u_0 - z\,W_{,x}$$
$$v = v_0 - z\,W_{,y} \qquad (1)$$
$$w = w_0$$

where u_0, v_0 and w_0 are inplane and transverse displacement functions, respectively in the middle surface.

The strains can be written as:

$$\{\varepsilon\} = \{\varepsilon_0\} - z\,\{\chi\} \qquad (2)$$

in which $\{\varepsilon\}$ = the generic strain vector at any point (x,y,z)
$\{\varepsilon_0\}$ = the plane strains at the reference surface
$\{\chi\}$ = the curvatures in the reference surface

The layered strip element stiffness is calculated as:

$$[K] = \sum_{m=1}^{r} \begin{bmatrix} K_{pp} & K_{pb} \\ K_{pb}^{T} & K_{bb} \end{bmatrix} \qquad (3)$$

in which K_{pp} = plane stiffness, K_{pb} = coupling stiffness and K_{bb} = bending stiffness.

MATERIAL PROPERTIES

The assumed failure envelopes for concrete in a biaxial state of stress and strain are given by (6) and are shown in Fig. 2. The yielding is governed by a yield criterion of the form:

$$F(\{\sigma\}^c) = 0 \qquad (4)$$

In accordance with the Von Mises yield criterion, the yield surface is defined as

$$F(\{\sigma\}^c) = [(\sigma_1^c)^2 - \sigma_1^c\,\sigma_2^c + (\sigma_2^c)^2]^{1/2} - \sigma_0^c \qquad (5)$$

where σ_0^c = uniaxial yield stress of concrete and σ_1^c, σ_2^c = the principal stresses of concrete.

To simulate the behavior of reinforced concrete under reversed loading, six crack modes for an element of concrete are allowed in the analytical model (6,7). These are shown in Fig. 3. The "Initial Stress Approach" is used (9).

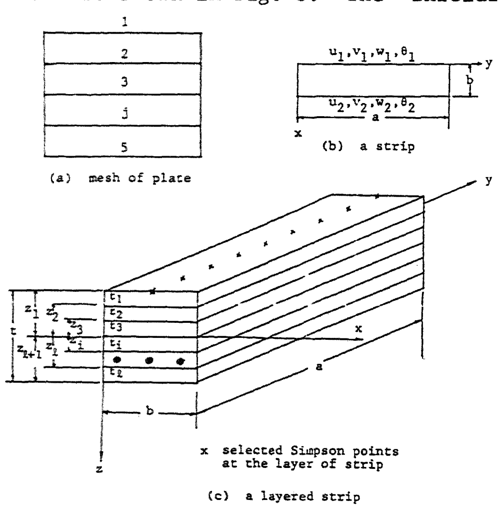

Fig. 1 The Idealization of Finite Layered Strip Element

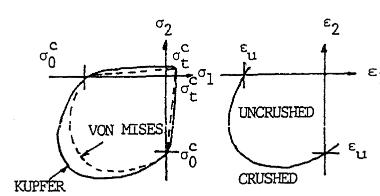

Fig. 2 Yield Surface for Concrete

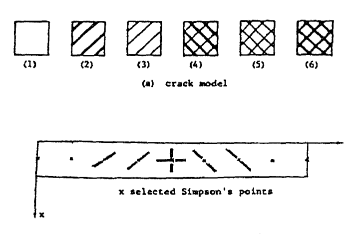

Fig. 3 Crack Model of Reinforced Concrete

A "smeared" composite material property matrix is generated by adding the constitutive matrix for steel reinforcement to that of plain concrete. This can be written as:

$$[D]_e = [D]_e^c + [D]_e^s \quad . \tag{6}$$

NUMERICAL PROCEDURE

The equations of static equilibrium are

$$[K]\ \{\Delta U\} = \{\Delta P\} + \{\Delta F\} \tag{7}$$

where [K] = tangent stiffness matrix; $\{\Delta U\}$ = generalized increment nodal displacement vector; $\{\Delta P\}$ = generalized increment external force vector and $\{\Delta F\}$ = equivalent increment nodal loads that account for the plastic flow.

The equation of motion for the implicit integration option is

$$[M]\ \{\Delta \ddot{U}\} + [K]\ \{\Delta U\} = \{\Delta P\} + \{\Delta F\} \tag{8}$$

in which [M] = consistent mass matrix. The expression for L02 rectangular strip is given in Ref. 5. The numerical procedure was programmed using the layered finite strip element method.

NUMERICAL RESULTS

A number of numerical examples were investigated to test the validity of the layered finite strip method.

Example 1: Elastic Simply Supported Square Plate: In order to illustrate the accuracy and convergence of the layered finite strip element, an elastic simply supported square plate subjected to uniformly distributed load was tested. The results were compared with those obtained by the finite strip element and with exact solution in Table 1.

Table 1. Convergence Test of the Layered Strip Method (Term 1)

V = 0.3	Finite Layered Strip			Finite Strip Method (1)		
Total Number of Layers	W_{max} (1)	Mx_{max} (2)	My_{max} (3)	W_{max} (4)	Mx_{max} (5)	My_{max} (6)
4	0.00411	0.0442	0.0436	0.00411	0.0502	0.0502
6	0.00411	0.0458	0.0452			
8	0.00411	0.0464	0.0458			
10	0.00411	0.0467	0.0461			
Exact (10)	0.00406	0.0479	0.0479	0.00406	0.0479	0.0479
Multiplier	qa^4/D	qa^2		qa^4/D	qa^2	

Example 2: Natural Frequencies of a Simply Supported Plate: Table 2 shows the natural frequence obtained by the layered finite strip method in comparison with the finite strip method and the exact solution.

Table 2. Natural Frequency for Simply Supported Square Plate

No.	Finite Layered Strip Method	Finite Strip Method (2)	Exact (11)
1	19.737	19.74	19.739
2	49.399	49.32	49.348
3	-	78.91	78.956
4	98.369	98.68	98.696
5	127.398	128.17	128.305
($a = b = 1$, $D = 1$, $\rho h = 1$, $\nu = 1/6$)			

Example 3: Isotropic Elasto-Plastic Simply Supported Square Plate: An elasto-plastic analysis of a simply supported plate is performed using the layered finite strip method. There are 6 layers and 8 Simpson's points for each layer. The results were compared with heterosis element in Ref. (12) shown in Fig. 4.

Example 4: Reinforced Concrete Slab: An analysis was carried out of a reinforced concrete slab S1 tested by Taylor (Ref. 13). The test performed by Taylor is a simply supported square plate subjected to uniformly distributed load. The properties of the specimen are summarized in Table 3.

Table 3. The Properties of the Concrete Slabs

Slab	Concrete					Steel		
	E_c (psi)	σ_c (psi)	σ_t (psi)	ε_c (in/in)	ν_c	E_s (psi)	σ_y (psi)	ds (in)
S1 (13)	3.3×10^6	5940	550	0.0025	0.18	3.0×10^7	54500	0.1875
ID2 (16)	3.0×10^6	3310	624	0.0025	0.18	3.0×10^7	50000	0.2500

The load-deflection curve is shown in Fig. 5. It would be noted that flexural cracking began on the underside at 4.5 ton, and that cracks appeared first in the central region and spread towards the corners under increasing load. The extent and directions of the cracks are shown in Fig. 6. As the load increased, the yielding of reinforcement steel occurred at $p = 13$ ton. At $p = 16.5$ ton crushing of concrete occurred. A comparison of the theoretical and experimental load-deflection curves shows that the experimental stiffness is approximately 75 percent of the theoretical. This result confirms the Bell's analysis (14). The discrepancy could be in the assumption of the concrete properties.

The layered strip method takes less computer time and has a smaller bandwidth of total stiffness matrix than the finite element method (see Table 4).

Table 4. Comparison of the Layered Finite Strip Method with the Finite Element Method

Method	No. of Joints	No. of Elements	No. of Equations	Band-Width	Total No. of Load Steps	No. of Iterations per Load Step	CPU Time
L.F.S.	5	4	20	20	27	2.1	488
F.E.M. (15)	19	24	72	30	16	8.9	709

Example 5: Dynamic Response of Concrete Slab: An initial investigation of the layered finite strip method on the dynamic response of a concrete slab ID2 is

shown in Fig. 7. The material properties are listed in Table 3. The analysis simulates the dynamic response by using a constant step load 9 psi. The difference shown in Fig. 7 perhaps could be due to the different loads used in the theoretical analysis and strain rate effects.

CONCLUSION

The layered finite strip method is developed and programmed. The theory gives very good results for elastic-plastic isotropic plates and for computing the natural frequencies. The method predicts the crack extent and directions of crack patterns agreed with the experiment quite well.

The method uses less computer time, and the bandwidth of the total stiffness matrix than that in the finite element method. Also, less input data is required.

Further efforts must be made to improve the accuracy analysis for concrete problems.

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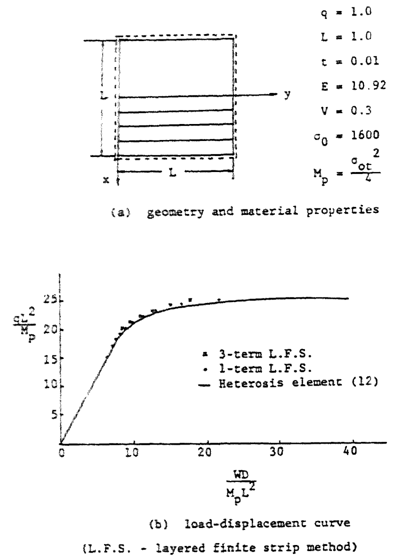

Fig. 4 Layered Elasto-Plastic Finite Strip

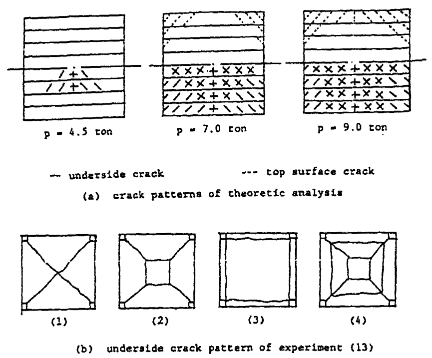

Fig. 6 Crack Pattern of Concrete Slab S1

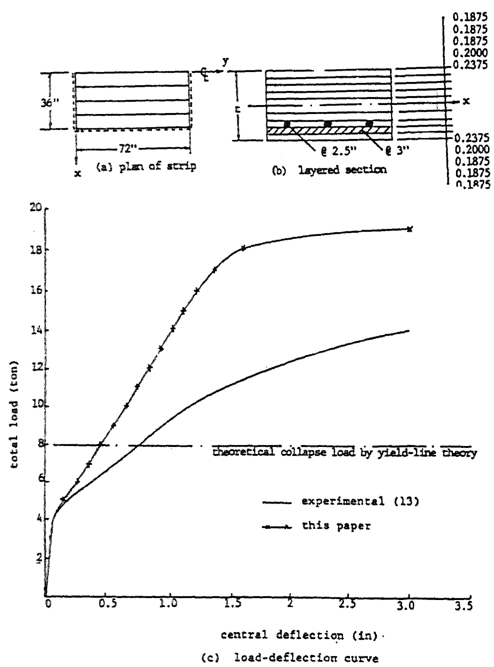

Fig. 5 Static Response of Slab S1

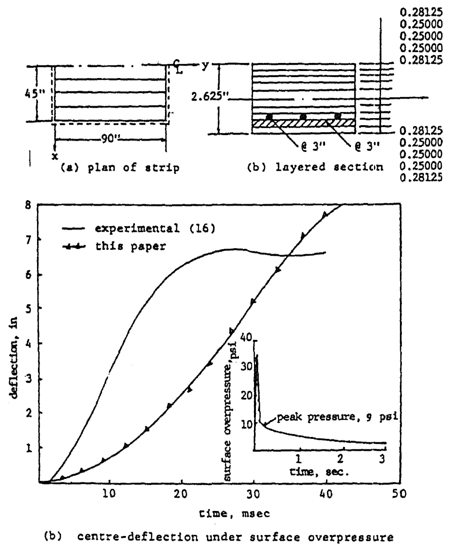

Fig. 7 Dynamic Deflection for Slab ID2