NEW RESULTS ON THE HORIZONTAL STRENGTH EVALUATION OF MASONRY BUILDINGS AND MONUMENTS

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SUMMARY

Presented in this paper is a method to evaluate the limit strength of masonry buildings and monuments subjected to seismic actions modelled as static horizontal loads. The behaviour of single walls with openings is firstly considered and the influence, on the collapse load, of the various geometrical and mechanical parameters is examined. The horizontal strength of masonry buildings with inplane rigid floors is then studied. It is shown that torsional effect can occur only for strongly asymmetric plans. Finally, the lateral strength of masonry structures, composed by walls and barrel vaults, typical in many monuments, is evaluated.

INTRODUCTION

The aim of the paper is to present a new method for the lateral strength evaluation of masonry structures loaded by horizontal forces. The problem, of course, is of worthy interest in the analysis of the behaviour of masonry buildings and monuments under earthquake shaking.

Collapse of masonry walls and vaults under horizontal forces occurs with the formation of cracks due more to tensile failures than to compression crushings. The masonry material exhibits, on the other hand, very low tensile strength. It can be useful therefore to assume, as constitutive model, the elastic material with zero tensile strength. Actually, this model has been examined in some theoretical and applied studies (Ref. 1). The finite compression strength on the masonry can also be taken into account.

In this framework the paper firstly analyzes the collapse behaviour of the plane multistory walls, the essential resistant elements of the masonry buildings. The lateral strength of the wall is controlled by the strength of the piers—mainly due to the effect of the vertical dead loads—and by the flexural strength of the architraves. The influence of the various factors influencing the lateral strength of the wall—as the intensity of the vertical dead loads, the flexural strength of the architraves or platbands, the geometry of the openings—is thoroughly examined. Finally the lateral strength of masonry structures composed by barrel vaults and walls, typical in many churches and monuments, is evaluated.
The lateral strength of the multistory masonry wall Let us consider the plane multistory masonry wall with \( n_p \) stories and \( n_c \) spans (Fig. 1). The considered wall has a regular scheme with piers of width \( b \) and architraves of height \( a \). The wall is subjected to vertical fixed dead loads \( G \) and to the gradually increasing horizontal forces \( \lambda F \), with \( \lambda \) the loading parameter, applied at the various stories along the architraves axes.

The lateral strength of the wall depends on two different effects. The first is due to the lifting of the vertical dead loads during the pier rotation. The second effect is due to the plastic dissipation developing in the architraves, that we assume reinforced.

According to the assumption of elastic material with zero tensile strength and to the analysis developed in (Refs. 2,3), the collapse condition of the single pier can be represented in term of the resulting bending moment \( M \) and axial force \( N \). The strength of the reinforced architrave is represented, on the other hand, by the plastic bending moment \( M_0 \). The corresponding shear force is therefore

\[
T_0 = \frac{2M_0}{\lambda}
\]  

(1)

where \( \lambda \) is the span length of the architrave. As shown (Ref. 4), according to the values of the characteristic ratio

\[
Z = \frac{T_0 \lambda}{Gb}
\]  

(2)

the collapse of multistory walls occurs with various mechanisms. Fig. 2a depicts the "architrave sidesway" mechanism, Fig. 2b the "pier sidesway" mechanism and Fig. 2c the "turn over" mechanism. For small values of \( Z \), i.e. when

\[
Z < Z_3 = \frac{1}{2(1 + K)}
\]  

(3)

with

\[
K = \frac{b}{\lambda}
\]  

(4)
the collapse of the multistory wall develops with the "architrave sidesway" mechanism of Fig. 2a. The corresponding collapse multiplier \( \lambda_c \) in adimensional form

\[
\Lambda_c = \frac{\lambda_c^\text{PH}}{G_b}
\]

Fig. 2 Failure mechanisms of the wall (a = architrave sidesway, b = pier sidesway, c = turn over)

is given by

\[
\Lambda_c = \frac{1 + 2Z(1 + X)}{(n_c + 1) \frac{n_c}{n_c + 1}} \frac{1 + n_p(1 + 2t)}{1 + n_p(1 + 2t)}
\]  

(6)

with

\[
t = \frac{a}{2H}
\]

(7)

The influence of the finite compression strength of the masonry can be also taken in account by multiplying the second member of eq. (6) by the factor

\[
\left(1 - \frac{\sigma_v}{\sigma_k}\right)
\]

(8)

where \( \sigma_v \) is the mean compression stress at the base of the piers, produced by the vertical dead loads \( G \), and \( \sigma_k \) the compression strength of the masonry. Of course, this factor \( \approx 1 \) for \( \sigma_v/\sigma_k \ll 1 \), as frequently it occurs in unreinforced masonry dwellings.

It is also rare to meet large values of the ratio \( Z \), i.e. values of \( Z \) larger than \( Z_1 \) given in eq. (3). As a rule, in fact, the shear \( T_o \) is very lower than \( G \), that is the increment, at the story, of the vertical dead loads, while the span length \( l \) and the width \( b \) of the pier are, more or less, of the same order of magnitude.

In the seismic analysis of walls it is usual to make reference to a conventional mean shear at the base sections of the piers. According to eq. (6), this ultimate shear is then given by

\[
\tau_c = \frac{G}{bHs} b \frac{n_p}{1 + n_p(1 + 2t)} \left[1 + 2Z(1 + X) \frac{n_c}{1 + n_c}\right]
\]

(9)

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where $s$ is the thickness of the piers. It can be recognized that:

i - the number of the stories and of the openings has not relevant influence on $T_C$

ii - the ultimate shear is proportional to the piers width and to the mean specific gravity $G/bH$s of the wall

iii - noteworthy influence on $T_C$ is due to the bending strength of the architraves by means the ratio $Z$.

A quantity more sharpening to represent the seismic lateral strength of the multistory wall is the specific lateral strength $r_C$, i.e. the ratio between the total horizontal limit force and the weight of the whole wall. This quantity can represent the mean value of the horizontal seismic acceleration corresponding to the horizontal collapse load for the building. We have

$$ r_C = \frac{b}{H} \frac{1}{1 + n_p (1 + 2t)} \left[ 1 + 2Z(1 + K) \frac{n_C}{n_C + 1} \right]; $$

(10)

of course, the higher is the wall, i.e. larger the number of stories, smaller is $r_C$.

**Collapse of masonry buildings accompanied by torsional rotations of the floors.** In presence of irregular plans, asymmetric locations of the walls, the collapse mechanism of the whole building will be characterized by both translations and torsional rotations of the floors. The floors will be represented by rigid diaphragms that tie together around the walls. Furthermore, we will assume that suitable reinforcements - in the architraves, at the floors, at the corners etc. - will be able to prevent all advance local failures so that the whole resistant structure of the masonry building will be able to exhibit at the collapse all its lateral ultimate strength.

The so defined masonry structure of the building will be subjected to:

i - vertical fixed dead loads $G$ acting, at any story, along the axes of the piers

ii - gradually increasing horizontal forces $\lambda F$, acting at the centers $C$ of the various floors (Fig. 3).

![Fig. 3 The model of the multistory masonry building](image-url)
The collapse of the building is controlled by the lateral strength of the multistory vertical walls. The collapse mechanism of the building is characterized or by a single rotation of the floors around a vertical axis passing through the intersection of two orthogonal walls or by a simple translation of the floors.

As for a multistory wall also for a masonry building the above defined $T_C$ and $r_C$ parameters can be evaluated. Fig. 4 shows, for two examples of buildings, the variation of the average ultimate shear $T_C$ and the global specific strength of the building $r_C$ with the number of stories, the flexural strength $M_o$ of the architraves.

![Diagram showing variations of $T_C$ and $r_C$ with the number of stories $n_p$.]

**Fig. 4** Variations of $T_C$ and $r_C$ with the number of stories $n_p$.

From these results it is possible to show that the torsional rotation of the floors is present in the collapse mechanism only for relevant eccentricities between the center of gravity of masses and the center of the lateral strength of the walls (Ref. 4).

**Lateral strength of some typical masonry monumental structures** According to the above defined approach – extending therefore to the case of the horizontal forces the Heyman (Ref. 5) analysis – it has been possible to evaluate the lateral strength of masonry structures composed by barrel vaults and walls. The first elementary scheme, in this case, is composed by a single barrel vault and two lateral walls. This simple scheme is present in many examples of monumental systems composed by buttressed vaults (Fig. 5).

The collapse of this simple structure can be easily examined by means of the limit analysis. The collapse mechanism can be represented, by or a partial mechanism in the vault or a global mechanism involving the overturning of a wall (Fig. 6).

In this line more complexe masonry structures have been analyzed (Refs. 6, 7).
Fig. 5 Some typical monumental masonry systems

The local collapse mechanism of the vault

The global collapse mechanism with the overturning of the right wall

Fig. 6

REFERENCES


