NONLINEAR DYNAMIC ANALYSIS OF STRUCTURES
INCLUDING HYDRODYNAMIC AND FOUNDATION INTERACTION EFFECTS

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SUMMARY

Presented in this paper is a method of analyzing nonlinear dynamic response
of structures considering the effects of foundation and hydrodynamic interactions.
The method is a finite element/finite difference hybrid scheme in which finite
difference part computes the radiation of waves into infinity. It is shown from
numerical results of simple scalar equation that the proposed method gives better
results than other numerical methods. The combined effects of nonlinearity and
interaction with foundation and fluid on the dynamic response of structures are
investigated by the proposed methods.

INTRODUCTION

In the analysis of the dynamic responses of systems which include infinite
domain using finite element or finite difference method, it is necessary to
introduce the boundary which does not reflect the radiating waves. The non-
reflecting boundary can be solved fairly well in numerical procedures which work
in the frequency domain. But for non-linear dynamic analysis of structure, it is
desired to develop the efficient scheme which works in time domain and absorbs all
types of waves.

This paper reports a method of time domain analysis for semi-infinite system.
The method can deal with general wave propagation problems of semi-infinite
structure-fluid-foundation system in one and two dimension and easily be extended
to three dimensional problem. The method is firstly applied to simple scalar wave
equations and the results are compared with those of other methods which make use
of direct integration in the time domain. Next, the method is applied to two-
dimensional dynamic response analysis of a concrete dam and the combined effects
of nonlinearity and interaction with foundation and fluid on the response are
investigated.

NUMERICAL METHOD

The proposed method is a hybrid finite element/finite difference method. The
finite element part deals with the near field part and the finite difference part
deals with the far field part (Fig. 1). In the near field part, ordinary finite
element scheme is used. Joint elements are incorporated to deal with crack
opening or slip/separation boundary conditions. Newmark's $\beta$-scheme is used for
time integration. The scheme used in the far field part is described below. At
first, the equation of motion for visco-elastic solid and water are transformed
into first order hyperbolic systems (Ref. 1).
\[ \frac{\partial \Phi}{\partial t} = A_x (\partial \Phi / \partial x) + A_z (\partial \Phi / \partial z) \]
\[ \frac{\partial V}{\partial t} = B_x (\partial V / \partial x) + B_z (\partial V / \partial z) \]
\[ \Phi = (\partial \Phi / \partial t, \partial \Phi / \partial x, \partial \Phi / \partial z)^t \]
\[ V = (u, v, \sigma_x, \tau, \sigma_z)^t \]  
(1)

where \( \Phi \) is velocity potential, \( u \) is horizontal velocity, \( V \) is vertical velocity, \( \sigma_x \) and \( \sigma_z \) are normal stress, and \( \tau \) is shear stress. Then, the transformation of coordinates are applied to the system to map the unbounded domain into a rectangle through the following relation.

\[
x = \begin{cases} 
0 & 1/a(\xi) \, d\xi \\
L_x & 1/b(\eta) \, d\eta 
\end{cases} 
\]

for \(|x'| < L_x\) and \(|\eta'| < L_z\).

\[ a(\xi) = 1 \quad \text{for} \quad |x'| < L_x \]

\[ b(\eta) = 1 \quad \text{for} \quad |\eta'| < L_z \]  
(2)

where \( x \) and \( z \) are the original coordinates, \( x', \eta' \) are transformed coordinates and \( a(\xi) \) and \( b(\eta) \) vanish as \(|\xi| \) and \(|\eta| \) approaches \( L_x \) and \( L_z \). The resulting equation are given as follows.

\[ \frac{\partial \Phi}{\partial t} = a(x') A_x (\partial \Phi / \partial x') + b(z') A_z (\partial \Phi / \partial z') \]

\[ \frac{\partial V}{\partial t} = a(x') B_x (\partial V / \partial x') + b(z') B_z (\partial V / \partial z') \]  
(3)

These equations are solved by Strang's formula for Lax-Wendroff differentiating scheme (Ref. 2).

\[ \Phi(t + k) = L_x(k; H; B_x)L_z(k; Hw; B_z) \Phi(t) \]

\[ V(t + k) = L_x(k; A_x)L_z(k; Hw; A_z) V(t) \]  
(4)

where \( L_x \), \( L_z \) are Strang's one step difference operator in \( x \) and \( z \) direction, \( k \) is time increment, \( H \) is mesh size for solid and \( Hw \) is mesh height for water. The stability limit is approximately given as follows.

\[ k \leq \min(H/V_x, Hw/C) \]  
(5)

where \( V_x \) is the velocity of dilatational wave in solid and \( C \) is the velocity of sound in water. The input of seismic wave is treated as follows. The total motion is composed of incident wave \( V_i \) and radiation wave \( V_x \).

\[ V = V_i + V_x \]  
(6)

The incident wave \( V_i \) is assumed to be known and is inputted at boundary \( A \) in Fig. 2. Only \( V_x \) is computed on and outside \( A \) by Eq. (4), while \( V \) is computed on and inside \( A \) in Fig. 2, where \( A \) is located just one mesh width within \( A \). In the computation of \( V_x \) on \( A \) by Eq. (4), \( (V - V_i) \) instead of \( V \) is used on \( A^\prime \) and \( (V_x + V_i) \) on \( A \) is used to obtain \( V \) on \( A \).

**COMPARISON WITH OTHER METHOD**

The proposed method is applied to the following simple scalar equation.

\[ \frac{\partial^2 \Phi}{\partial t^2} + q(\partial \Phi / \partial t) = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \]  
(7)
The results are compared with those by other two numerical methods in time domain. One is the viscous boundary method, where viscous dashpots are used to absorb radiating waves. The other is the superposition method, where the boundary reflection are cancelled out by superposing the solutions corresponding to constant-velocity and constant-stress boundary conditions incrementally at the boundaries. The initial and boundary conditions are as follows.

$$\Phi(t, x) = 0 \quad (t = 0 \text{ and } 0 < x < \infty)$$

$$\Phi(t, 0) = f(t) \quad (t > 0) \quad (8)$$

The mesh width and time increment are fixed to be 0.1 and 0.08, respectively. Artificial boundary is set at x = 1.0. For superposition method, the length to superposition zone is set to be 0.5. Results are shown in Figs. 3 and 4. When no internal damping exists (\(q = 0\)), viscous boundary method gives good results, while several percent of error arises when internal damping exists (\(q = 0.2\)). The results of superposition method shows drifting and does not converge to true solution. The proposed method gives good results both with and without internal damping. Numerical example of the application of the method to the semi-infinite solid to impulsive surface loading demonstrates the good performance of the method in two dimensional wave propagation problem (Figs. 5, 6).

**RESPONSE OF CONCRETE GRAVITY DAM**

Proposed method is applied to the dynamic response analysis of gravity dam. Analytical model is shown in Fig.7. Height of the dam is 140m. Only FEM part is shown in the figure. The material properties are shown in Table 1. The slip/separation boundary conditions of construction joint and contact plane between dam body and bedrock are as follows.

$$\sigma_n = K_n \cdot \varepsilon \cdot n$$

$$\sigma_n = 0$$

$$\tau = K_s (\varepsilon_{n} - \gamma_s)$$

$$\tau = K_s (\gamma_s - \tau_o) \leq \tau$$

$$\tau = \tau (\gamma_s - \tau_o)/|\gamma_s - \tau_o| \quad \text{if } K_s (\gamma_s - \tau_o) > \tau$$

$$\quad \text{if } K_s (\gamma_s - \tau_o) > \tau \quad (9)$$

where \(\tau\) is shear stress, \(\sigma_n\) is normal stress, \(\gamma\) is friction angle, \(K_s\) and \(K_n\) are appropriate coefficients, and \(\varepsilon_n, \sigma_n, \varepsilon, \gamma, \gamma_s\) are relative displacements in tangential and normal direction, strength of joints in normal and tangential direction, and residual slip. respectively. For non-linear analysis the values of \(\tan \phi\) c, c, \(\gamma\) are assumed to be 0.8, 30kgf/cm² and 20kgf/cm². For linear analysis, c and \(\gamma\) are set to very large value. Safety factor n for sliding is assumed to be as follows.

$$n = \int_{\tau_v ds} \frac{\tau ds}{\tau} \quad (10)$$

Results are shown in Table 2. From the results, the followings are observed.

1. Water-Dam interaction increases the response (decreases safety factor) for both linear and nonlinear case.
2. As the Young's modulus of bedrock decrease, the response decreases for both linear and nonlinear case.
3. Slip and separation do not significantly affect the value of safety factor.

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CONCLUSIONS

A method for treating wave propagation in infinite media in time domain is proposed. The method is compared with other two existing time domain methods. The proposed method gives better results than the other two methods for both with and without internal damping. The method is now applicable to dynamic response analysis of two dimensional semi-infinite structure-water-foundation system. It is applied to earthquake response of concrete gravity dam. It is shown that non-linearity due to slip and separation does not affect the response significantly.

REFERENCES


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<th>Density(kgf⋅sec²/cm⁴)</th>
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<td>Bedrock</td>
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Table 1 Material Properties

Table 2 Safety Factors for Sliding

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<th>Case</th>
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<th>Water Level (m)</th>
<th>Young's Modulus of Bedrock (kgf/cm²)</th>
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Fig. 1 Two-dimensional Model Problem of Structure-Water-Foundation

(a) Superposition Method

(b) Viscous Boundary Method

(c) Proposed Method

Fig. 3 Response of One-dimensional Model (No Damping)

Fig. 2 Seismic Wave Input

(a) Superposition Method

(b) Viscous Boundary Method

(c) Proposed Method

Fig. 4 Response of One-dimensional Model (with Damping)
Fig. 5 Surface Loading on Semi-infinite elastic body

\[ P = P_0 \sin(2\pi \frac{t}{T}) \quad \text{Pa} = 3 \]

\[ \lambda = 1.0 \]
\[ \mu = 0.5 \]
\[ P = 1.0 \]  
Lame's Constant  
Density

Fig. 6 Response to Surface Loading

Fig. 7 Concrete Dam Model