PROCEDURES FOR ESTIMATING THE NONLINEAR EARTHQUAKE RESPONSE OF EARTH DAMS

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SUMMARY

Computational models are employed to investigate the nonlinear earthquake induced response of earth dams. Based on the shear slice vibration concept, a series of models are proposed to study upstream–downstream (UD), longitudinal (L) and vertical (V) earth dam dynamic response. These models range from simple 1–dimensional (1D) to elaborate 3D which account for gravitational effects and actual canyon configuration. Soil behavior is represented by an incremental plasticity constitutive relation. A multi–surface kinematic plasticity model is used so as to generate hysteretic yielding response. Step–by–step solutions are constructed using Time Integration procedures. Numerical results pertaining to the performance of La Villita Dam (Mexico) during the September 19, 1985 earthquake are shown and compared to actual observed response.

INTRODUCTION

Civil engineers are usually faced with various complexities when an analysis of stress and deformation in an earth embankment is attempted. In dynamic analyses, it is often necessary to undergo a broad parametric–type investigation in order to define a final elaborate numerical model. Simplified procedures are particularly appropriate for such situations and, in view of economical and/or practical considerations, may be the only available tool.

Simple linear dynamic analysis was originally proposed by Mononobe et al. [1] as early as 1936 for UD vibration of earth dams. Since then, similar assumptions led to the proposal of other linear models of UD, L and V vibration [2–4] of earth dams in idealized rectangular canyons. More recently, the above–mentioned UD vibration models [1,2] were extended to incorporate nonlinear hysteretic elasto–plastic constitutive soil relations [5–8]. In these nonlinear models, a Galerkin implementation of the method of weighted residuals is adopted. The solution of the equations of motion is expanded using a set of basis functions defined over the spatial domain occupied by the earth dam system. A semi–discrete matrix equation is obtained which is further discretized in time by the use of step–by–step integration.

In this paper, based on the simplified 2D linear models presented in Refs. 2–4 a 3D nonlinear coupled vibration model is developed. The model is also extended to account for own weight gravitational effects and represent arbitrary canyon geometry and foundation alluvial layers (if present). An incremental multi–yield surface plasticity model is employed to describe the dynamic behavior of soil thus generating hysteretic damping and permanent deformation effects. The dynamic response of La Villita dam (Mexico) during
the September 19, 1985 earthquake \((M = 8.1)\) is investigated using the developed 3D model. The numerically calculated acceleration response is used in a Newmark sliding-block-type approach \([9]\) to account for localized deformation observed along the dam crest.

**Formulation:** The initial/boundary value problem governing the response of the dam (Fig. 1) may be written as:

\[
\bar{\nabla} \cdot \bar{\tau} + \rho \ddot{\bar{y}} = \rho (\ddot{\bar{y}}_g + \ddot{\bar{y}})
\]
on \(\Omega\)

along with the boundary conditions

\[
\bar{y} = 0
\]
on \(\Gamma_u\)

\[
\bar{y} \cdot \bar{n} = 0
\]
on \(\Gamma_h\)

where \(\bar{\nabla} \cdot\) is the spatial divergence operator; \(\bar{\tau} = \bar{\tau}(\bar{x}, t)\) = generalized stress tensor; \(x = \{x, y, z\}^T\) = spatial coordinates; \(t\) denotes time; \(\rho = \rho(\bar{x})\) = mass density of the dam material; \(\bar{y} = \bar{y}(\bar{x}, t) = \{u(\bar{x}, t), v(\bar{x}, t), w(\bar{x}, t)\}^T\) = relative displacement vector, \(\ddot{\bar{y}} = \{g, 0, 0\}^T\) = body force due to gravitational own weight, \(g\) = gravitational acceleration; \(\Omega = \Omega(t)\) = spatial domain occupied by the dam; \(\bar{y}_g = \bar{y}_g(\bar{x}, t) = \{u_g(\bar{x}, t), v_g(\bar{x}, t), w_g(\bar{x}, t)\}^T\) = vector of input motion; \(\Gamma_u\) = part of boundary along which displacement is prescribed; \(\Gamma_h\) = part of boundary along which stress is prescribed; \(\bar{n}\) = unit outward normal to \(\Gamma_h\); a superposed dot denotes time differentiation, and \(\{\}^T\) denotes the transpose of a vector.

A simplifying assumption will be adopted in the following to reduce the above general 3D problem to an equivalent 2D problem. The displacement and stress will be assumed stationary along the \(z\) or upstream–downstream direction of the dam (i.e., \(\partial/\partial z = 0\)). This assumption, originally proposed by Mononobe et al. \([1]\) in 1936 was also adopted by \([2–8]\) for UD, L and \(V\) earth dam vibration models.

A set of simplified equations may now be obtained by integrating the stress and displacement over the \(z\)-direction (see Figure 1) and equating forces in directions \(x\), \(y\) and \(z\).

\[
\rho x \ddot{u} - \frac{\partial}{\partial x} (x \tau_{xx}) - \frac{\partial}{\partial y} (x \tau_{xy}) = - \rho x (\ddot{u}_g - g)
\]

\(1a\)

\[
\rho x \ddot{v} - \frac{\partial}{\partial x} (x \tau_{xy}) - \frac{\partial}{\partial y} (x \tau_{yy}) = - \rho x \ddot{v}_g
\]

\(1b\)

\[
\rho x \ddot{w} - \frac{\partial}{\partial x} (x \tau_{xz}) - \frac{\partial}{\partial y} (x \tau_{yz}) = - \rho x \ddot{w}_g
\]

\(1c\)
in \(\Omega\), along with the boundary conditions: \(\bar{y} = \bar{y}_g\) along the dam–canyon interface, and the stress–free crest boundary as shown in Figure 2.

**Solution:** In the present derivation, it is proposed to express the solution \(\bar{y}\) in the form

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\[ y = y_B + \sum_{n=1}^{N} \phi_n(x,y)q_n(t) \] (2)

in which \( \phi_n(x,y) \) are any set of comparison or admissible functions which satisfy homogeneous boundary conditions, and \( q_n(t) = \{q_{x_n}, q_{y_n}, q_{z_n}\}^T \) are generalized displacements.

The displacement \( y \) is to be represented to the desired accuracy by a finite number of functions \( N \). Following the Galerkin procedure of the method of weighted residuals, equation 1 along with its boundary conditions may thus be written in the following matrix form:

\[ \ddot{\mathbf{M}}\mathbf{Q} + \mathbf{F} = \mathbf{E} \] (3)

in which \( \mathbf{M} \) is the resulting mass matrix, \( \mathbf{Q} \) is the vector of unknown generalized displacements, \( \mathbf{F} \) is the nonlinear internal force vector and \( \mathbf{E} \) is the external gravitational and dynamic (earthquake) load vector.

In this investigation, shape functions \( (\phi_n, n = 1, 2, \ldots, N) \) which span the global domain \( \Omega \) are chosen. The functions depict a sine wave along the longitudinal direction of the dam superposed on a Bessel Function along the vertical direction of the dam (Fig. 3). Higher order functions will depict a higher frequency sine or Bessel. These global shape functions are chosen herein because of their close similarity to the linear mode shapes of uncoupled UD, L, or V oscillation of earth dams in rectangular canyons. The choice of the above-mentioned shape functions is by no means unique and any set of functions which satisfy the associated boundary conditions is perfectly appropriate.

Remarks: 1) Alluvium layers which may exist between the dam and the canyon bedrock can be modeled as a shear beam following Ambroseys [2]. The alluvium zone is accounted for using the same formulation (Eq. 1) with \( x \) equal to the height of the overlying dam at the particular location.

2) A diagonal mass matrix would result if the solution (Eq. 3) is expanded in terms of the linear mode shapes corresponding to the earth dam model. Analytically derived mode shapes were employed in the context of UD analysis [5–8].

3) A constitutive model based on the flow or incremental theory of plasticity is used. The model is capable of generating the Bauschinger effect exhibited by soil under cyclic loading. A J2 (Von Mises) set of nested yield functions which obey a kinematic hardening rule [5] are utilized (Fig. 4). Generated damping is purely hysteretic and independent of strain rate. Associative plasticity is adopted. The constitutive model described above may be substituted by any other nonlinear model deemed more accurate in representing cyclic soil response.

Earthquake Response of La Villita Dam [11]: La Villita, an earth and rockfill dam, is located in Mexico 13 kilometers upstream from the mouth of the Balsas River which empties into the Pacific Ocean. It was constructed in the period between 1964 and 1968. An alluvial layer of varying thickness (maximum thickness of 70 meters or 230 ft) lies between the embankment and bedrock. The dam is 440 meters (1467 ft) in length and 60 meters (196.8 ft) in height with a clay core and rockfill shells (Figure 5).

The 3D model described above is used in the section to study the coupled UD, L, and V response of La Villita Dam during the September 19, 1985 (M = 8.1) earthquake. The dam geometry and material properties are described in Refs. 10–12. A rock outcrop acceleration record (3–components) is taken as the input ground motion (peak input is .12g in T, .11g in L and .06g in V). The numerical response at the dam crest (center location) is shown in Fig. 6. Computed peak accelerations in the 3–directions were found to closely
match those actually measured during the earthquake (.7g in T, .31g in L and .30g in V), except for the UD where significant spikes have appeared (Fig. 7) in the measured acceleration record. The asymmetric appearance of the UD measured acceleration is attributed [10–12] to Newmark sliding—block—type [9] deformation. Using the crest UD computed acceleration record (Fig. 6) as input to a Newmark—sliding—block analysis the response of Fig. 8 is obtained. As shown in Fig. 8 a permanent deformation of about 5 cms is calculated (matches the value actually measured) and an acceleration response which is asymmetric in appearance (similar to Fig. 7) is also obtained.

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REFERENCES

Figure 1. Typical Three-Dimensional Earth Dam Configuration.

Figure 2. Geometric Configuration of Proposed Earth Dam Model.

Fig. 3. Shape Functions Defined Over the Global Domain.

Fig. 4. Yield Surfaces in 3D Stress-Space.

Fig. 5. La Villita Dam; (a) Maximum Cross-Section, (b) Longitudinal Section.
Fig. 6a. Computed Crest (Center) Longitudinal Acceleration

Fig. 6b. Computed Crest (Center) Vertical Acceleration

Fig. 6c. Computed Crest (Center) Transverse Acceleration

Fig. 7. Recorded Crest (Center) Transverse Acceleration

Fig. 8. Newmark Sliding-Block Analysis; Computed Transverse Acceleration and Resulting Relative Displacement.