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## PROPOSAL FOR A DISTRIBUTION OF SEISMIC SHEAR COEFFICIENT ALONG THE HEIGHT OF A BUILDING

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### SUMMARY

This paper deals with the distribution of seismic forces along the height of a building. Five mass model structures of shear, shear-flexural and flexural types with six different mass-stiffness distributions were analyzed by modal analysis using a simplified response spectrum. As a result of the analysis a new expression for the distribution of seismic shear coefficient is proposed. The proposed distribution was compared with a time history analysis of the same series of model structures subjected to recorded earthquake ground motions.

### INTRODUCTION

Since the inverted triangular distribution of seismic forces was first suggested by the Structural Engineering Association of California (Ref. 1), many seismic codes in the world (Ref. 2) have adopted this method to define design lateral seismic forces. However, this method does not give an appropriate distribution for tall buildings. Therefore, some of the codes stipulate a parabolic distribution of seismic coefficient or the application of some concentrated force at the top. These methods still are not satisfactory for many types of buildings. This paper proposes a new expression for the distribution of seismic shear coefficient.

### DISTRIBUTION OF SEISMIC SHEAR COEFFICIENT BY SRSS ANALYSIS

In this paper, the seismic shear  $Q$  is defined as the shear which is transmitted to the level concerned of the building and is given by the product of the shear coefficient  $C$  and the weight of the building above the level concerned. Since this paper only deals with distributions of these parameters,  $Q$  is normalized by the base shear and  $C$  is normalized by the base shear coefficient.

In order to study the effect of shear versus flexural deformations and mass-stiffness distributions on the distribution of seismic shear coefficient, five-mass model structures of shear (S), shear-flexural (SF) and flexural (F) types fixed at the bases were analyzed, consisting of six different mass-stiffness distributions (I~VI) as shown in Figs. 1 and 2. The shear-flexural model was chosen as having shear and flexural parts are simply connected by rigid rods (Fig. 1(a)). Each part is assumed rigidly connected to the foundation and has the same deflection; i.e., the top deflections of shear and flexural parts are equal when subjected to the same gravity forces applied

laterally (Figs. 1(b) and 1(c)). All story heights are assumed to be equal.

The model structures were analyzed by modal analysis using a simplified response spectrum as shown in Fig. 3 where the period ratio  $t$  is defined as the period  $T$  of the building divided by the critical period  $T_c$  of the spectrum. Fig. 3 shows that the acceleration response is constant up to the critical period and decreases hyperbolically. The maximum values were evaluated as the square root of the sum of the squares (SRSS) from the first through the fifth modes. Some analytical results are shown as dashed lines in Fig. 5 where the ordinate indicates normalized weight  $\alpha$  which is defined as the ratio of the weight above the level concerned to the total weight above ground level and the abscissa indicates normalized shear coefficient  $C$  or normalized shear  $Q$ . In Fig. 5 the solid lines indicate a proposal which will be discussed in the subsequent section. (Ref. 3.)

The results of the SRSS analysis indicate the following general features:

- 1) The seismic shear coefficient of the upper portion is always larger than that of the lower portion, except for Type VI buildings (a tower-on-a-base type) in the case of the larger period ratio for which the seismic shear coefficient of the middle part is smaller than the base shear coefficient.
- 2) The seismic shear of the upper portion is not greater than that of the lower portion, although an exception may occur for flexural models if higher modes are more strongly excited than the first mode.
- 3) The distributions of seismic shear coefficient or of seismic shear for Type V buildings (one with a penthouse) are identical with those of Type II buildings (regular one), as long as the distribution is expressed by  $\alpha$  and the term of  $1/\sqrt{\alpha}$  is included in the seismic shear coefficient.

For shear type models:

- 4) The distribution of seismic shear coefficient is almost uniform from top to first story for Type S-III buildings (soft-first-story shear type).
- 5) The distribution of Type S-IV buildings (stiff-first-story shear type) is almost identical with the inverted triangular distribution of seismic coefficient in the case of smaller period ratio. In the case of larger period ratio, the distribution is close to the distribution of shear type structure subjected to white noise excitation ( $C \propto 1/\sqrt{\alpha}$ ).
- 6) The distributions for Type S-I or S-II buildings (uniform or regular shear types) fall between the distributions of Type S-III building (soft-first-story shear type) and Type S-IV building (stiff-first-story shear type).

For shear-flexural type models:

- 7) The distributions of seismic shear coefficient and seismic shear of shear-flexural models fall between those of shear models and flexural models although the distributions are closer to those of flexural models.

For flexural type models:

- 8) The distribution of seismic shear coefficient and seismic shear vary only little with the mass-stiffness distributions.
- 9) The distributions for smaller period ratio are very close to the Japanese  $A_i$  distribution for longer period.
- 10) The higher mode effect causes a concave distribution of seismic shear at the lower portion of long period flexural type buildings.

For Type VI buildings (a tower-on-a-base type):

- 11) As to all Type VI buildings, the distribution for smaller period ratio is close to the Japanese  $A_i$  distribution for longer period. For larger period ratio, however, the distribution is completely different from any existing distributions; i.e., the seismic shear coefficient of the lower portion of the tower becomes almost equal to or smaller than the base shear coefficient.

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For the purpose of the following analysis, the seismic shear is expressed as a linear combination of: a) the uniform distribution of seismic coefficient; b) the inverted triangular distribution of seismic coefficient; c) the distribution of shear type structure subjected to white noise excitation; d) the distribution by higher mode effects, especially for flexural models. If the normalized seismic shears of these four distributions are  $Q_a$ ,  $Q_b$ ,  $Q_c$  and  $Q_d$ , respectively (Fig. 4), then the normalized seismic shear  $Q$  can be expressed as (Ref. 3):

$$Q = Q_a + k_1(Q_b - Q_a) + k_2(Q_c - Q_a) + k_3(Q_d - Q_a) \quad (1)$$

where  $k_1$ ,  $k_2$  and  $k_3$  are the coefficients determined by the characteristics of buildings and ground motions. Eq. (1) gives:

$$Q = \alpha + k_1 \alpha (1 - \alpha) + k_2 \alpha \left( \frac{1}{\sqrt{\alpha}} - 1 \right) + k_3 \alpha (0.2 - \alpha)(1 - \sqrt{\alpha})^2 \quad (2)$$

The last term is empirically proposed by the author. Dividing Eq. (2) by  $\alpha$  gives the normalized seismic shear coefficient  $C$  as:

$$C = 1 + k_1(1 - \alpha) + k_2 \left( \frac{1}{\sqrt{\alpha}} - 1 \right) + k_3(0.2 - \alpha)(1 - \sqrt{\alpha})^2 \quad (3)$$

There are many ways to determine the values of  $k_1$ ,  $k_2$  and  $k_3$ . In order to satisfy the summarized items from 1) to 11) in the previous section and the analytical results (Fig. 5), the following relationships are proposed:

$$k_1 = \frac{0.05}{0.05 + r} \frac{s^2}{0.5 + s^2} \frac{4}{4 + t^2} + \frac{2}{3} \frac{r}{0.05 + r} \frac{1.5 + s^2 + t^2}{1 + s^2 + t^2} \quad (4)$$

$$k_2 = \frac{0.05}{0.05 + r} \frac{s^2}{0.2 + s^2} \frac{t^2}{4 + t^2} + \frac{2}{3} \frac{r}{0.05 + r} \frac{s^2 + t^2}{1 + s^2 + t^2} \quad (5)$$

$$k_3 = \frac{r}{0.2 + r} \frac{s}{0.1 + s} \frac{30 t^2}{9 + t^2} \quad (6)$$

where,  $r$  is the ratio of the shear deflection  $\Delta s$  to the flexural deflection  $\Delta f$  when each part is individually subjected to the lateral load equal to the gravity forces pertaining to masses of the structure (i.e.,  $r = \Delta s / \Delta f$  in Fig. 1;  $r = 0$  for shear type and  $r = \infty$  for flexural type),  $s$  is the ratio of the first story stiffness to the average stiffness of the building, and  $t$  is the period ratio (the ratio of the fundamental period of the building  $T$  to the critical period  $T_c$  of the response spectrum of Fig. 3). Substitution of Eqs. (4), (5) and (6) into Eq. (3) yields the new proposed normalized seismic shear coefficient:

$$C = 1 + \left( \frac{0.05}{0.05 + r} \frac{s^2}{0.5 + s^2} \frac{4}{4 + t^2} + \frac{2}{3} \frac{r}{0.05 + r} \frac{1.5 + s^2 + t^2}{1 + s^2 + t^2} \right) (1 - \alpha) + \left( \frac{0.05}{0.05 + r} \frac{s^2}{0.2 + s^2} \frac{t^2}{4 + t^2} + \frac{2}{3} \frac{r}{0.05 + r} \frac{s^2 + t^2}{1 + s^2 + t^2} \right) \left( \frac{1}{\sqrt{\alpha}} - 1 \right) + \frac{r}{0.2 + r} \frac{s}{0.1 + s} \frac{30 t^2}{9 + t^2} (0.2 - \alpha)(1 - \sqrt{\alpha})^2 \quad (7)$$

Further analysis on buildings other than five stories has shown that the equivalent stiffness  $s_e = s^{2/N}$  should be used for N story buildings.

Incidentally, the  $A_i$  distribution of the Japanese code can be given by Eq. (3) with the following relationships:

$$k_1 = k_2 = \frac{2T}{1 + 3T}, \quad k_3 = 0 \quad (8)$$

Substituting above relationships into Eq. (3) yields:

$$C = 1 + \left( \frac{1}{\sqrt{\alpha}} - \alpha \right) \frac{2T}{1 + 3T} = A_i \quad (9)$$

The proposed distribution was compared with the analytical results by time history analysis for the same series of model structures (damping ratio = 0.05 of critical) and gave satisfactory comparisons as shown in Fig. 6, except for tower-on-a-base type buildings. In Fig. 6 solid lines indicate the proposed distribution, and (+) and (o) marks connected by dashed lines refer to El Centro NS and EW, connected by dotted lines to Taft NS and EW, and connected by dash-dotted lines to Miyagi-ken-oki NS and EW, respectively.

#### CONCLUSIONS

A new distribution of seismic shear coefficient is proposed which gives satisfactory results for any types of structures except for Type VI (tower-on-a-base type) buildings. This type of buildings should be designed by dynamic analysis because the distribution of seismic shear coefficient can differ from the other types of buildings and is not well represented by any existing nor by the proposed distribution.

#### ACKNOWLEDGEMENT

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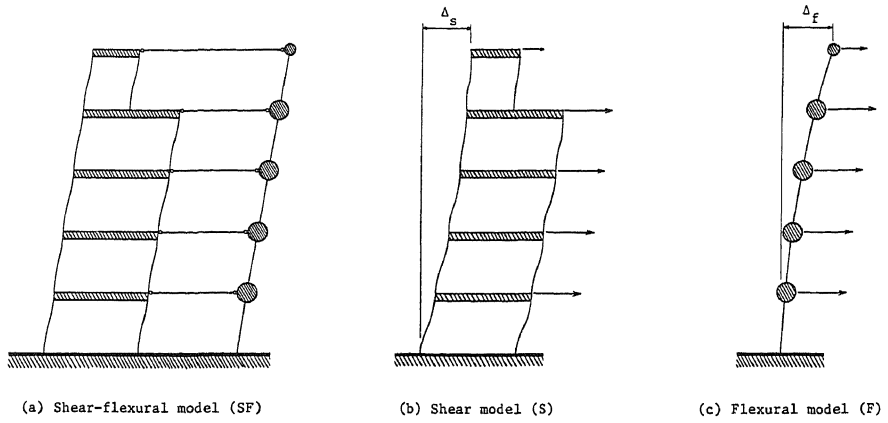


Fig. 1 Shear-flexural, Shear and Flexural Models

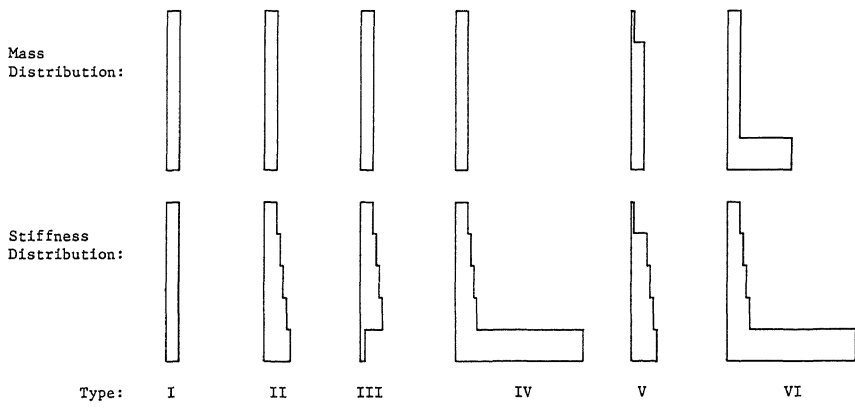


Fig. 2 Mass and Stiffness Distributions of Types of Model Structures

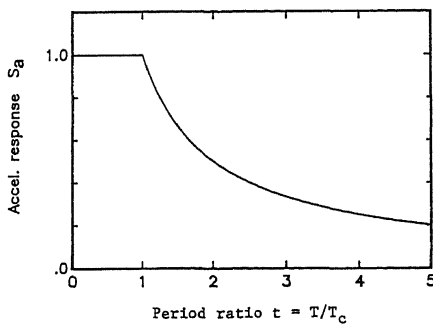


Fig. 3 Simplified Response Spectrum

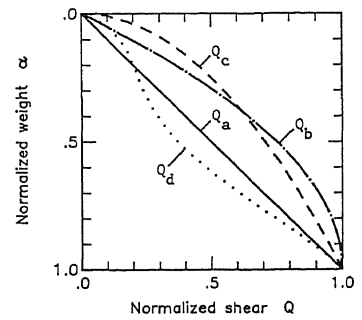


Fig. 4 Four Basic Distributions of Normalized Shears

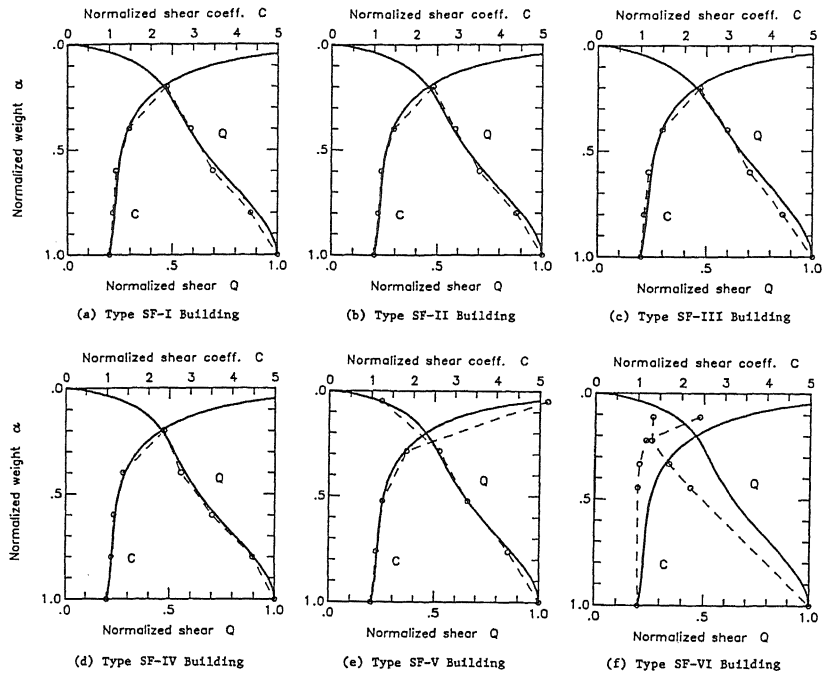


Fig. 5 Comparison of Proposed Distributions (solid lines) with SRSS Analyses (dashed lines) for Shear-flexural Type Models ( $t=3$ )

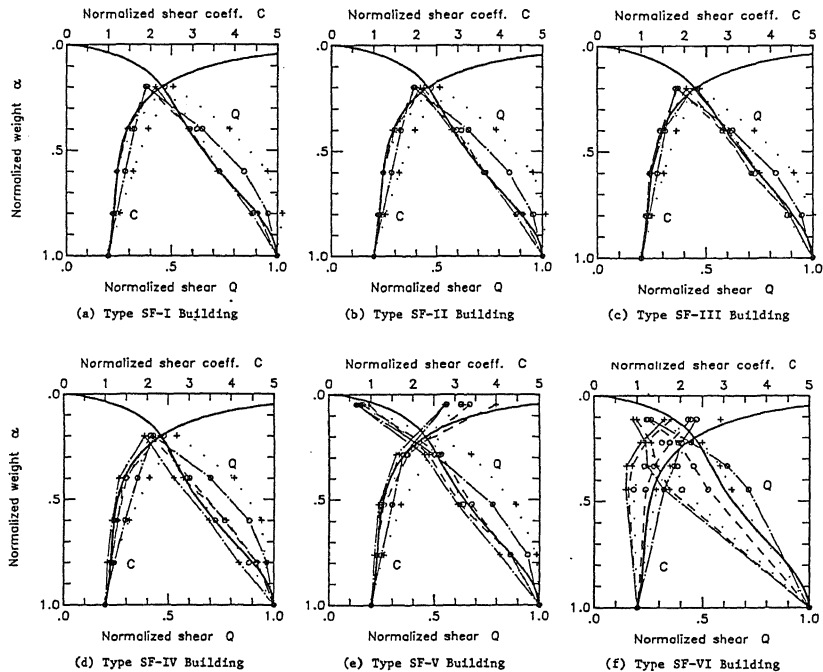


Fig. 6 Comparison of Proposed Distributions (solid lines) with Time History Analyses (dashed lines, etc.) for Shear-flexural Type Models ( $t=3$ )