SIMPLIFIED 3-D SEISMIC ANALYSIS OF EMBANKMENT DAMS

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SUMMARY

A numerical procedure for seismic response analysis of embankment dams accounting for canyon geometry and variable material properties in plan and elevation is presented. Material nonlinearities are treated by the linear equivalent method. The proposed procedure is a straightforward application of the shear beam concept incorporating canyon wall restraint by means of horizontal springs inserted along the height of the shear wedge. Some numerical examples performed with actual dam sections show good agreement with more elaborate F.E. models, thus making it attractive for calculations at design stage.

INTRODUCTION

Seismic response analysis of embankment dams on narrow valleys requires consideration of canyon geometry and variation of material properties both in plan and elevation. At present, this can be best accomplished using 3-D finite element models as proposed in (4) and more recently in (6), allowing accurate representations of the dynamic properties of the complete embankment. While this approach is desirable, its application at design stage may be limited by the computational effort required, rendering more attractive other approximate procedures that account for both horizontal and vertical shear transfer from the base rock.

Various simplified procedures to account for 3-D effects have been proposed in the past. Abdel-Ghaffar et al. (8) developed explicit expressions for the natural frequencies of a homogeneous shear wedge on rectangular canyons, both for longitudinal and transverse displacement modes, coupling exact shear wedge expressions with trigonometric expansions across the valley. Dakoulas et al. (2) have extended a similar concept to dams on semi-cylindrical canyons. They have also analyzed a certain class of inhomogeneous sections (1) using a semi-analytical approach.

Response calculations of embankment dams, however, must allow for variable properties within the section due to variations in confinement pressures and
materials. This paper is concerned with a generalization of a numerical integration procedure (5) for analysis of embankment dams under horizontal seismic excitations that is capable of accounting for canyon cross section and variable material properties in plan and elevation. The procedure, that follows the linear equivalent method due to Seed and co-workers (7), has been shown to furnish a reasonably accurate representation of earthquake-induced shear stresses as compared to more elaborate finite element models using QUAD 4 program in long dams. A simplified representation of the 3-D effects is accomplished by coupling the shear wedge model with a series of horizontal shear beams of rectangular cross section supported at the opposite sides of the valley. Calculations are performed numerically, thus allowing consideration of arbitrary valley cross section.

SIMPLIFIED MODEL

An idealized form of valley and dam geometry is given in Fig.1. Canyon walls are assumed to be prismatic with a vertical plane of symmetry, but of otherwise arbitrary shape. The embankment is also assumed bounded by a prismatic surface with horizontal generatrix lines normal to those of the canyon. The dam cross section at the vertical plane of symmetry depicted in Fig.2 is subdivided into horizontal layers at evenly spaced elevations. Equivalent shear modulus within each layer is evaluated as a weighted average of values for segments of the layer in terms of the mean effective confinement pressure and of earthquake-induced shear strains. Dynamic equilibrium of the shear wedge in the vertical plane is governed by:

\[
\frac{d}{dy} \left[ b \left( G \dot{Y} + c \ddot{Y} \right) \right] - \gamma b \ddot{U}_a = 0
\]  (1)

in which \( \dot{Y} = \partial u/\partial t \) = the shear strain, \( u \) = the horizontal displacement along the canyon; \( \dot{Y} \) = the vertical coordinate; \( \ddot{Y} = \partial \dot{Y} / \partial t \) = the shear strain rate; \( c \) = viscous damping coefficient; \( b \) = width of the shear beam as a function of \( y \); \( \gamma \) = mass density of the layer; \( \ddot{U}_a = \ddot{U}_s + \ddot{U} \) = the absolute acceleration at elevation \( y \), and \( \ddot{U}_s \) = the acceleration at the base.

To introduce the restraining effect on the shear wedge provided by the horizontal shear beams, it is assumed that the horizontal displacement has a parabolic distribution across the canyon:

\[ u(x, y) = u(0, y) \left( 1 - (y/B)^2 \right) \]  (2)

This approximation implies a linear variation of shear with \( x \) and a constant change of shear between adjacent wedges. The restraining effect on the central section, \( T \), is then given by:

\[ T = \frac{2G}{B^2} b u(0, y) \]  (3)

This term is then introduced into eq. (1):

\[
\frac{d}{dy} \left[ b \left( G \dot{Y} + c \ddot{Y} \right) \right] - 2 \frac{Gb}{B^2} u - \gamma b \ddot{U}_a = 0
\]  (4)

For a base acceleration of amplitude \( \ddot{U} e^{i\omega t} \), assumed uniform along canyon walls, eq.(4) reduces to:

\[
\frac{d}{dy} \left( bc^* \ddot{Y} \right) + \left( \gamma bu^2 - 2 \frac{Gb}{B^2} \right) \ddot{U} - \gamma b \ddot{U}_s = 0
\]  (5)

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Eq. (5) is now solved numerically by introducing piecewise linear interpolation expressions for $U$ and $\Gamma$, leading to the following recurrence relations:

$$
\Gamma_{i+1} = \frac{1}{D} [(G^*b)_i \Gamma_i - \varphi b_m \Delta_1 \omega^2 (U_i + \frac{\Gamma_i \Delta_1}{4}) + \varphi b_m \Delta_1 U_s + \frac{2b_m}{B_m} G^* \Delta_1 U_i] \tag{6a}
$$

$$
U_{i+1} = U_i + (\Gamma_i + 1 + \Gamma_i) \frac{\Delta_1}{2} \tag{6b}
$$

in which $D = (G^*b)_{i+1} + \varphi b_m \omega^2 \Delta_1^2 / 4$; $b_m$ and $B_m$ are mean values of $b$ and $B$ between layers $i$ and $i+1$; $\Delta_1 = y_{i+1} - y_i$. The solution vector $V = \{\Gamma_i, U_i\}$ is expressed in terms of a particular solution for the acceleration input $\ddot{U}_s$ designated $V_0$ and a homogeneous solution $V_1$ obtained for $\ddot{U}_s = 0$:

$$
V = V_0 + c V_1 \tag{7}
$$

The integration procedure is completed introducing the boundary conditions of the problem, whereby the shear strain at $y = 0$ is zero, and the relative displacement at the base ($y = H$) is also zero. These conditions lead to the actual value of $c$. Once $U$ is obtained, the absolute acceleration response is given by:

$$
\ddot{U}_a = \ddot{U}_s - \omega^2 U \tag{8}
$$

The equilibrium equation in the longitudinal direction of the dam is similar to eq. (5); the restraining effect term $T$ is similar to that given by eq. (3), but $G$ is replaced by $E$ therein; displacement and all associated variables refer to the longitudinal axis of the dam.

The solution in the time domain is obtained through the inverse FT of response variables $U$, $\ddot{U}$ and $\Gamma$.

Following the linear equivalent method, the maximum strain $\gamma_{\text{max}}$ for each layer is obtained, and the shear modulus and damping is recalculated from experimental curves due to Seed and co-workers (7) as function of the equivalent shear strain $\gamma_{\text{eq}} = 0.65 \gamma_{\text{max}}$. The complete integration procedure is recycled until convergence of the shear modulus and damping is reached in all layers.

**NUMERICAL RESULTS**

To test the accuracy of the proposed procedure, the acceleration transfer function at the apex of a homogeneous triangular wedge is calculated. Results for various canyon shapes and for the indefinite section (plane) are given in Fig. 3. Table 1 shows natural frequencies associated with the peaks and those given by analytical expressions (3). A total of 60 layers was used in the model. Good agreement is found, assessing the ability of the proposed procedure to incorporate 3-D effects into account in seismic dam response. Further numerical examples are presently under way to compare results of the proposed model to actual field response measured at the Infierriillo Dam and also to response calculations performed with a 3-D finite element model.
CONCLUSIONS

A simplified procedure for seismic response analysis of embankment dams accounting for 3-D effects has been presented. The procedure is simple, yet is capable of representing main features of the restraint provided by narrow canyons. Numerical examples performed in idealized cases show good agreement with semi-analytical solutions, but further testing is required to assess numerical results obtained with actual measurements in the field. The proposed scheme is well suited for calculations at design stage since it requires very modest programming effort affordable with small computing equipment.

REFERENCES


TABLE 1. COMPARISON OF FUNDAMENTAL PERIODS
Vibration along canyon axis

<table>
<thead>
<tr>
<th>Case</th>
<th>Analytical (3)</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane shear beam</td>
<td>$T_o = 2.61 \frac{H}{v_s}$</td>
<td>$T_o = 2.64 \frac{H}{v_s}$</td>
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<tr>
<td>Semi-cylindrical canyon</td>
<td>$T_o = 2 \frac{H}{v_s}$</td>
<td>$T_o = 2.10 \frac{H}{v_s}$</td>
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<td>Rectangular canyon B/H=1</td>
<td>$T_o = 2.19 \frac{H}{v_s}$</td>
<td>$T_o = 2.21 \frac{H}{v_s}$</td>
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<tr>
<td>Triangular canyon B/H=1</td>
<td>$T_o = 1.60 \frac{H}{v_s}$</td>
<td>$T_o = 1.65 \frac{H}{v_s}$</td>
</tr>
</tbody>
</table>

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Fig. 1 Canyon Geometry

Fig. 2 Cross section discretization

Fig. 3 Acceleration Transfer Functions at Midcrest