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### 3-D NON-LINEAR DYNAMIC ANALYSIS OF EFFECTIVE STRESSES OF TONGLING TAILINGS DAM

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#### SUMMARY

Presented in this paper is a 3-D non-linear dynamic analysis for the Tongling tailings dam of Anhui Province. The dam material is liable to liquefy. Based upon the principle of effective stress a method of 3-D dynamic analysis of the seismic response of saturated tailings sand is submitted. A detailed analysis has shown that a local and limited sliding failure may occur in the dam under VII earthquake.

#### INTRODUCTION

The tailings dam of Tonling Copper Mine is located in the south of Anhui province, about 30 km east of Tongling city. The tailings reservoir is formed by the construction of a major dam and a minor dam nearly at right angle to each other in plan. The max. height of minor dam is 51 m with a max. length of 150 m. Upstream and downstream slopes are 1:6.5 and 1:4 respectively. Owing to the topography and length to height ratio of the dam (equals to 3), the stress state of minor dam must be considered as a three dimensional one. Fig. 1 shows the plan

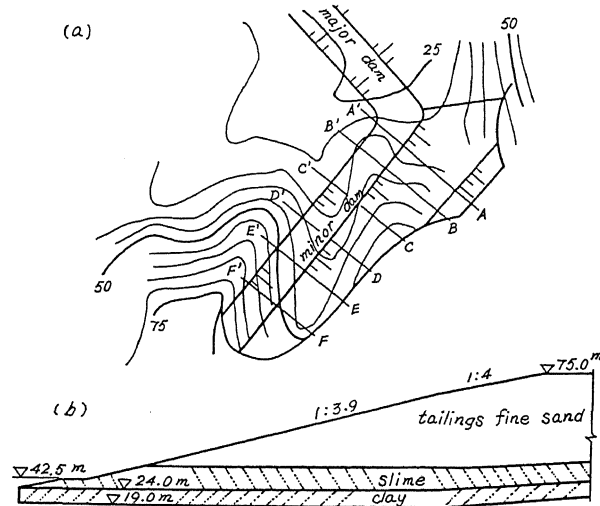


Fig. 1. Tonling Minor Tailings Dam: (a) plan;  
(b) main cross section

of minor dam and its simplified main cross section. The dam material consists of tailings fine sand, slime and clay. The bedrock is covered with 5 m thick soil deposits. The fine sand has a uniform gradation and large void ratio. It is very liable to liquefy under vibration. The dam is located in an earthquake area with intensity VI of modified Mercalli scale. Since seismic liquefaction failure of tailings dam in major earthquakes are well know, it is necessary to investigate the anti-liquefaction stability of this dam under earthquake motion.

#### MATERIAL PROPERTIES AND COMPUTATION PARAMETERS

Dynamic Properties If tailings is regarded as equivalent to visco-elastic medium, the following empirical relationships proposed by Hardin and Drnevich for shear modulus G and damping ratio D may be used (Ref.1):

$$G = \frac{G_{max}}{1 + \gamma_k} \quad (1)$$

$$D = D_{max} \frac{\gamma_k}{1 + \gamma_k} \quad (2)$$

where

$$G_{max} = 6920 k_{2max} (\sigma'_m)^{1/2} \quad (p_a) \quad (3)$$

$$\gamma_k = \frac{\gamma}{\gamma_r} \left[ 1 + a \exp\left(-b \frac{\gamma}{\gamma_r}\right) \right] \quad (4)$$

in which  $\gamma$  is amplitude of dynamic shear strain,  $\gamma_r = \tau_{max}/G_{max}$  is reference strain,  $\tau_{max}$  is shear strength,  $k_{2max}$  is max. shear modulus coefficient of tailings,  $D_{max}$  is max. damping ratio, a and b are parameters related with tailings type and loading frequency. The values of  $k_{2max}$ ,  $D_{max}$  and other index properties of tailings dam materials are summarized in Table 1.

Table 1 Index Properties of Tailings Dam Materials

| Material  | fine sand            | slime                | clay                  |
|---|----------------------|----------------------|-----------------------|
| Saturated Unit Weight $\gamma$ (KN/m <sup>3</sup> ) | 21.9                 | 21.2                 | 19.0                  |
| Specific Gravity $G_s$                              | 3.27                 | 3.25                 | 2.7                   |
| Uniformity Coefficient U                            | 5.8                  | 9.1                  | —                     |
| Median Grain Size $d_{50}$ (mm)                     | 0.04                 | 0.02                 | —                     |
| Effective Cohesion $C'$ (kPa)                       | 0                    | 0                    | 9.8                   |
| Angle of Internal Friction $\phi'$ (°)              | 30                   | 28                   | 24                    |
| Possion's Ratio $\mu$                               | 0.35                 | 0.37                 | 0.40                  |
| Coefficient of Permeability k (m/s)                 | $5.2 \times 10^{-6}$ | $4.8 \times 10^{-7}$ | $8.7 \times 10^{-10}$ |
| Max. Shear Modulus Coefficient $k_{2max}$           | 4.7                  | 43                   | 30                    |
| Max. Damping Ratio $D_{max}$                        | 0.30                 | 0.29                 | 0.28                  |

Formula for Pore Water Pressure Induced by Vibration In this paper, the following empirical formula based upon the test data of tailings from Tonling Copper Mine is used:

$$\Delta p_e = 12 M \bar{N}^{-0.4} \frac{\tau_d}{N_L} \Delta N \quad (5)$$

where  $\Delta p_e$  is increment of seismic pore pressure in  $\Delta N$  cycles;  $\Delta N$  is loading cycles in  $\Delta t$  time;  $N_L$  is number of loading cycles to cause liquefaction;  $\tau_d$  is dynamic shear stress in 45° plane,

$$M = 1 - (\tau_o/\sigma'_{mo})^2 / (\tau_d/\sigma'_{mo}) \quad \bar{N} = (N/N_L)(\tau_d/\sigma'_{mo})$$

in which  $\tau_0$  is initial shear stress in failure plane;  $\sigma'_{m0}$  is initial mean stress; N is number of loading cycles.

#### COMPUTATION THEORY AND BASIC EQUATIONS

Governing Equations Generally speaking, the generation of pore water pressure under earthquake is always accompanied with its diffusion in the tailings dam. If we consider the tailings skeleton as a linearly elastic porous medium and introduce the generation of pore pressure into the Biot's equations, it is possible by using geometric equation, Hooke's law and Darcy's law, to obtain the following governing equations to describe the process of 3-D consolidation deformation of tailings dam with the consideration of seismic pore water pressure (Ref.2)

$$E_1 \frac{\partial^2 u}{\partial x^2} + E_3 \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + (E_2 + E_3) \frac{\partial^2 v}{\partial x \partial y} + (E_2 + E_3) \frac{\partial^2 w}{\partial x \partial z} - \frac{\partial p}{\partial x} + \frac{\partial p_g}{\partial x} + X = 0 \quad (6)$$

$$(E_2 + E_3) \frac{\partial^2 u}{\partial x \partial y} + E_1 \frac{\partial^2 v}{\partial y^2} + E_3 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right) + (E_2 + E_3) \frac{\partial^2 w}{\partial z \partial y} - \frac{\partial p}{\partial y} + \frac{\partial p_g}{\partial x} + Y = 0 \quad (7)$$

$$(E_2 + E_3) \frac{\partial^2 u}{\partial x \partial z} + (E_2 + E_3) \frac{\partial^2 v}{\partial y \partial z} + E_1 \frac{\partial^2 w}{\partial z^2} + E_3 \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - \frac{\partial p}{\partial z} + \frac{\partial p_g}{\partial z} + Z = 0 \quad (8)$$

$$\frac{k_x}{\gamma_w} \frac{\partial^2 p}{\partial x^2} + \frac{k_y}{\gamma_w} \frac{\partial^2 p}{\partial y^2} + \frac{k_z}{\gamma_w} \frac{\partial^2 p}{\partial z^2} = \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \quad (9)$$

where  $p_g$  is the seismic pore water pressure,  $p$  is the residual pore water pressure,  $u$ ,  $v$  and  $w$  are displacements of tailings skeleton in  $x$ ,  $y$  and  $z$  directions,  $X$ ,  $Y$  and  $Z$  are body forces of tailings,  $k_x$ ,  $k_y$  and  $k_z$  are coefficients of permeability in  $x$ ,  $y$  and  $z$  directions,  $\gamma_w$  is unit weight of water

$$E_1 = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)}, \quad E_2 = \frac{E\mu}{(1+\mu)(1-2\mu)}, \quad E_3 = \frac{E}{2(1+\mu)},$$

where  $E$  and  $\mu$  are Young's modulus and Poisson's ratio.

Computation Formula The equations (6) - (9) can be solved numerically under giving boundary and initial condition by FEM. In formulating finite element equations the weighted residual method and 3-D isoparametric element with eight nodes will be used. Let  $\Delta u_i$ ,  $\Delta v_i$ ,  $\Delta w_i$  and  $\Delta p_i$  be the increments of node variable  $u_i$ ,  $v_i$ ,  $w_i$  and  $p_i$  in node  $i$ , during time increment  $\Delta t$ ,  $u_{i0}$ ,  $v_{i0}$ ,  $w_{i0}$  and  $p_{i0}$  be the initial value of these variables at the beginning of this time increment,  $p_{i0} + \Delta p_i / 2$  be the mean value of  $p_i$ , the following set of finite difference equation is obtained from Eqs. (6) - (9).

$$\sum_{j=1}^n \left[ k_{ij}^1 \Delta u_j + k_{ij}^2 \Delta v_j + k_{ij}^3 \Delta w_j + k_{ij}^4 \Delta p_j \right] = \Delta F_i \quad (10)$$

$$\sum_{i=1}^n [k_{ij}^{11} \Delta u_j + k_{ij}^{22} \Delta v_j + k_{ij}^{33} \Delta w_j + k_{ij}^{44} \Delta p_j] = \Delta F_i^1 \quad (11)$$

$$\sum_{i=1}^n [k_{ij}^{21} \Delta u_j + k_{ij}^{32} \Delta v_j + k_{ij}^{43} \Delta w_j + k_{ij}^{44} \Delta p_j] = \Delta F_i^2 \quad (12)$$

$$\sum_{i=1}^n [k_{ij}^{31} \Delta u_j + k_{ij}^{42} \Delta v_j + k_{ij}^{43} \Delta w_j + \Delta t k_{ij}^{44} \Delta p_j] = \Delta F_i^3 \quad (13)$$

where  $k_{ij}^{11} \sim k_{ij}^{44}$  are elements of coefficient matrix,  $\Delta F_i^1$ ,  $\Delta F_i^2$  and  $\Delta F_i^3$  are increments of nodal loading in x, y and z directions,  $\Delta F_i^4$  is increment of nodal seepage discharge, during time increment  $\Delta t$ ,  $i=1, 2, \dots, n$ ,  $j=1, 2, \dots, n$ ,  $n$  is number of nodes.

#### DYNAMIC ANALYTICAL PROCEDURES

Among the right side terms of Eqs. (10) - (12) there are the seismic-induced increments of pore water pressure  $\Delta p_g$ , which may be obtained from dynamic analysis of effective stresses. The basic equation for dynamic analysis is

$$[M] \{\ddot{\delta}\} + [C] \{\dot{\delta}\} + [K] \{\delta\} = \{F(t)\} \quad (14)$$

where  $[M]$ ,  $[K]$  and  $[C]$  are mass, stiffness and damping matrix respectively,  $\{F(t)\}$  is nodal earthquake load vector, and  $\delta$ ,  $\dot{\delta}$ ,  $\ddot{\delta}$  are nodal displacement, velocity and acceleration vector relative to bedrock motion. In order to consider the nonlinearity of material properties such as the strain-dependent variable shear modulus  $G$  and damping ratio  $D$  which change during the earthquake, an iteration procedure is used (Ref.3). In this paper the effective stress method is adopted. The whole period of earthquake motion is subdivided into several time steps. For each time step the non-linear dynamic response analysis is carried out and  $\Delta p_g$  in each individual element is evaluated. This  $\Delta p_g$  is converted into equivalent nodal force and added into the loading terms of Eqs. (10) - (12). Then the resulted simultaneous equations are solved to find the increments of displacement and residual pore water pressure from which a static effective stress field is obtained. New values of modulus are compatible with these new effective stresses. In addition, since the modulus and damping ratio are also strain dependent, several iterations are needed in the dynamic analysis for each time step, and for the first iteration value are assigned equal to those in the last time step. The procedure is carried out step by step until the end of earthquake motion. Several time steps are also needed for post-earthquake static analysis to search for the residual pore water pressure dissipation.

As residual pore water pressure  $p$  for any point at any time can be computed,  $r_u = p/\sigma'_{v0}$  is defined as pore water pressure ratio. If the value of  $r_u$  reaches unity, then it would be liquefied, otherwise liquefaction will not be happened at this point.

#### COMPUTED RESULTS AND ANALYSIS

The finite element idealization of minor dam for analysis is shown in Fig.2. An accelerogram recorded at Qian-an, Tangshan during a major after-shock (magnitude 6.3) on August 31, 1976, is used as input motion (Fig. 3).

Fig.4 shows the computed time history of response acceleration  $a_x$  at dam crest of main cross-section. Fig.5 represents the computed time history of shear stress  $\tau_{xy}$  at dam crest. Similar curves for  $a_y$ ,  $a_z$ ,  $\tau_{yz}$  and  $\tau_{zx}$  have also been obtained, but these responses are much smaller than  $a_x$  and  $\tau_{xy}$ .

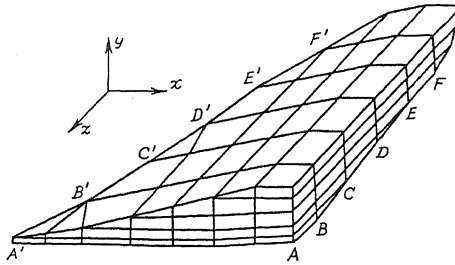


Fig. 2. Finite Element Idealization of Minor Dam

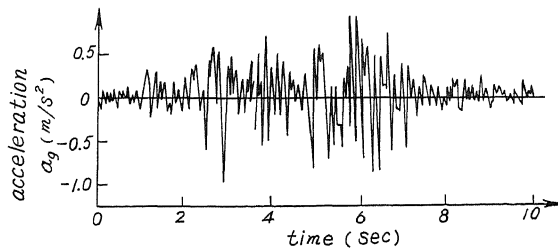


Fig. 3. Input Motion

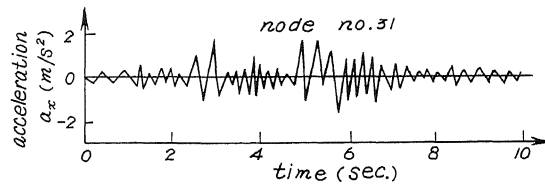


Fig. 4. Time History of Response Acceleration at Crest (nodal no. 31)

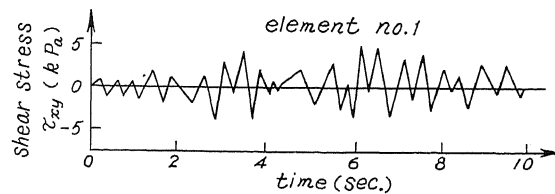


Fig. 5. Time History of Shear Stress  $\tau_{xy}$  at Crest (element no. 1)

Fig. 6 shows distribution of residual pore pressure and liquefied region in the dam at the end of earthquake motion. It is clear from the figure that there exists a high pore water pressure region together with a restricted region of liquefaction of a thickness about 4 m near the crest (shaded area). No liquefaction would occur in other cross sections of dam, but with high pore water pressure ratio, the stability condition of dam is not good.

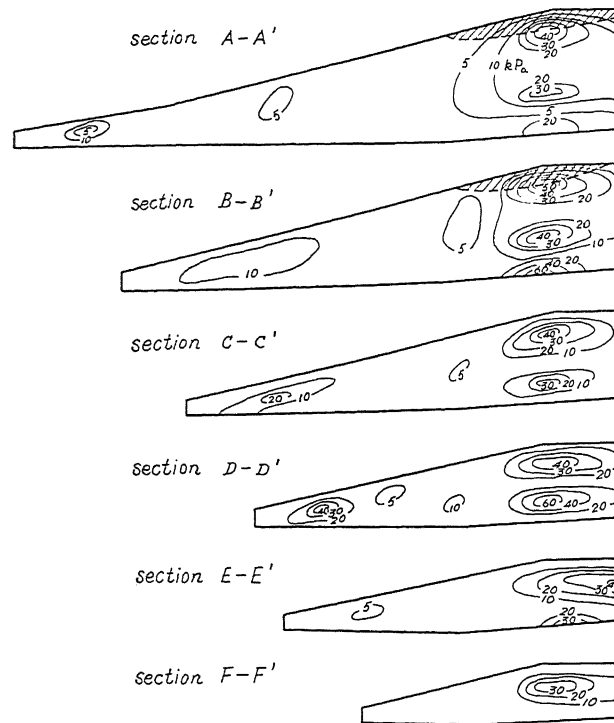


Fig. 6. Distribution of Residual Pore Water Pressure at the End of Earthquake Motion

#### CONCLUSION

A 3-D dynamic analysis of effective stresses of Tongling tailings minor dam has shown that under an earthquake with intensity VI of modified Mercalli scale, residual pore water pressure of different magnitudes in the dam may happen, especially near the dam axis and crest of sections A-A' and B-B'. Owing to the high water pressure there may be a thickness of 3~4 m of liquefied zone within a restricted region near the crest, at the end of earthquake motion. It is possible that a local and limited sliding due to liquefaction may occur at first, and then would induce successive failure of dam under earthquake.

#### REFERENCES

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