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# ARCH DAM SYSTEM IDENTIFICATION USING FREQUENCY RESPONSE TEST DATA AND NONCLASSICAL MODES

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## SUMMARY

A systematic method is developed for the interpretation of frequency test results of arch dams. The method is based on a nonclassical modal synthesis formulation with the classical formulation as a special case and seeks to identify the modal parameters using the frequency response data. The parameter identification process contains a single mode method for initial parameter estimation and a simple criterion to find the optimal number of modes. Results of the application of the method to the test data of four arch dams indicate that the nonclassical formulation is an accurate model for the arch dam system.

## INTRODUCTION

One of the most effective methods to study the dynamic characteristics of an arch dam system is to conduct steady-state forced vibration tests on existing dams (Refs. 1,2,3). The frequency response curves obtained from these tests can be utilized in different ways. An often used approach is to find the natural vibration frequencies and damping ratios by the simple half-power bandwidth method, in which the modal interference effect is neglected. In one instance, it was reported that a classical modal synthesis approach, which included the modal interference effect, was used to identify the modal parameters from the frequency response data (Ref. 4). More recently, the frequency responses were used to verify finite element models of the arch dam system (Refs. 1,2) In view of the great accuracy achieved in recent tests, however, it is felt that the frequency response data obtained has not yet been fully utilized. For example, the data may be used to identify the best mathematical model for the dam system and to compare the merits of different assumed models without having to set up the detailed equations of motion and identifying the equation coefficients. In this study, a nonclassical modal synthesis approach is adopted and a systematic procedure to identify the modal parametes is developed.

The nonclassical modal synthesis is derivable from a set of constant-coefficient linear differential equations. By comparing the predictions based on the nonclassical modal synthesis with the measured frequency response values, the adequacy of the constant-coefficient equations approach may be assessed. On the other hand, if the damping matrix is assumed to satisfy certain conditions, the solution of the constant-coefficient equations is represented by the classical modal synthesis. Thus, by comparing the results from classical and nonclassical models, the adequacy of the classical model may also be assessed.

## SINGLE-MODE METHOD AND NONCLASSICAL MODAL SYNTHESIS

Before a multiple-mode parameter identification procedure is carried out on a set of given data, it is useful to obtain some initial estimates on the values of the natural frequency and the damping of the system. Traditionally such initial estimates are obtained by the half-power bandwidth (HPB) method. One serious drawback of the HPB method, however, is the requirement that the frequency response curve must be sufficiently well defined to show a clear bandwidth at the half-power level below the peak. This requirement is often not met because of the modal interference effect. Described in the following is an alternative simple method that requires a minimal computational effort and a minimal number of data points near a peak on the frequency response curve for the determination of the modal parameters of an equivalent single-mode system.

It is assumed that near a peak on the frequency response curve the behavior of the dam can be approximated by a single-mode system represented by

$$\ddot{q} + 2\omega\xi\dot{q} + \omega^2 q = P\, e^{i\Omega t} \tag{1}$$

where $q$ is the displacement quantity, $\omega$ is the modal frequency, $\xi$ is the modal damping ratio, $P$ is the participating factor, and $\Omega$ is the exciting frequency of a driving force. The steady-state solution for the amplitude of $q$ at exciting frequencies $\Omega_k, k=1,2,...L$, is then

$$A_k = \frac{P}{\sqrt{(\omega^2-\Omega_k{}^2)^2 + (2\xi\omega\Omega_k)^2}} \quad , \qquad k = 1, 2,...L \tag{2}$$

The task is to determine $\omega$, $\xi$, and $P$ from the measured amplitude $\bar{A}_k$ and measured frequencies $\Omega_k$, $k=1,2,...L$. It is clear that the right-hand-side of Eq.2 is a nonlinear function in the three modal parameters. The nonlinear expression may be transformed into a linear one by squaring both sides and rearranging terms to obtain

$$-\Omega_k{}^4 = \omega^4 + \Omega_k{}^2(4\xi^2\omega^2 - 2\omega^2) - \frac{1}{A_k{}^2}P^2, \qquad k=1, 2,...L \tag{3}$$

and by defining three new unknown parameters: $x_1 = \omega^4$ , $x_2 = 4\xi^2\omega^2 - 2\omega^2$ , and $x_3 = P^2$. Then, Eq.3 becomes linear in the new parameters $x$'s. The least square solution for the minimization of the error in the quantity $-\Omega_k{}^4$ is easily obtained as the solution of three simultaneous equations in the $x$'s (Ref. 5). The original modal parameters $\omega$, $\xi$, and $P$ can be obtained once the $x$'s are known. This single-mode parameter identification method has been used in a recent thesis (Ref. 5). During the course of preparing this paper, it is learned that the method has been independently developed and applied to the vibration test data of an arch dam (Ref. 6).

Experimentation with measured data indicates that using five data points near the peak always gives satisfactory results. In the following computation, the single mode method is used to obtain initial estimates as the input to the multiple mode identification procedure..

The governing equations of an N-degree-of-freedom system can be put into a matrix form as

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C}\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{f}(t) \tag{4}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices, respectively, each of size NxN, $\mathbf{q}$ is the displacement vector, and $\mathbf{f}(t)$ is the generalized force vector. In a general nonclassical modal synthesis, the N equations are transformed into a set of 2N first order equations and the solutions for the displacement vector $\mathbf{q}$ and the velocity vector $\dot{\mathbf{q}}$ may be written as

$$\begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} = \Sigma \begin{Bmatrix} \alpha_n U_n \\ U_n \end{Bmatrix} y_n(t) \tag{5}$$

where the $\alpha_n$ is the complex-valued eigenvalues in the 2N system and may be expressed in terms of the equivalent natural frequencies $\omega_n$ and the equivalent damping ratios $\xi_n$.

$$\alpha_n = -\omega_n \xi_n \pm i\, \omega_n \sqrt{1 - \xi_n^2}\ , \qquad j = 1, 2, \ldots N \tag{6}$$

The $\mathbf{U}_n$ is the complex-valued displacement mode shape and the $y_n$ is the generalized coordinate. For vibration tests, the forcing function is sinusoidal with a driving frequency $\Omega$, $\mathbf{f}(t) = \mathbf{F}\, e^{i\Omega t}$, and the generalized coordinates also assume the form $y_n(t) = y_n\, e^{i\Omega t}$, where the amplitude $y_n$ is complex. By simple substitution, the solution for $y_n$ can be easily obtained and the original displacement vector $\mathbf{q}$ becomes

$$\mathbf{q} = \Sigma \left[ \left( \mathbf{p}_{an} \frac{w_{1n}}{w_{0n}} + \mathbf{p}_{bn} \frac{w_{2n}}{w_{0n}} \right) - i \left( \mathbf{p}_{an} \frac{w_{3n}}{w_{0n}} + \mathbf{p}_{bn} \frac{w_{4n}}{w_{0n}} \right) \right] e^{i\Omega t} \tag{7}$$

where the w's are functions of $\omega_n, \xi_n$, and $\Omega$

$$\begin{aligned} w_{0n} &= (\omega_n^2 - \Omega^2)^2 + 4\omega_n^2 \xi_n^2 \Omega^2, & w_{1n} &= \omega_n \xi_n (\Omega^2 + \omega_n^2) \\ w_{2n} &= \omega_n (\Omega^2 - \omega_n^2) \sqrt{1 - \xi_n^2}, & w_{3n} &= \Omega\, [\Omega^2 - \omega_n^2 (1 - 2\xi_n^2)] \\ w_{4n} &= -2\Omega \omega_n^2 \xi_n \sqrt{1 - \xi_n^2} \end{aligned} \tag{8}$$

and $\mathbf{p}_{an}$ and $\mathbf{p}_{bn}$ are the linear combinations of the real and the imaginary parts of the displacement mode shape. In the case of classical damping, the imaginary part of the eigenvectors vanishes, and it can be shown easily that $\mathbf{p}_{an} = \mathbf{0}$. For a particular displacement component $\mathbf{q}$ in the vector $\mathbf{q}$, the corresponding terms in $\mathbf{p}_{an}$ and $\mathbf{p}_{bn}$ can be combined into a new parameter $S_n = p_{an}/p_{bn}$. Then the real part of the displacement component $\mathbf{q}$ at a point becomes

$$q = \Sigma\, p_{bn} \left[ \left( S_n \frac{w_{1n}}{w_{0n}} + \frac{w_{2n}}{w_{0n}} \right) \cos\Omega t + \left( S_n \frac{w_{3n}}{w_{0n}} + \frac{w_{4n}}{w_{0n}} \right) \sin\Omega t \right] \tag{9}$$

This is the equation to be used later for parameter identification. It is seen that for each mode included, there are four modal parameters to be identified, $\omega_n$, $\xi_n$, $p_{bn}$, and $S_n$. If an additional measurement is made at a different point, an additional pair of parameters $p_{bn}$ and $S_n$ will be included for identification. If the structural response is represented by classical modes, the same equation can be used by simply making $S_n = 0$.

## SYSTEM IDENTIFICATION APPLICATIONS

The displacement at any point can be calculated according to the nonclassical or classical system equations from Eq. 9. The parameters involved are selected in such a way that an objective function representing the difference between the calculated and measured displacements is minimized. The objective function is defined as the sum of squares of the vector differences in displacements (Ref. 5). As the calculated displacements are the superposition of the modal contributions, it is essential that the number of modes included be determined appropriately. A simple method that determines the optimal number of parameters was used with success by McVerry and Beck (Ref.7) in the parameter identification involving the time history of earthquake records. This method is adopted in the present study to determine the optimal number of modes in the frequency domain applications. Details of the application of this method is given in a report (Ref.8). The system identification procedure involves the minimization of the objective function with a gradient search method and applying the minimization to nonclassical as well as classical system models. The results of forced vibration tests conducted in recent years on four arch dams are utilized for system identification studies.

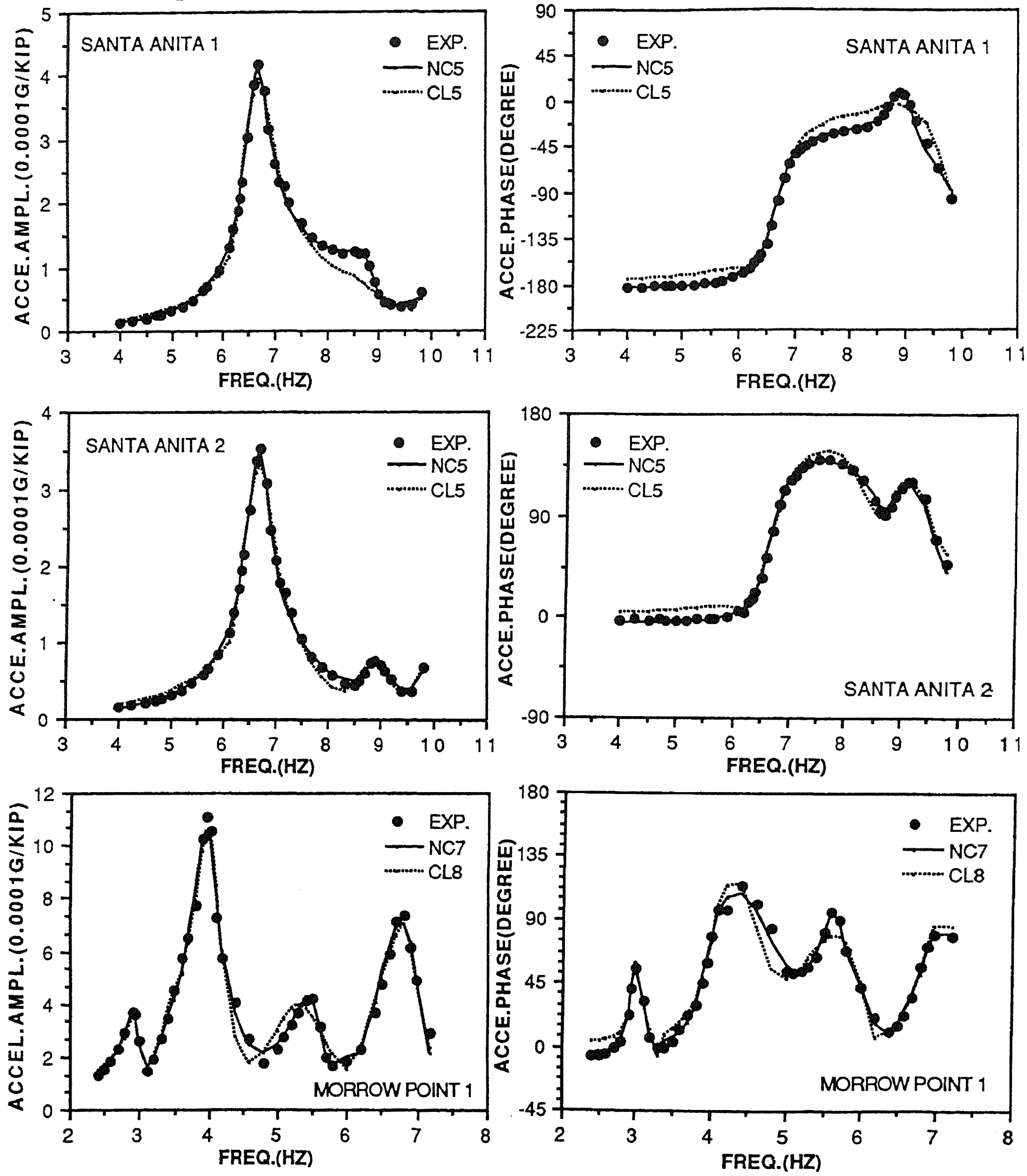


Fig. 1 Optimal Solutions for Two Sets of Santa Anita Dam Data and One Set of Morrow Point Dam Data.

Santa Anita Dam and Morrow Point Dam Duron and Hall (Ref.3) tested the Santa Anita Dam while the reservoir was nearly empty and the Morrow Point Dam while the reservoir was almost full. Their data include both the amplitude and phase responses. Three sets of radial responses at three points on the Santa Anita Dam crest are used in this study. The frequency range is from 4 Hz to 10 Hz, covering the first three modes. The optimal number of modes to fit the Santa Anita Dam data is 5 for both the classical and nonclassical models. Two sets of radial response data recorded on the Morrow Point Dam crest are available. The frequency ranges from 2.5 Hz to 7 Hz covering the first six modes. The optimum number of modes for the nonclassical case is 7, while for classical model is 8. A part of the optimal solutions for the two dams are shown in Fig.1.

Quan Shui Dam and Xiang Hong Dian Dam The Quan Shui Dam and the Xiang Hong Dian Dam were tested by a joint team of American and Chinese reseachers for the purpose of studying the hydrodynamic pressure caused by the vibration of the dam. (Refs.1,2). The dam crest responses were given in the form of amplitude response data only. Three sets of amplitude response data are used in this study. The frequency range used is from 2 Hz to about 10 Hz, covering the first five modes. The optimal number of modes is 8 for both the classical and the nonclassical models. The radial amplitude responses at two points on the Xiang Hong Dian Dam crest are available from 3 Hz to 24 Hz. The frequency responses up to 15 Hz, covering the first ten modes, are used in this study. The optimal number of modes is 11 for the nonclassical model and twelve for the classical model. A part of the optimal solutions for the two dams are shown in Fig.2. More details on the solutions for the four dams are contained in Ref.8.

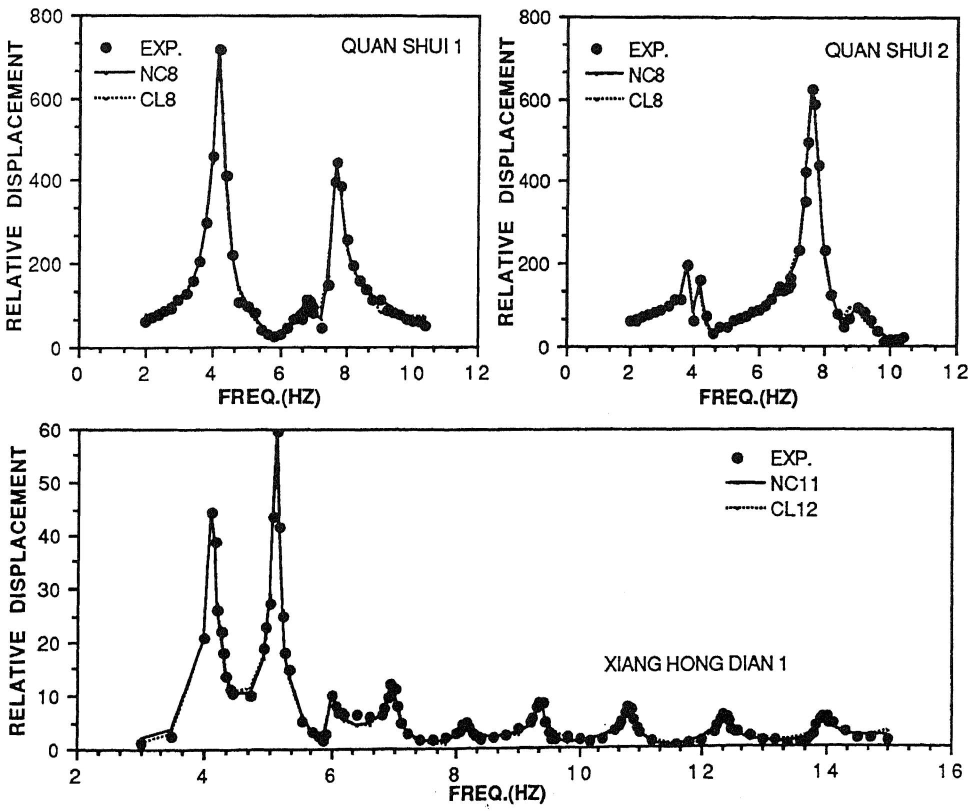


Fig. 2 Optimal Solutions for Two Sets of Quan Shui Dam Data and One Set of Xiang Hong Dian Dam Data.

## CONCLUSIONS

Based on the results of this study,the following conclusions may be reached.

1. A systematic method for the identification of modal parameters from forced vibration test data is developed. The method is capable of treating multiple-mode data measured at more than one point. The method has been applied to cases containing both amplitude and phase data and to cases with as many as ten modes and more than seventy unknown parameters.
2. For the four arch dam data, the nonclassical solution fit consistently better than the classical solution. For the two cases with both amplitude and phase data available, the nonclassical solution is clearly much better than the classical solution.
3. Comparison of the Santa Anita Dam result and the Morrow Point Dam result indicates a small discrepancy between the test data and the nonclassical solution when the reservoir effect is present. This discrepancy seems to suggest the presence of frequency dependent characteristics of the dam-foundation-reservoir system caused by the reservoir effect.
4. The nonclassical modes have more effects on the phase response than the amplitude response. Thus, for accurate determination of modal parameters, it is advisable to have both the amplitude and the phase data available.

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