OPTIMUM ASEISMIC DESIGN OF EQUIPMENT SUPPORT-STRUCTURE SYSTEMS

Navin C. NIGAM and Gagan P.S. BAINS

1Thapar Institute of Engineering and Technology, Patiala, INDIA
2Department of Civil Engineering, University of California, Irvine, USA

SUMMARY

The dynamic behaviour of an idealised equipment support-structure (E S-S) system to earthquake excitation is investigated. The E S-S system is modelled as a coupled 4-d system, and the support-structure is optimised to minimise the response of equipment under harmonic base excitation. The capacity of ductile materials to dissipate large amount of energy during inelastic excursions is utilised to reduce the level of the excitation transmitted to the equipment by the support-structure during an earthquake. It is shown that support-structures can be designed to support a standard equipment in regions of different seismicity. The proposed approach is illustrated through an example.

INTRODUCTION

Uninterrupted functioning of equipment systems is critical for maintaining the life-line of modern industrial infrastructure during and after an earthquake. The extensive damage to utilities during Koyna (1967) and San Fernando (1971) earthquakes and the economic loss due to disruption of life-line stimulated a worldwide review of criteria for aseismic design of equipment (Refs. 1, 2, 3 and 4). The recommendations based on these studies have generally resulted in more severe design loading conditions.

An equipment is generally mounted on a support-structure, and may have special features such as: light weight, brittle components, internal resonances and constraints on relative displacement and acceleration response. To achieve economy of scale, it is desirable to develop standard equipment which can be manufactured in large numbers and installed on suitably designed support-structure at sites of different seismicity. This paper deals with the optimum aseismic design of such E S-S systems. The optimization is carried out in two stages. First the mass and stiffness properties of E S-S system are fixed to minimise the equipment response for harmonic base excitation. The yield level of the support-structure is then chosen to limit the equipment response within the upper limits specified by the manufacturer. Design curves are presented to guide the choice of yield strength. An example is included to illustrate the application of proposed design approach.

EQUIPMENT SUPPORT-STRUCTURE MODEL

We shall consider the equipment support-structure model shown in Fig. 1. It consists of an equipment idealised as a rigid mass (m_e) supported on a single
column, and mounted on a support-structure which is idealised as a simple space-
frame consisting of a rigid mass \( m_E \) supported on four identical columns. The
columns are rigidly clamped at the top and bottom ends, and their principal axes
lie in the directions 1-1 and 2-2. The mass and shear centres of the frame
coincide with the point of attachment of the equipment, and both the equipment
and support-structure have two translational degrees of freedom each in the
directions 1-1 and 2-2. The base of the support-structure rests on the ground
and is free to move in the horizontal plane. The effect of vertical component
of earthquake ground motion is neglected. The damping is assumed to viscous in
both the equipment and the support-structure.

![Diagram of E S-S System Model and Support-Structure Column Profile and Cross-section](image)

The equipment is assumed to remain elastic. The support-structure is design-
ed in such a way that it remains elastic during 'small' earthquakes, undergoes
'limited' plastic deformation during 'medium' earthquakes, and may undergo large
plastic deformation without collapse during a 'large' earthquake. During inelas-
tic excursions yielding occurs simultaneously at the top and bottom sections of
the support-structure columns. The forces acting at these sections are: the axial
force; the shear forces and the bending moments in the directions 1-1 and 2-2.
The yield behaviour will depend on the interaction between these forces (Ref.5).
We shall neglect the effect of axial force and shear forces and, therefore,
yielding shall be assumed to depend only on the interaction between the bending
moments in the directions 1-1 and 2-2 (EPI). The response of E S-S system in
which interaction effects are neglected (EP) is also investigated.

**OPTIMUM DESIGN OF SUPPORT-STRUCTURE**

The support-structure is designed to minimise the peak force and peak dis-
placement of the equipment, and to limit the ductility ratio \( \mu \) in the support-
structure within specified bounds during earthquakes. The optimization is carried
out in two stages:

(i) For a given equipment, the design parameters of the E S-S system (the mass
ratio \( c = m_E/m_i \); and frequency ratio \( \beta_i = \omega_E/\omega_i \), \( i = 1,2 \)) are chosen such that
the peak values of the displacement and the force on the equipment are minimised
for resonant harmonic excitation and elastic support-structure behaviour. Bains
(Ref.6) has shown that the optimum values of these parameters are: \( 0.5 < c < 0.95 \),
and \( 0.5 < \beta_i < 0.8 \). A reduction in the force on the equipment generally results in
an increase in the displacement of the support-structure. The optimum choice of
the design parameters is governed by a balance between the response of the equipment and the support-structure.

(ii) The yield strength of the support-structure optimized in stage-(i) is chosen for each seismic zone such that the force and displacement of the equipment are within acceptable limits specified by the manufacturer, and the support-structure remains elastic during small earthquakes, undergoes limited plastic deformation ($1 < \mu < 3$) during medium earthquakes, and may undergo large plastic deformation ($3 < \mu < 6$) during a large earthquake. The design yield strength is chosen from the design curves obtained by detailed dynamic analysis discussed in the next section.

**EARTHQUAKE RESPONSE OF E S-S SYSTEMS**

The equations of motion of a symmetrical E S-S system can be expressed in the following dimensionless form:

\[
\begin{align*}
\ddot{u}_i + 2(\tau_{si} + \tau_{ei} \alpha / \beta_i) \dot{u}_i - (2 \tau_{ei} \alpha / \beta_i) \dot{u}_{ei} + p_{si} - p_{ei} &= -\gamma g_i (\tau / \omega_s) \\
\alpha \ddot{u}_i + (2 \tau_{ei} \alpha / \beta_i) \dot{u}_i + (2 \tau_{si} \alpha / \beta_i) \dot{u}_{si} + p_{ei} &= -\alpha \gamma g_i (\tau / \omega_s)
\end{align*}
\]

where \(i = 1, 2\),

\[
\begin{align*}
u_{si} &= x_{si} / x_s, \quad u_{ei} = x_{ei} / x_s, \quad \tau_{si} = C_{si} / m, \quad \tau_{ei} = C_{ei} / m, \quad \omega_{s1} = \omega_{s2} = \omega_s, \\
q_{ys} &= k_s x_s, \quad a_{ys} = q_{ys} / m, \quad p_{si} = q_{si} / q_{ys}, \quad p_{ei} = q_{ei} / q_{ys}, \\
\tau_i(t) &= r_1 g_i(t), \quad \gamma = (r_1^2 + r_2^2)^{1/2} / a_{ys}; \quad \tau = \omega_s t.
\end{align*}
\]

in which subscripts \(e\) and \(s\) refer to equipment and support-structure respectively; \(C\) represents the lateral stiffness; \(k_s\) represents the lateral displacement; \(\dot{Z}(t)\) represents the ground acceleration; \(r\) represents the rms value of the ground acceleration; \(x_{ys}\) represents the yield displacement, \(q_{ys}\) represents the lateral yield force; \(a_{ys}\) represents the yield acceleration, and \(\gamma\) is the acceleration ratio.

In Eq. (1), the non-dimensional forces \(p_{si}\) and \(p_{ei}\) depend on the instantaneous behaviour of the support-structure and can be expressed as:

For elastic behaviour (E):

\[
\begin{align*}
p_{si} &= u_{si} \\
p_{ei} &= \alpha / \beta_i (u_{ei} - u_{si})
\end{align*}
\]

For elasto-plastic behaviour without interaction (EP)

\[
\begin{align*}
p_{si} &= u_{si} - (u_{si})_0 \\
p_{ei} &= \alpha / \beta_i (u_{ei} - u_{si}), \text{ if } |p_{si}| < 1, \text{ or } |p_{si}| = 1 \text{ and } \dot{W}_{si} > 0 \quad (3a) \\
p_{si} &= 1 \\
p_{ei} &= \alpha / \beta_i (u_{ei} - u_{si}), \text{ if } |p_{si}| = 1, \dot{W}_{si} < 0 \quad (3b)
\end{align*}
\]

For elasto-plastic behaviour with interaction (EPI)

\[
\begin{align*}
p_{si} &= u_{si} - (u_{si})_0.
\end{align*}
\]
\[ p_{ei} = \alpha / \beta_i (u_{ei} - u_{si}), \text{ if } \phi(p_{s1}, p_{s2}) < 1; \text{ or } \phi(p_{s1}, p_{s2}) = 1, \text{ and } \dot{\gamma}_s^p < 0 \] (4a)

\[ P_{s1} = p_{s2}^2 u_{s1} - p_{s1} p_{s2} u_{s2} \]

\[ P_{s2} = -p_{s1} p_{s2} u_{s1} + p_{s1}^2 u_{s2} \]

\[ p_{ei} = \alpha / \beta_i (u_{ei} - u_{si}), \text{ if } \phi(p_{s1}, p_{s2}) = 1, \text{ and } \dot{\gamma}_s^p > 0 \] (4b)

where \( \phi(p_{s1}, p_{s2}) = p_{s1}^2 + p_{s2}^2 \); and \( \dot{\gamma}_s^p \) represents the rate of plastic work.

The response of ES-S system to earthquake excitation is determined by step-by-step integration of Eq.(1) using the third order Runge-Kutta method for the following four cases: i) equipment resting directly on the ground (GR); the behaviour of support-structure is ii) elastic (E); iii) elasto-plastic without interaction (EP); and iv) elasto-plastic with interaction (EPI). Following system parameter values and excitations are considered:

\[ \zeta_e = 0.3, 0.5, \text{ and } 0.7; \zeta_s = 0.03; \zeta_y = 0.05 \]

\[ \alpha = 0.9; \beta_1 = \beta_2 = 0.6; \gamma = 0.2 \text{ and } 0.6 \]

Excitation: Ensemble of artificial earthquakes (Ref.7)

Relative displacement and force spectra of equipment response are plotted in Figs. 2a and 2b, and the ductility ratio of the support-structure in Fig. 2c. Comparison with the response of ground based equipment shows that a support-structure can be designed to reduce the intensity of shaking of the equipment significantly. The capacity of the support-structure to dissipate energy through plastic deformation provides effective and practical means for reducing the intensity of shaking and the interaction effects are favourable in this respect. The simpler elasto-plastic model EP, is generally conservative and may be used for design purposes.

Fig. 2(a) Relative Displacement Spectra; (b) Force Spectra, (c) Ductility Ratio. Average of Artificial Earthquake Ensemble. \( \zeta_{ei} = 0.03, \zeta_{si} = 0.05, \alpha = 0.9 \text{ and } \beta_i = 0.6 \).
Design curves (Figs. 2a, b and c) can be conveniently used to choose the yield strength of support-structure to meet the operational constraints specified by the manufacturer of the equipment, and the design constraints on the support-structure as shown in the example to follow.

**EXAMPLE**

India is divided into five seismic zones. In this example we shall design the support-structure for a standard equipment to be located in each of these zones. Following design data is used:

Equipment: \( m_e = 1300 \text{ kg} \); \( T_e = 0.3 \text{s} \); and \( \zeta_e = 0.03 \)

Maximum relative displacement \( \leq 0.6 \text{ cm} \).

Maximum force \( \leq 7000 \text{ kg m/s} \)

Support-Structure: \( \zeta_s = 0.05 \)

Maximum relative displacement \( \leq 7 \text{ cm} \).

Ductility ratio: Medium earthquake \(( \leq 3)\), Large earthquake \(( \leq 7)\)

Design Earthquakes: Small, medium and large earthquakes specified by rms values given in Table 1

Choosing \( \alpha = 0.9 \) and \( \beta_1 = \beta_2 = 0.6 \), the mass \( m_s = 1445 \text{ kg} \) and period \( T_s = 0.5 \text{ secs} \). A stepped mild steel column shown in Fig. 1(b) is used which permits changes in yield strength for a fixed value of stiffness given by

\[
k_s = \frac{12 E I_1 I_2}{I (L-2L_1) + 6 I L_1 L_2 (L_2 - 2 L_1)}
\]

The geometric and yield properties of the column are chosen from the design curves to meet the design constraints and are given in Table 2. Young's modulus and yield strength of steel column are \( 2 \times 10^{11} \text{ N/m}^2 \) and \( 2.1 \times 10^{8} \text{ N/m}^2 \) respectively.

<table>
<thead>
<tr>
<th>Zone</th>
<th>RMS Gr. Accln. in cm/sec²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Earthquakes</td>
</tr>
<tr>
<td></td>
<td>Small</td>
</tr>
<tr>
<td>I</td>
<td>2.0</td>
</tr>
<tr>
<td>II</td>
<td>4.0</td>
</tr>
<tr>
<td>III</td>
<td>8.0</td>
</tr>
<tr>
<td>IV</td>
<td>10.0</td>
</tr>
<tr>
<td>V</td>
<td>16.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zone</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>250</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>L_1</td>
<td>2.5</td>
<td>5.0</td>
<td>15.0</td>
</tr>
<tr>
<td>D_1</td>
<td>2.9</td>
<td>2.9</td>
<td>3.0</td>
</tr>
<tr>
<td>D_2</td>
<td>6.6</td>
<td>6.3</td>
<td>6.0</td>
</tr>
<tr>
<td>d_2</td>
<td>0.0</td>
<td>0.0</td>
<td>3.0</td>
</tr>
<tr>
<td>a_{ys}</td>
<td>0.1g</td>
<td>0.116g</td>
<td>0.185g</td>
</tr>
</tbody>
</table>

The optimum support-structure described in Table 2 remains elastic and within permissible design constraints in Zones I and II. The response of the E S-S system in Zones III, IV and V is given in Table 3. To demonstrate the effectiveness of the support-structure, the maximum response of the equipment resting directly on the ground (GR) is indicated. For comparison the force on the
equipment obtained by a method due to Biggs (Ref. 8), and in accordance with I.S. code is also included. It is seen that these values compare well with the force for medium earthquake and are nonconservative for a large earthquake. It is concluded that proposed approach can be used to design support-structures to support a standard equipment in regions of varying seismicity.

Table 3 Response of an Optimised Equipment Support-Structure System

<table>
<thead>
<tr>
<th>Response</th>
<th>Earthquake Size</th>
<th>Zone</th>
<th>Zone</th>
<th>Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>IV</td>
<td>V</td>
</tr>
<tr>
<td>Force on Equipment (Kg.m/s²)</td>
<td>Small</td>
<td>GR</td>
<td>1082</td>
<td>1350</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SS</td>
<td>910</td>
<td>1138</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>GR</td>
<td>2164</td>
<td>2706</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SS</td>
<td>1515</td>
<td>1924</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>GR</td>
<td>6492</td>
<td>8118</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SS</td>
<td>3380</td>
<td>4160</td>
</tr>
<tr>
<td></td>
<td>Biggs Method</td>
<td>SS</td>
<td>1735</td>
<td>2169</td>
</tr>
<tr>
<td></td>
<td>I.S. Code</td>
<td>SS</td>
<td>1691</td>
<td>2114</td>
</tr>
<tr>
<td>Ductility Ratio Support-Structure</td>
<td>Medium</td>
<td>SS</td>
<td>2.87</td>
<td>2.87</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>SS</td>
<td>3.40</td>
<td>4.25</td>
</tr>
</tbody>
</table>

GR: Equipment on ground. SS: Equipment on support-structure

REFERENCES