Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.VI)

10-1-6

STOCHASTIC ANALYSIS OF SEISMIC STABILITY OF EARTH DAMS

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SUMMARY

This paper presents a practical method for the analysis of seismic stability of earth dams considering the three-dimensional shape of the dam particularly paying attention to the shear strain in the horizontal cross-section due to relative displacement of the dam along its longitudinal axis. The method takes into consideration both spatial variabilities in the dam material and geometry and randomness of the earthquake motion. The proposed method is used to determine the depth-wise distribution of shear stress in the horizontal cross-section of the dam as well as that in the transverse cross-section. Such a distribution of shear stress is in turn used to examine their effect on the seismic stability of the dam.

INTRODUCTION

The stability analysis of earth dams during earthquake is usually performed utilizing a two-dimensional model consisting of a representative transverse cross-section under plane strain condition. However, when the cross-section is not uniform as in the case where the dam is constructed in a narrow canyon and bounded by sloping canyon walls, the effect of the three-dimensional shape of the dam can be significant.

This three-dimensional effect is analyzed here under the assumption that the difference of response characteristics at each transverse cross-section of the dam and the time lag of input motion are primarily responsible for the effect. The dam is modelled as an assembly of one dimensional shear wedges which allow for shear deformation in the upstream-downstream direction. Then the relative displacement between adjacent wedges is estimated. The results are given in terms of the (a)variance of the relative displacement, (b)expected maximum value of the strain and (c)expected value of local safety factor during earthquake.

DESCRIPTION OF THE METHOD

Equation of Motion Consider an earth dam located in a triangular canyon as shown in Fig. 1. Input earthquake wave is considered to be vertically incident S-H wave with its amplitude in the upstream-downstream direction. The arrival time of the wave at the base varies according to its elevation. Herein the dam is modelled as an assembly of shear wedges as also shown in Fig. 1. The equation of motion of the shear wedge is then derived as follows (Ref. 1).

$$\rho B(y,z)(\partial^2/\partial t^2)[u_b(y,t) + u_r(y,z,t)] = (\partial/\partial z)[B(y,z)T_s] \qquad (1)$$

where ρ is the mass density, B is half the width of the wedge, T_s is the shear stress in the x-y plane of the wedge, and u_b is the displacement at the base of the wedge and u_r is the relative displacement to the base. The displacement of the wedge is expressed as the sum of the modal components as

$$u_r(y,z,t) = \sum_{i=1}^{\infty} J_0(\lambda_i z / L(y))\, x_i(t) \qquad (2)$$

where J_0 is Bessel function of the first kind of order 0, λ_i is the i-th root satisfying $J_0(\lambda) = 0$, $L(y)$ is the height of the transverse cross-section at y and x_i is the generalized coordinate.

Spatial Distribution of Predominant Frequency and Damping   Response characteristics of shear wedge represented by the resonance frequency and modal damping ratio are dependent on the wedge location specified by y. The spatial variability of resonance frequency $\omega^*_i(y)$ and modal damping ratio $h^*_i(y)$ as functions of y are expressed as follows (Ref. 2).

$$\omega^*_i(y) = \omega_i(y)[1 + f(y)], \qquad h^*_i(y) = h_i(y)[1 + h(y)] \qquad (3)$$

where $\omega_i(y)$ and $h_i(y)$ are the mean values of $\omega^*_i(y)$ and $h^*_i(y)$, and $f(y)$ and $h(y)$ are homogeneous stochastic fields which are considered to result from the randomness of material properties. In Eq. (3), it is assumed $E[f^2(y)] \ll 1$ and $E[h^2(y)] \ll 1$ with $E[\cdot]$ indicating expectation operator. It should be noted that although f(y) and h(y) are homogeneous stochastic fields, $\omega^*_i(y)$ and $h^*_i(y)$ are non-homogeneous because the mean values $\omega_i(y)$ and $h_i(y)$ are dependent on y. Relationships between the variance $\sigma_{\omega^*\omega^*}$ of $\omega^*_i(y)$ and that of f(y), and between the variance $\sigma_{h^*h^*}$ of $h^*_i(y)$ and that of h(y) are

$$(\sigma_{\omega^*\omega^*})^2 = \omega^2_i(y)(\sigma_{ff})^2, \qquad (\sigma_{h^*h^*})^2 = h_i^2(y)(\sigma_{hh})^2 \qquad (4)$$

where σ_{ff} and σ_{hh} are the standard deviations of f and h respectively.

Spatial variability of Response   In estimating seismic stability of earth structures such as earth dams, effect of the first mode is predominant. Hence, the first mode is considered hereafter. Taking the first term of Eq. (2), the relative displacement u_r of the dam to the ground considering spatial variability is given as follows.

$$u^*_r(y,z,t) = -\beta J_0(\lambda_1 z / L(y)) \int_{-\infty}^{\infty} I^*(y,\tau)\, u_b(t-\tau-\eta(y)/C_{SR})\, d\tau \qquad (5)$$

where β is the modal participation factor, $I^*(y,\tau)$ is the impulse response function considering spatial variability, u_b is input acceleration at the base of the dam, η is the elevation of the dam base and C_{SR} is the shear wave velocity associated with the bedrock. The time-space correlation function Q_{UU} of the total displacement $u(y,z,t)$ $(= u^*_r(y,z,t) + u_b(t-\eta(y)/C_{SR}))$ is defined by

$$Q_{UU}(y,z,t,\xi,\tau) = E[u(y,z,t)u(y+\xi,z,t+\tau)] \qquad (6)$$

where ξ and τ are the spatial and time separations. Temporally spectral and spatially correlational function P_{UU} and spatial correlation function of u, R_{UU} are defined as

$$P_{UU}(y,z,\xi,\omega) = 1/(2\pi) \int_{-\infty}^{\infty} \exp(-i\omega\tau)\, Q_{UU}(y,z,\xi,\tau)\, d\tau \qquad (7)$$

$$R_{UU}(y,z,\xi) = Q_{UU}(y,z,\xi,0) = \int_{-\infty}^{\infty} P_{UU}(y,z,\xi,\omega)\, d\omega \qquad (8)$$

where the stationarity of u with respect to time is assumed. For the stochastic estimation of relative displacement of the dam during earthquake, P_{uu} is derived following Ref. 2 and 3. Expanding the unit impulse response function into Taylor series with respect to ω^* and h^* around their mean values ω_0 and h_0. And truncating higher terms beyond the second, P_{uu} is obtained as follows.

$$P_{uu}(y,z,\xi,\omega) = S_{\ddot{u}_b\ddot{u}_b}(\omega)\exp(-i\omega/C_{SR}[\eta(y+\xi)-\eta(y)])$$
$$\times\{[1/\omega^2+\beta J_0(\lambda_1 z/L(y))H(-\omega,y)][1/\omega^2+\beta J_0(\lambda_1 z/L(y+\xi)H(\omega,y+\xi)]$$
$$+4R_{ff}(\xi)\beta^2 J_0(\lambda_1 z/L(y))J_0(\lambda_1 z/L(y+\xi)H^2(-\omega,y)H^2(\omega,y+\xi)$$
$$\times\omega_1{}^2(y)\omega_1{}^2(y+\xi)[1+i\omega(h_1 y+\xi)/\omega_1(y+\xi)-h_1(y)/\omega_1(y)]\} \quad (9)$$

where $S_{\ddot{u}_b\ddot{u}_b}(\omega)$ is the power spectral density of input ground acceleration, $R_{ff}(\xi)$ is the correlation function of f(y) and $H(\omega,y)$ is the first modal frequency response function for relative displacement.

Estimation of Shear Strain Consider relative displacement u_D along the longitudinal (y) axis between two points specified by y and y + D at a depth z from the crest,

$$u_D(y,z,D,t) = u(y+D,z,t) - u(y,z,t) \quad (10)$$

Then space-time correlation function $Q_{u_Du_D}$ of u_D is given by

$$Q_{u_Du_D}(y,z,D,\xi,\tau) = E[u_D(y,z,D,t)u_D(y+\xi,z,D,t+\tau)] \quad (11)$$

and spatial correlation function $R_{u_Du_D}$ (= $Q_{u_Du_D}(y,z,D,\xi,0)$)) is given by substituting Eq. (10) into Eq. (11) and using Eq. (8) as

$$R_{u_Du_D}(y,z,D,\xi) = R_{uu}(y+D,z,\xi) + R_{uu}(y,z,\xi) - R_{uu}(y,z,D+\xi)$$
$$- R_{uu}(y+D,z,\xi-D) \quad (12)$$

The spatial correlation function $R_{u_Du_D}$ with respect to relative displacement along vertical (z) axis is derived in the same way using spatial correlation function R_{uu} with respect to z.

As for the strain, consider the local average γ_D of shear strain γ defined by

$$\gamma_D(j) = (1/D)\int_j^{j+D}\gamma(j)dj = (1/D)u_D(j), \quad (j = y \text{ or } z) \quad (13)$$

Then the variance $\sigma^2_{\gamma_D\gamma_D}$ of γ_D is given by

$$\sigma^2_{\gamma_D\gamma_D} = (1/D^2)\sigma^2_{u_Du_D} \quad (14)$$

After the variance of shear strain is obtained, one can estimate expected maximum value of shear strain using peak factor, PFA, derived from probability distribution for extreme values (e.g. Ref. 4) as follows.

$$(\gamma_D)_{max} = PFA\cdot\sigma_{\gamma_D\gamma_D} \quad (15)$$

$$PFA \fallingdotseq \sqrt{2\ln(2\nu T)} + \gamma/\sqrt{2\ln(2\nu T)} \quad (16)$$

where ν is the apparent frequency of the process, T is the duration time of the process and γ is Euler's constant (= 0.5772···).

Input Earthquake Motion As the power spectrum $S_{\ddot{u}_b\ddot{u}_b}(\omega)$ of input acceleration, the filtered Kanai-Tajimi spectrum (Ref.5) given below is used in which singularity at $\omega = 0$ is removed to make the estimation of displacement variance possible.

$$S_{\ddot{u}_b\ddot{u}_b}(\omega) = S_A(\omega)(\omega/\omega_f)^4/[\{1-(\omega/\omega_f)^2\}^2 + 4\zeta_f^2(\omega/\omega_f)^2] \qquad (17)$$

with $S_A(\omega)$ being the Kanai-Tajimi spectrum (Ref.6) given as

$$S_A(\omega) = S_0[1 + 4\zeta_g^2(\omega/\omega_g)^2]/[\{1-(\omega/\omega_g)^2\}^2 + \{2\zeta_g(\omega/\omega_g)\}^2] \quad (18)$$

where S_0 is the intensity of white noise, ζ_g and ω_g are damping ratio and natural frequency when the ground is considered as a SDOF system, and ζ_f and ω_f are damping and frequency parameters determined to give desired filter characteristics. Fig. 2 shows the power spectrum of input earthquake acceleration given by Eq. (17), which is used in the following analyses. Parameters used are $\omega_g = 8\pi$ (rad/sec), $\zeta_g = 0.6$ (ω_g and ζ_g for bedrock; Ref. 7), $\omega_f = \pi/2$ (rad/sec) and $\zeta_f = 0.6$.

Estimation of Local Safety Factor In the stability analysis of earth dams, evaluation of local safety factor is carried out by which potential sliding surfaces are assumed and occurrence of local fracture is estimated. Local safety factor is defined as the ratio of available shear strength of the soil to shear stress at each portion of the dam. Using Mohr-Coulomb's failure criterion for the strength of the soil and coefficient of lateral stress at rest K_0, the local safety factors $(F_s)_{xz}$ and $(F_s)_{xy}$ in the x-z plane (transverse cross-section) and x-y plane (horizontal cross-section) are respectively given as follows.

$$(F_s)_{xz} = [C\cos\phi + \{(1+K_0)/2\}\sigma_z\sin\phi]/[(\sigma_z/2)^2(1-K_0)^2 + (\tau_{xz})_d^2]^{1/2}$$
$$(F_s)_{xy} = (C\cos\phi + K_0\sigma_z\sin\phi)/(\tau_{xy})_d \qquad (19)$$

where C is the cohesion, ϕ is the internal friction angle and $(\tau_{xz})_d$ and $(\tau_{xy})_d$ are the dynamic shear stresses due to earthquake.

RESULT OF ESTIMATION

Local Safety Factor In the estimation of local safety factor $C = 9.8 \times 10^4$ N/m^2 and $\phi = 40^\circ$ are used for the calculation of shear strength. The correlation function used in the analysis is of negative exponential form given by

$$R_{ff}(\xi) = \sigma_{ff}\exp(-(\xi/b)^2) \qquad (20)$$

where b is the correlation distance. Figs. 3 and 4 show distribution of local safety factor, respectively for the cases in which the slopes of the canyon wall are 1/1 and 1/4 at a depth of 10m from the surface. In the former case, local safety factor in the horizontal cross-section is smaller than that in the transverse cross-section in every portion of the dam, while in the latter case, in the upper part of the dam, local safety factor in the horizontal cross-section is smaller. These results show that as the level in the dam increases local safety factor in the transverse cross-section increases while that in the horizontal cross-section decreases. And in the portion closer to the surface, local safety factor in both cross-sections decreases, where local safety factor in horizontal cross-section tend to be smaller than that in the transverse cross-section. This trend becomes more conspicuous when the canyon wall becomes steeper reflecting the corresponding spatial distribution of shear strain under these circumstances.

CONCLUSIONS

This paper presented a practical method for estimating the relative displacement and the strain in the earth structures such as earth dams constructed in a triangular narrow canyon. It was shown that shear strain in the horizontal cross-section is an important factor in the estimate of seismic stability of the dam especially when the dam is located in a narrow canyon.

ACKNOWLEDGEMENT

This work was partially supported by the National Center for Earthquake Engineering Research under NCEER Contract Number 87-3008 under NSF Master Contract Number ECE-86-07591. The authors also acknowledge the support provided by the Central Research Institute of Electric Power Industry, Japan.

REFERENCES

1. Okamoto, S., Introduction to Earthquake Engineering, University of Tokyo Press, Tokyo (1984)
2. Harada, T. and Shinozuka, M., Ground Deformation Spectra, Proc. of the 3rd U.S. National Conference on Earthquake Engineering (1986)
3. Harada, T. and Shinozuka, M., Stochastic Analysis of Ground Response Variability for Seismic Design of Buried Lifeline Structures, Proc. of the 7th Japan Earthquake Engineering Symposium (1986)
4. Davenport, A.G., Note on the Distribution of Largest Value of a Random Function with Application to Gust Loading, Proc. Inst. Civil Eng., Vol. 28, (1964)
5. Clough, R.W. and Penzien, J., Dynamics of Structures, McGrawhill Inc. (1975)
6. Tajimi. H., A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake, Proc. 2nd WCEE (1960)
7. Ellingwood, B.R. and Batts, M.E., Characterization of Earthquake Forces for Probability-Based Design of Nuclear Structures, Nuclear Regulatory Commission Report NUREG/CR-2945, BNL-NUREG-51587 (1982)

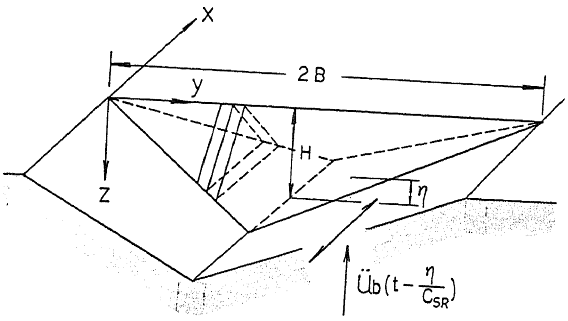

Fig. 1 Three-Dimensional View of Earth Dam in Triangular Canyon and Modelling of the Dam in the Present Analysis

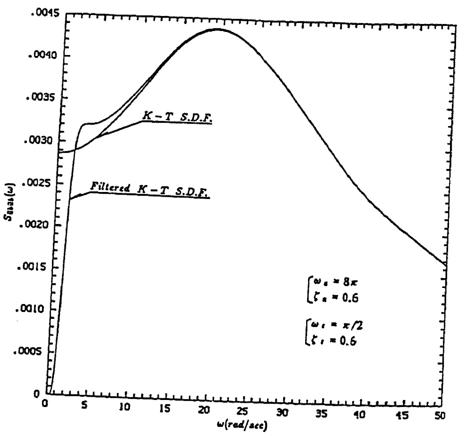

Fig. 2 Power Spectral Density of Input Acceleration (Filtered and Non-Filtered Kanai-Tajimi Spectrum)

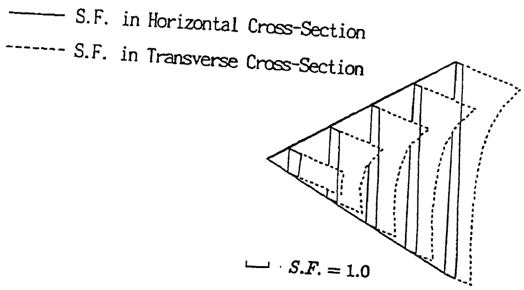

Fig. 3 Distributioin of Local Safety Factor (B/H = 1/1 10m from the Surface)

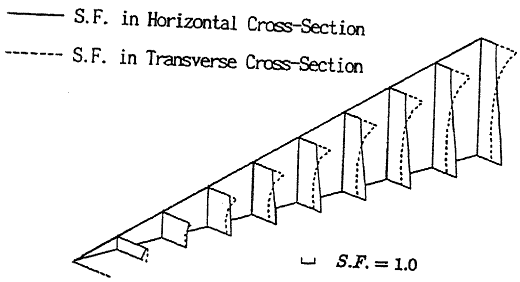

Fig. 4 Distributioin of Local Safety Factor (B/H = 1/4) 10m from the Surface)