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## A SIMPLE METHOD FOR THE FULL-SCALE 3-D DYNAMIC ANALYSIS OF ARCH DAM

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### SUMMARY

A Lagrangian method which could be easily implemented in standard codes is developed for the complicated 3-D time domain seismic interaction analysis of concrete dams. Development of special elements for the dam body and its reservoir interface has enabled a precise formulation of the arbitrary solid-structure system. Water is assumed as compressible, inviscid, and irrotational. Three dimensional viscous conditions are applied to the reservoir and foundation boundaries. The method proves very accurate and agrees with the results of modern Euler-Lagrangian analyses. Besides, site observations coincide surprisingly well with such computational results.

### INTRODUCTION

It has been shown that the hydrodynamic interaction could never be overlooked for arch dams located in areas of high seismicity (Ref.1). Due to the slenderness of modern arch dams simplified methods could not be applied anymore. On the other hand compressibility of water still affects the response of arch dams and is by no means negligible (Ref.2). Furthermore the three dimensional arbitrary shape canyon suggests FEM techniques to deal with the reservoir, and analytically known solution is seldom applicable.

For the purpose of a time domain seismic analysis method meeting the above desires and for which the extension to nonlinear cases be possible, contrary to the most of works done so far, a full Lagrangian formulation is proposed. Euler-Lagrangian methods are numerically tedious due to both the stability limits and the non-symmetry characteristics. In Ref.3 an efficient frequency domain analysis in which Fourier synthesis is needed to get the time history is devised. That method is not capable of nonlinear extension (e.g. due to joint opening and large deformations). Besides, the structural engineer would preferably like to be able to use his standard FEM code for such interaction analysis. However although the full Lagrangian approach could satisfy these needs a few problems have prevented it from getting fully successful. First, the mathematical model of the arbitrary shape fluid-structure interface has been immature or inaccurate, and this has introduced significant error in the analysis. Second, abundance of degrees of freedom vis-a-vis shortage of strain relations introduces some zero-energy modes in the response. These modes could almost be captured by rotational constraints using reduced integration order techniques (Ref.4). Third, if low approximating elements are used for the fluid domain, and if the model should be extended to a large distance, the number of degrees of freedom exceeds the economical limits.

The aforesaid shortcomings are eliminated by the methods suggested here. A special 3-D interface element similar to what employed in rock mechanics (Ref.5) has guaranteed the fluid-structure interface conditions which are of prime importance for correct interaction. As for the reservoir, application of the fluid elements with the rotational constraints has minimized the number of zero-energy modes in a way that even the impulsive modes of the system become possible to obtain. Furthermore the high precision of the fluid elements along with the radiation and refraction boundaries in three dimensions has helped considerable reduction in the sizes of the reservoir and foundation models. Shorter extension of the foundation model is justified with the virtue of a 3-D viscous boundary as an extension of its 2-D model (Ref.6). And this for the first time leads to a correct dynamic interaction analysis of arch dam for which the realistic flow of energy out of the system for all frequency contents of waves is guaranteed. The combination of the above techniques has made it possible to analyse such systems by merely adding a few new elements to the library of existing standard structural codes.

## METHODS

Fluid Model Isoparametric 27-node elements whose shape function is a Lagrange family polynomial of second order are employed (Ref.4). Element stiffness matrix has two parts. The volume contribution  $K_w$  is based on compressible and frictionless water with very small displacements, and is calculated using reduced integration order.

$$K_w = \int \tilde{B}^T C_w \tilde{B} dv$$

as  $\tilde{B}$  is the strain-displacement transformation matrix, and  $C_w$  the fluid elasticity matrix. The second part of stiffness is contributed from the free surface linear gravity waves. The fluid element mass is essentially consistent diagonal matrix as

$$M_{ii} = \frac{M\rho}{\sum m_{kk}} \int N_i^T N_i dv, \text{ provided } M = \rho \int dv$$

where  $M$ ,  $M_{ii}$ , and  $m_{kk}$  stand for element total mass, diagonal mass, and consistent mass coefficients. Matrix  $N_i$  is the shape function of node  $i$ , and  $\rho$  is the water mass density. Frictionless fluid is assumed and the only damping sources are radiation and refraction. The reservoir upstream truncation boundary should allow the travelling waves of any frequency to pass through it. Indeed this is feasible by introducing the 3-D Sommerfeld boundary over there. In other words a viscous traction is applied to such boundaries as

$$\sigma_n = \rho c \dot{U}_n$$

where  $c$  is the velocity of sound in water and  $\dot{U}_n$  the particle velocity in normal direction. The underlying theory of this boundary is based on the one-dimensional wave propagation with normal incidence on the boundary. Such an assumption is reasonably met at large enough distances.

With the same analogy wave refraction happens at the reservoir bottom and walls. The one-dimensional refraction for all wave types could be expressed as

$$\frac{d\phi}{dr} = -\frac{1}{r} \left( \frac{m-1}{2} \right) \phi - \frac{1}{c} \left( \frac{1+\alpha}{1-\alpha} \right) \dot{\phi}, \text{ where } \alpha = \frac{\rho_b V_b - \rho c}{\rho_b V_b + \rho c}$$

in which  $V_b$ , and  $\rho_b$  are the reservoir banks material compressional wave velocity and mass density.  $\alpha$  is the wave reflection coefficient,  $\phi$  is the wave amplitude,  $r$  is the propagation direction and  $m$  is a characteristic parameter whose value is equal to unity for plane waves. Then the viscous boundary traction for the reservoir banks is defined as

$$\sigma_n = \rho c \left( \frac{1+\alpha}{1-\alpha} \right) \dot{U}_n$$

The above two tractions contribute to the damping force  $F$  as

$$F = -\int N^T \sigma_n ds = -C_{ext} \dot{a}$$

where  $C_{ext}$  is the external damping matrix and  $\dot{a}$ , the nodal velocity vector.

Structure Model For consistency and ideal compatibility between the dam and its reservoir the upstream face of the dam elements should have 9 nodes. Therefore by adding two extra nodes on two opposite faces of the parabolic serendipity element we get a 22-node element whose shape function for corner nodes as well as for mid-layer nodes ( $i=1-8$  and  $17-20$ ) is expressed as

$$N_i = t^2 r s r_i s_i (1+rr_i)(1+ss_i)(1+tt_i)/8 - (1+rr_i)(1+ss_i)(1-t^2)(3t_i^2-2)/8$$

and for other mid-edge nodes ( $i=9-16$ ) as

$$N_i = (rr_i + ss_i)(1-r^2s_i^2 + rr_i)(1-r_i^2s^2 + ss_i)(1+tt_i)/4$$

and finally for mid-face nodes ( $i=21,22$ ) as

$$N_i = (1-r^2)(1-s^2)(1+tt_i)/2$$

where  $r, s$  and  $t$  stand for the curvilinear coordinates with  $t$  along the dam thickness direction. The corresponding integration order is 3 for the shell surface and 2 for the thickness direction.

Fluid-Structure Interface Model Three conditions (Ref.7) should be met on the interface namely;

- 1) Identity of normal displacements for both fluid and solid.
- 2) Absence of any tangential forces.
- 3) Identity of normal forces for both fluid and solid.

An isoparametric 3-D surface interface element (Ref.5) is modified to satisfy the above conditions. This element has no independent shape function and its only contribution is a stiffness matrix  $K^e$  defined as

$$K^e = \int B^T D B ds, \quad \text{where } B = \theta^T [N_{Top}, -N_{Bot}] \quad \text{and,} \quad D = \begin{bmatrix} C_n & 0 & 0 \\ 0 & C_{s1} & 0 \\ 0 & 0 & C_{s2} \end{bmatrix}$$

The coordinate transformation matrix  $\theta$  consists of three vectors  $s_1, s_2$ , and  $n$  as the two tangential and the normal direction cosines of the surface.  $N_{Top}$ , and  $N_{Bot}$  are the shape functions of fluid and structure elements respectively (as in Fig.1). Coefficients of elasticity  $C_n, C_{s1}$ , and  $C_{s2}$  also correspond to the normal and the tangential directions for relative displacement formulation. The interface force vector is obtained as  $f = D \Delta$ , with its components  $f_n = C_n \delta_n$ , and  $f_1 = C_{s1} \delta_1$  etc. Now by choosing  $C_n = \infty$  one gets  $\delta_n = 0$  to satisfy the first condition. Also  $C_{s1} = C_{s2} = 0$  is an appropriate choice to meet the slip condition and  $f_1 = f_2 = 0$ . The last condition is also automatically fulfilled.  $K^e$  should be calculated by an integration order equal to three.

Foundation Model Linear elasticity is applied for the 3-D foundation body. An extension radius of  $0.5H-0.8H$  was found sufficient when 20-node 3-D parabolic elements along with viscous non-reflective boundaries are employed. Furthermore the compatibility with the elements of dam are ensured. It is important to mention that due to the usage of boundary input motion the foundation mass is also accounted for. Damping consists of two parts; the internal viscous part and the external

radiation part. The non-reflective viscous boundary used to account for radiation could be expressed by a traction boundary condition as

$$f^b = \rho_f \bar{V} \dot{U}^b$$

where  $f^b$  is the traction in natural coordinates of the external boundary, and  $\rho_f$  is the rock unit mass.  $\bar{V}$  is a diagonal matrix of wave velocities (two shears and one dilatational), and  $\dot{U}^b$  is the particle velocity vector at the boundary. In fact this viscous boundary is the only frequency-independent non-reflective boundary that is applicable to the three dimensional geometry of arch dam foundation (Ref.8). Indeed by few investigations it was understood that the latter is very efficient in absorbing almost all types of waves involved in elastodynamics. The corresponding damping is found as follows.

$$f = \theta f^b$$

where  $\theta$  is already defined and  $f^T = [f_x, f_y, f_z]$  is the global traction. To find the global velocity on the boundary we use the relations below

$$\dot{U}^b = \theta^T \dot{U}$$

$$f = \theta \rho_f \bar{V} \theta^T \dot{U}$$

$$F = - \int N^T f ds = -C_{ext} \dot{a}, \text{ where } C_{ext} = \int N^T \theta \rho_f \bar{V} \theta^T N ds$$

is the foundation contribution to the external damping matrix. After adding the internal damping a non-proportional total damping matrix is obtained.  $F$  is a nodal force.

Equation of Motion The global equation of motion is obtained in a standard way as

$$\bar{K} a + \bar{C} \dot{a} + \bar{M} \ddot{a} = \bar{F}$$

where  $\bar{F}$  corresponds to the three dimensional seismic uniform motion of the system.  $\bar{K}$ ,  $\bar{C}$ , and  $\bar{M}$  are the global property matrices. For such a fluid-structure system formulation direct time integration of the coupled equation is suggested.

## RESULTS

Several cases were investigated and some of them are presented here. Dynamic pressure on a rigid wall excited by a harmonic horizontal motion is illustrated in Fig.2. A comparative study of response histories based on the results of the present method with that of a modern Euler-Lagrangian method (Ref.3) for the crest motion of Morrow Point arch dam due to the Taft ground motion reveals great similarities (Fig.3). Finally an actual assessment of the effectiveness of the method was carried out. The deep abutment motion of arch dam during an actual earthquake was used for input of the overall system. Then the calculated response of a mid-crest point was compared with its measured counterpart. This process was carried out for two Japanese arch dams. Significant correlations are observed as in Figures 4, 6, and 7. In Fig.6 the very low frequency components disagreements are probably due to the nonlinearities of rock and old concrete. On the other hand the very high frequency components (larger than about 6.5 Hz) are influenced by the mesh limitations (Fig.5). But in the interval in which the few lower modes of arch dam exist good agreement is observed.

## CONCLUSIONS

A general and relatively simple fluid-structure-foundation interaction analysis method is devised. This full Lagrangian approach is applied to the complicated system of arch dam. The accuracy of the method is verified by considering simple

cases and by comparing with Euler-Lagrangian methods. Great effectiveness is achieved when the method is applied to realistic arch dams. This enables a precise and simple full-scale 3-D analysis of such dynamic systems by standard structural analysis FEM programs. Besides, such methods could be extended to the nonlinear level of analysis without much difficulties.

#### ACKNOWLEDGMENTS

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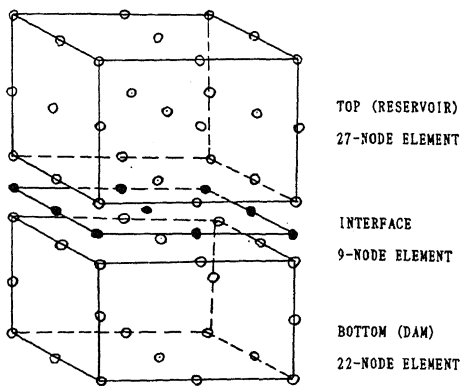


Fig.1-Combination of Parent Elements of Fluid, Interface and Structure, as Suggested for Lagrangian Formulation of Arbitrary Interfaces.

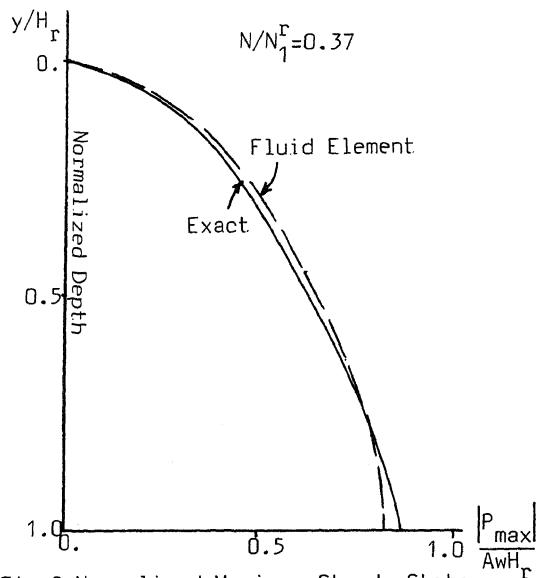


Fig.2-Normalized Maximum Steady-State Hydrodynamic Pressure on a Flat Rigid Dam

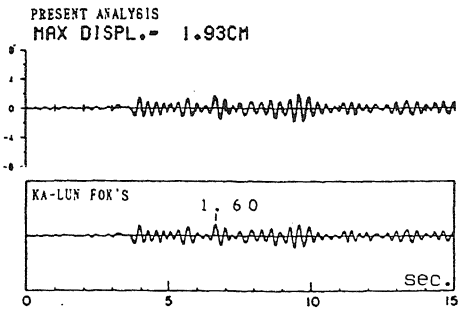


Fig.3- Displacement Response of Crest Center of Morrow Point Arch Dam due to Stream Component Earthquake (Taft) with Full Reservoir.  $\alpha = 0.5$

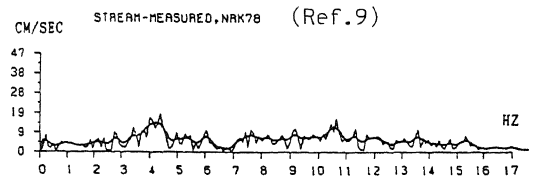
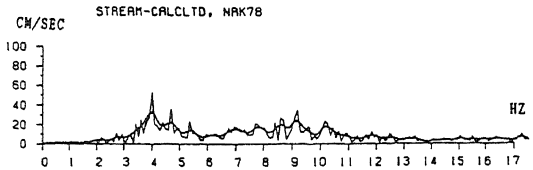


Fig.4- Fourier Amplitude Spectra of Stream Component Accelerations of a Point in the Center of Gallery of Naruko Arch Dam due to the 3-D Earthquake of 1978.

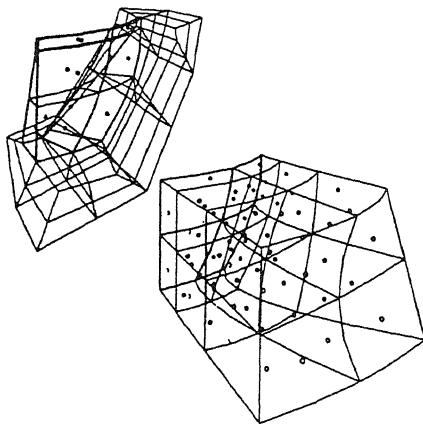


Fig.5- Finite Element Model of Yuda Arch Dam, Foundation and Reservoir.

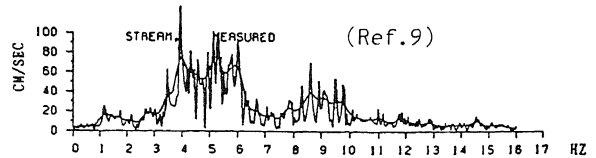
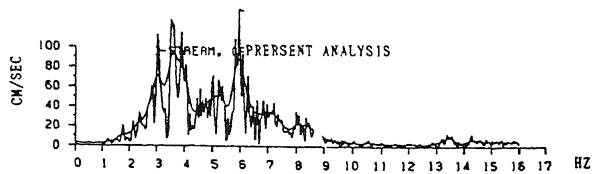


Fig.6- Fourier Amplitude Spectra of Stream Component Accelerations of Crest Center of Yuda Arch Dam due to Simultaneous Vertical and Stream Components of 1978 Earthquake.

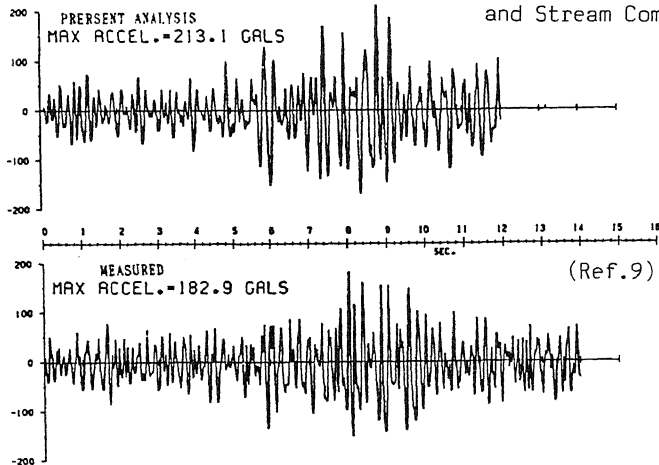


Fig.7- Stream Component Accelerations of Crest Center of Yuda Arch Dam due to Simultaneous Vertical and Stream Components of 1978 Earthquake.