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FINITE RESONANCE RESPONSE ANALYSIS OF ASYMMETRIC STRUCTURES

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SUMMARY

This paper deals with an analysis of earthquake response behaviors of torsionally coupled buildings with nonlinear resisting elements (columns or walls) under dynamic excitations by using FINITE RESONANCE RESPONSE ANALYSIS (FRRA) method[1]. In this paper, an example of earthquake response analysis for a mono-eccentric building subjected to a unidirectional earthquake excitation is also carried out.

INTRODUCTION

FRRA is an evaluating method for the maximum amplitude of response displacement amplitude and absorbed energy of single degree of freedom system (Fig.1). In the method, it is assumed that the system be in a steady-resonance-state with a hysteretic cyclic loop shown in Fig.2, and its response rate β is a finite value given by Eq.(1)[1].

$$\beta = 0.6\pi/(h_e \cdot \pi + 0.4) \tag{1}$$

in which h_e = the equivalent viscous damping ratio, given by Eq.(2) [2].

$$h_{ex} = \frac{1}{4\pi} \cdot \frac{Axa\ (\text{loop area})}{1/2 \cdot F_{xa} \cdot X_a} \tag{2}$$

On the other hand, the response rate (e.g. velocity) can be expressed as

$$\beta = V_{xm}/V_o \tag{3}$$

in which V_{xm}, V_o = the velocity amplitudes of the response , the ground motion, and X_a, F_{xa} and A_{xa} = the displacement amplitude, restoring force amplitude and the area of the one-cyclic hysteretic loop (Fig.2). Then the Finite Resonance Capacity (FRC) (Velocity) C_{rv}' may be deducted from Eqs.(1)-(3).

$$C_{rv}' = m\sqrt{V_o} = \frac{5}{6\pi}\frac{A_{xa}}{\sqrt{F_{xa} \cdot X_a}} + \frac{2}{3\pi}\sqrt{F_{xa} \cdot X_a} \tag{4}$$

While, as shown in Fig.2, the equivalent natural period T_e is expressed as

$$T_e = 2\pi\sqrt{\frac{m \cdot Xa}{F_{xa}}} \tag{5}$$

The relationship between C_{rv}' and T_e may be considered as a spectrum-relation like as the earthquake excitation spectrum. The C_{rv}'-T_e spectrum is called as FRC Spectrum. Therefore, the results of the analysis approach may be given by the intersection of the earthquake excitation spectrum and the FRC spectrum, as shown in Fig.3 [3]. Thus, the analysis approach for the eccentrical buildings by means of FRRA may be said how to express the FRC of the eccentrical buildings and the cross-point of the two spectrum curves.

Analysis Model The idealized one-story structure in Fig.4 consists of a rigid deck supported by planar frame and/or wall assemblages situated at the periphery. To simplify the problem, the structure is assumed to be eccentric for excitation in the x-direction only. The system has two coupled degree of freedom, namely lateral displacement X_c of the mass center (CM) relative to the ground and rotation θ_c about the vertical axis.

Equations of Motion Let l_{xi} and l_{yi} represent the distances from the CM, f_{xi} and f_{yi} represent the lateral forces, and u_{xi} and u_{yi} represent the lateral displacements of ith resisting element along the principal axes of resistance x and y, respectively. And, u_{xi} and u_{yi} can be expressed as

$$u_{xi} = X_c - l_{yi}\cdot\theta_c \; ; \qquad u_{yi} = l_{xi}\cdot\theta_c \qquad (6)$$

Within the range of nonlinear behavior, the equations of motions for coupled lateral-torsional response of the system of Fig.4 to ground acceleration $\ddot{X}_g$ along the x-axis, can be written as

$$\left.\begin{aligned} M\cdot\ddot{X}_c + \Sigma f_{xi} &= -M\cdot\ddot{X}_g \\ I\cdot\ddot{\theta}_c - \Sigma f_{xi}\cdot l_{yi} + \Sigma^g f_{xi}\cdot l_{yi} &= 0 \end{aligned}\right\} \qquad (7)$$

in which M = the mass of the deck; I = the mass moment of inertia of the deck about the vertical axis through the CM. Then, let

$$Z_c = \theta_c\cdot r \qquad \text{(r: gyration radius)} \qquad (8)$$

$$l_{xi}' = l_{xi}/r; \qquad l_{yi}' = l_{yi}/r \qquad (9)$$

and let Fx, Fz denote the total forces of the resisting elements along the x-, z-directions, respectively,

$$F_x = \Sigma f_{xi}; \qquad F_z = \Sigma(f_{yi}\cdot l_{xi}' - f_{xi}\cdot l_{yi}') \qquad (10)$$

Then Eq.(7) can be written simply as

$$\left.\begin{aligned} M\cdot\ddot{X}_c + F_x &= -M\cdot\ddot{X}_g \\ M\cdot\ddot{Z}_c + F_z &= 0 \end{aligned}\right\} \qquad (11)$$

Absorbed Energy of System Taking the line-integration of both sides of Eq.(11) with respect to X_c and Z_c over one cycle, the integrations can be expressed as

$$\left.\begin{aligned} \oint F_x\cdot dX_c &= -\oint M\cdot\ddot{X}_g\cdot dX_c \\ \oint F_z\cdot dZ_c &= 0 \end{aligned}\right\} \qquad (12)$$

Because the system is in the state of the steady resonance, it can be assumed that X_c, Z_c and X_g are the sinusoidal waves with the amplitudes X_{ca}, Z_{ca} and α_x, respectively. Therefore,

$$\oint M\cdot\ddot{X}_g\cdot dX_c = \pi M\cdot\alpha_x\cdot X_{ca} \qquad (13)$$

Substituting Eqs.(6), (11) and (13) into Eq.(12) and adding them, Eq.(14) can be obtained.

$$\oint(\Sigma f_{xi}\cdot du_{xi} + \Sigma f_{yi}\cdot du_{yi}) = \pi\cdot M\cdot\alpha_x\cdot X_{ca} \qquad (14)$$

Then, let A_{xi} and A_{yi} represent the areas of the one-cycle hysteretic loop of ith element in x- and y-directions, respectively. Eq.(14) can be written as

$$\Sigma A_{xi} + \Sigma A_{yi} = \pi\cdot M\cdot\alpha_x\cdot X_{ca} \qquad (15)$$

Thus

$$A_{xa} = \Sigma A_{xi} + \Sigma A_{yi} \qquad (16)$$

in which A_{xa} = the area of the one-cyclic hysteretic loop of F_x-X_c, or the absorbed energy of the system in the one-cyclic vibration. Therefore, the total absorbed energy W_{ps} of the system during the ground motion can be expressed as

$$W_{ps} = n_c\cdot A_{xa} \qquad (17)$$

in which n_c = the number of the vibration-cycles. Then, let t_o represent the duration of the ground motion, and n_c can be expressed as

$$n_c = t_o/T_e \qquad (18)$$

Finite Resonance Capacity Let h_{ex} denote the equivalent viscous damping ratio of the x-direction, it may be defined as

$$h_{ex} = \frac{1}{4\pi} \cdot \frac{Axa}{1/2 \cdot F_{xa} \cdot X_a} \qquad (19)$$

in which F_{xa} = the amplitude of F_x. Substituting h_{ex} into Eqs.(16)-(12) and arranging them, Eq.(20) can be obtained.

$$1/2h_{ex} = F_{xa}/M \cdot \alpha_x \qquad (20)$$

In Eq.(20), $1/2h_{ex}$ may be considered as the RESONANCE RESPONSE RATE. For the FINITE-RESONANCE, replacing it with Eq.(1) in which h_e is replaced with h_{ex} shown in Eq.(19) and expressing the acceleration amplitude α_x with the velocity amplitude V_{ox} of the ground motion, the FRC (velocity) of the system subjected to the x-directional earthquake excitation can be obtained, and expressed as

$$C_{rvx}' = M\sqrt{V_{ox}} = \frac{5}{6\pi} \frac{A_{xa}}{\sqrt{F_{xa} \cdot X_{ca}}} + \frac{2}{3\pi}\sqrt{F_{xa} \cdot X_{ca}} \qquad (21)$$

FINITE RESONANCE RESPONSE ANALYSIS

Equivalent Natural Period Suppose that the system resonate steadily when it is excited by the components of the ground motion which have the same frequency with the natural frequency of the system. Here, the vibration frequency (or period) is called as the equivalent natural frequency $\dot{\omega}_e$ (or period T_e) for the system with the nonlinear resisting elements. And consider only the situation at the time that the responses of the system reach their amplitudes simultaneously, and because of the steady resonance, the ground motion acceleration should be 0 when the responses reach their amplitudes. Thus, Eq.(12) can be written as

$$\left.\begin{array}{l} -M \cdot \omega_e^2 \cdot X_{ca} + F_{xa} = 0 \\ -M \cdot \omega_e^2 \cdot Z_{ca} + F_{za} = 0 \end{array}\right\} \qquad (22)$$

in which Fza = the amplitude of Fz. From Eq.(22), the equivalent natural frequency and the equivalent natural period can be expressed respectively, as

$$\omega_e^2 = F_{xa}/M \cdot X_{ca} = F_{za}/M \cdot Z_{ca} \qquad (23)$$

$$T_e = 2\pi\sqrt{M \cdot X_{ca}/F_{xa}} = 2\pi\sqrt{M \cdot Z_{ca}/F_{za}} \qquad (24)$$

Eq.(23) or (24) means that the vibration of the system is lateral-torsional coupled.

Equivalent Eigenvalue Problem Let k_{exi}, k_{eyi} represent the equivalent lateral stiffnesses of ith element in x- and y-directions, respectively, at the time of the response amplitudes.

$$k_{exi} = f_{xai}/u_{xai}, \quad k_{eyi} = f_{yai}/u_{yai} \qquad (25)$$

in which f_{xai}, u_{xai} and f_{yai}, u_{yai} denote the amplitudes of f_{xi}, u_{xi} and f_{yi}, u_{yi}, respectively. And the total equivalent lateral stiffness of x-direction is then written as

$$K_{ex} = \Sigma k_{exi} \qquad (26)$$

Both the total equivalent torsional stiffness and the equivalent eccentricity e_{ey} are defined with respect to the mass center by

$$K_{ez} = \Sigma k_{eyi} \cdot l_{xi}' - \Sigma k_{xei} \cdot l_{yi}' \qquad (29)$$

$$e_{ey} = \Sigma k_{exi} \cdot l_{yi}/K_{ex} \qquad (30)$$

Therefore, the equations of motion (Eq.(22)) at the time of the response-amplitudes can be then written ,in the matrix form, as

$$-M\ \omega_e^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} X_{ca} \\ Z_{ca} \end{Bmatrix} + \begin{bmatrix} K_{ex} & -e_{ey} \\ -e_{ey} & K_{ez} \end{bmatrix} \begin{Bmatrix} X_{ca} \\ Z_{ca} \end{Bmatrix} = 0 \qquad (31)$$

Because of $(Xca, Zca)^T \neq 0$, Eq.(25) can be considered as the equivalent eigenvalue problem shown in Eq.(26).

$$\begin{vmatrix} K_{ex}/M - \omega_e^2 & -e_{ey} \\ -e_{ey} & K_{ez}/M - \omega_e^2 \end{vmatrix} = 0 \qquad (32)$$

Eq.(32) has the same solution with Eq.(23) or (24). It leads to the equivalent natural frequencies (or periods) of the vibration corresponding with the mode shapes of the vibration.

Analysis Flow Chart The relationship between the FRC (Eq.(21) and the equivalent natural period (Eq.(23), (24) or (32)) can be expressed as the FRC spectrum relative to the mode shapes of the vibration. At the cross-point of the FRC spectrum and the earthquake excitation spectrum, the solution of the response analysis (the max. displacements and absorbed energy) can be decided. The analysis procedure is simply expressed with a flow chart shown in Fig.5.

ANALYSIS EXAMPLE

Basic System Parameters It is apparent from the equations of motion (Eq.(7)) that the nonlinear responses of the idealized analysis model to specified ground acceleration $\ddot{X}_g$ along the x-principal axis of resistance, depend not only on the static eccentricity e_y and the plan geometry, but also on the number, location, and the hysteretic characteristics of the individual resisting elements. In order to simplify this analysis procedure, the basic system parameters for the analysis example are given as follows;

1) aspect ratio of plan shown in Fig.4 : $l_y/l_x=1$
2) uncoupled lateral period in x- and y-directions : $T_{eo}=0.5$ sec
3) uncoupled lateral yield force coefficient in x- and y-directions : 0.2
4) characteristics of ith resisting element: Fig.6
5) ratio of eccentricity e_y to l_y: Table 1
6) ratio of elastic stiffnesses of elements: Table 1

Ground Motion for Response Analysis As the earthquake ground motion input, the earthquake, May 18, 1940, El Centro (NS component) is employed. For comparing the analysis with a time-history numerical integration, the Runge-Kutta's method (R.K.) is used. For the two methods, the forms of the ground motion input are shown in Fig.7 a) and b), respectively.

Results of Response Analysis The Results of the response analysis obtained by the R.K. method and the FRRA method (only the selection of the 1st vibration-mode is considered in the FRRA procedure) are shown in Fig.8 for the maximum lateral displacements, Fig.9 for the torsional effects, and Fig.10 for the absorptions and the distributions of energy, respectively. In these figures, the abscissas denote the parameter e_y/l_y, and the ordinates denote the analytical responses by the R.K. method and the FRRA method. For comparing the results from the two methods, all analytical results are made non-dimensional, with the response displacements being divided by the max. lateral displacement X_{com} by the R.K. method, and the energy absorptions being divided by the absorbed energy W_{pom} by the R.K. method (Table 2).

CONCLUSIONS

Based on the results above, the following conclusions may be drawn:

1) Max. Lateral Displacements Fig.8 shows an increase in the lateral displacements of the mass center and the elements X1, X2 with the increase of e_y/l_y. And, as shown in Figs.8 b) and c), the increase of X1 is more remarkable than X2.

2) Torsional Effects With the increase of e_y/l_y, the max. torsional displacements of the system become large as shown in Fig.9. And the increase of torsional components of X1 is apparent than X2 as shown in Figs.9 b) and c).

3) Energy Absorptions As shown in Fig.10 a), the system shows a somewhat constant value in the absorbed energy in despite of e_y/l_y increasing. And while the distribution in the elements X1,X2 is getting descending, the one in the elements Y1,Y2 increasing, for e_y/l_y becoming large.

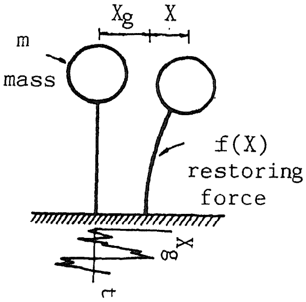


Fig.1 one mass oscillator

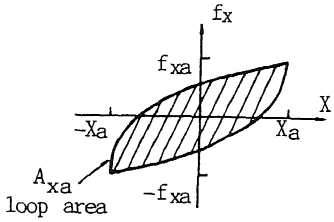


Fig.2 hysteretic loop

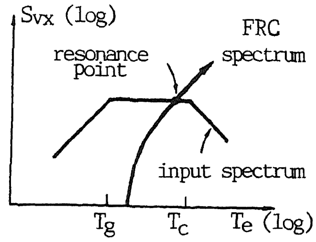


Fig.3 input spectrum and FRC spectrum

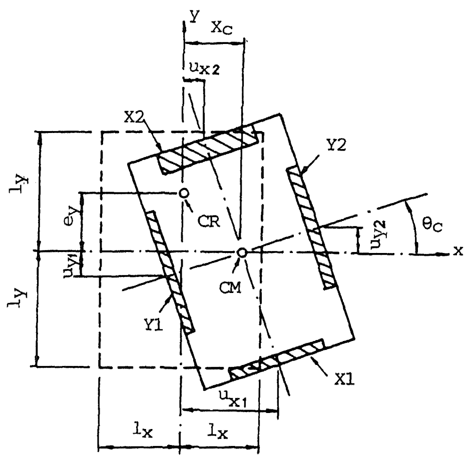
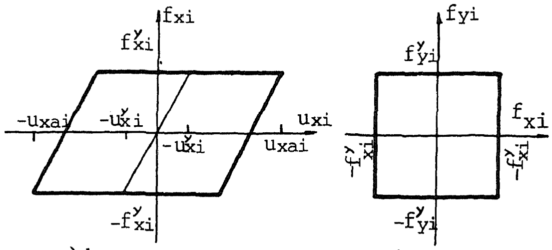


Fig.4 analysis model



a) hysteretic loop

b) yield surface

Fig.6 properties of resisting element

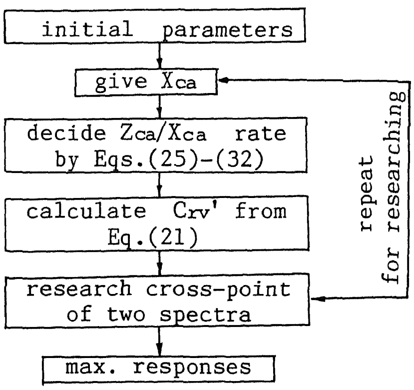
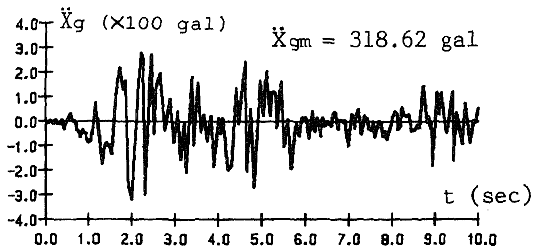
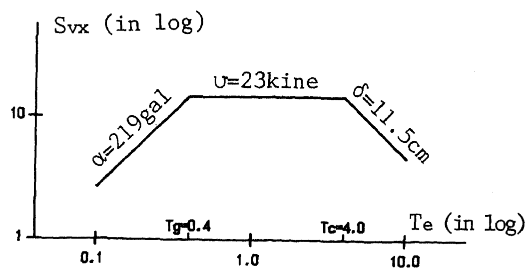


Fig.5 flow chart



a) Acceleration wave

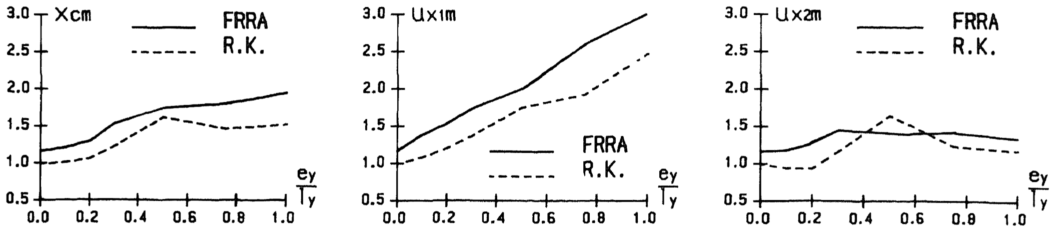


b) Pseudo-velocity spectrum

Fig. 7 ground motion for analysis

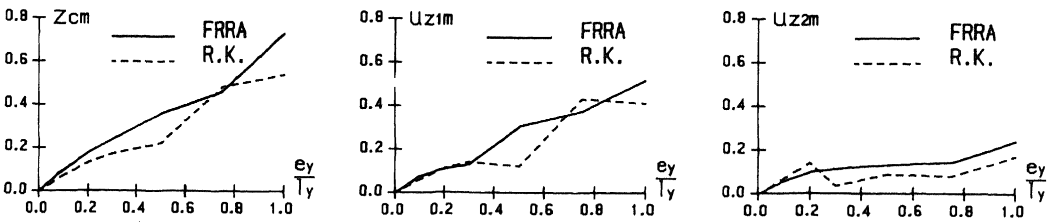
Table 1 parameter ey/ly and stiffness-rate

ly/ey	0.00	0.10	0.20	0.30	0.50	0.75	1.00
k_{x1}	1.00	1.11	1.20	1.33	1.50	1.75	2.00
k_{y1}, k_{y2}	1.00, 1.00	1.00, 1.00	1.00, 1.00	1.00, 1.00	1.00, 1.00	1.00, 1.00	1.00, 1.00
k_{x2}	1.00	0.99	0.80	0.67	0.50	0.25	0.00



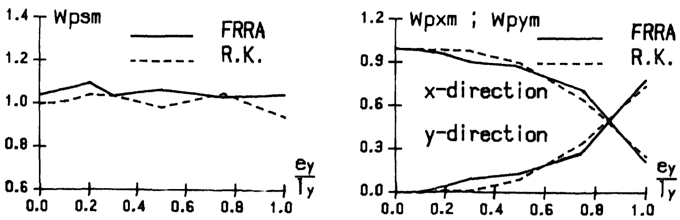
a) CM b) X1 c) X2

Fig. 8 max. lateral displacements in x-direction



a) CM b) X1 c) X2

Fig. 9 max. torsional displacements of system



a) total absorption b) distributions

Fig. 10 energy absorptions and distributions

Table 2 max. uncoupled responses

method / term	FRRA	R.K.
X_{com} (cm)	5.0271	4.2854
W_{pom} (cm·t)	7229.8	7067.2

4) Comparison of Two Methods From the analytical results and discussions above, it is indicated that the FRRA method proposed in this paper gives reasonable estimates of the responses for the asymmetric structures.

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