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STRENGTH ECCENTRICITY CONCEPT FOR INELASTIC ANALYSIS OF ASYMMETRIC STRUCTURES

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SUMMARY

In view of the relatively poor correlation between peak inelastic torsional responses and the eccentricity parameter based on uneven distribution of stiffness, this paper proposes the use of strength eccentricity as a more appropriate parameter to indicate the severeness of inelastic torsional responses for asymmetrical structures. The strength eccentricity takes into account the uneven distribution of strength, rather than stiffness of the structure. By means of an example, it is shown that much better correlation exists between inelastic torsional responses and strength eccentricity than the traditionally used stiffness eccentricity parameter.

INTRODUCTION

Considerable interest has been shown by earthquake engineers on the torsional behaviour of asymmetrical structures under seismic excitation, in view of the many asymmetrical buildings that were damaged during the 1985 earthquake in Mexico City [1]. Traditionally, the degree of asymmetry has been measured in terms of the structural eccentricity. It has been a useful parameter to correlate the seismic elastic response of asymmetrical structures, as shown by many studies. However, when the structural system is excited into the inelastic range, yielding of the resisting elements complicates the behaviour. Only a limited research has been carried out on the inelastic torsional behaviour of structures [2-6]. In all these studies, the structural eccentricity is still used as an index of asymmetry. However, the intensity of inelastic torsional response and structural eccentricity become less well correlated. For example, it was reported that the effect of structural eccentricity on inelastic torsional response is insignificant in one study [3], while another study suggested that the inelastic response varies linearly with eccentricity [6]. This dispersion of conclusions shows the need for a better index to denote the severity of inelastic torsional behaviour of asymmetrical structures subjected to ground motion excitation.

The present study proposes an alternative definition of eccentricity that is based on the yield strength properties of the structure. To assess the significance of the new eccentricity, denoted as the strength eccentricity, versus that of the traditionally used eccentricity based on stiffness distribution, the paper examines the inelastic seismic response of single story monosymmetric structural models with different stiffness and strength distributions. It is found that inelastic torsional deformations are strongly correlated with the magnitude of the strength eccentricity. Therefore, the strength eccentricity is a much better structural parameter to relate torsional response of asymmetrical structures when they are excited well into the inelastic range.

Definition of "Strength Eccentricity" Consider a single storey structure with lateral force resisting elements having elasto-plastic force-displacement characteristics. For the ith resisting element, its load deflection relation is characterized by the elastic lateral stiffnesses, k_{x_i} and k_{y_i}, and yield strength in transverse shear, V_{px_i} and V_{py_i}, in the global x and y directions, respectively. For such a structural plan, the structural eccentricity is commonly defined as the offset of the centre of stiffness CS from the centre of mass CM. The coordinates locating CS is given by

$$x_s = \sum_i k_{y_i} x_i / \sum_i k_{y_i}, \qquad y_s = \sum_i k_{x_i} y_i / \sum_i k_{x_i} \tag{1}$$

The centre of strength (CP) is defined as the centre of yield strengths of the resisting elements. The coordinates of the centre of strength can be found by taking the first moment of yield strengths and are given by:

$$x_p = \sum_i V_{py_i} x_i / \sum_i V_{py_i}, \qquad y_p = \sum_i V_{px_i} y_i / \sum_i V_{px_i} \tag{2}$$

The proposed "strength eccentricity" is then defined as the offset of the centre of strength (CP) from the centre of mass. The magnitude of the strength eccentricity depends on the strength distribution among resisting elements in relation to the mass distribution. The "strength eccentricity" e_p is also referred to as the plastic eccentricity in this paper and the subscript p is used to differentiate it from the structural or stiffness eccentricity e_s. If a structure is excited well into the inelastic range, it is expected that the strength distribution, rather than the stiffness distribution that will have a strong influence on the torsional response of such a system. As a result, one would expect a structure with a lower plastic eccentricity will experience less torsional deformation. An example on the inelastic seismic response of a single mass asymmetrical system is given in the following section to substantiate such a hypothesis.

EXAMPLE

Consider the simple single mass monosymmetric structural model shown in Fig. (1). It consists of a rigid square uniform deck of mass m and plan dimension D by D supported on four massless inextensible columns located at the extremities of a square of dimension a. The load-deflection relationship of each column is assumed to be of the elasto-plastic hysteretic type with initial stiffness k_i and yield strength V_{pi} along the two principal directions of resistance. The model is subjected to the two orthogonal horizontal components of ground motion $\ddot{u}_{gx}(t)$ and $\ddot{u}_{gy}(t)$. The deformation of the model can be described in terms of the two translational displacements q_x and q_y, and rotation q_θ at the mass center CM of the rigid deck. The equations of motion are then given by:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{q}_x \\ r\,\ddot{q}_\theta \\ \ddot{q}_y \end{Bmatrix} + 2\xi m \begin{bmatrix} \omega & 0 & 0 \\ 0 & \omega_\theta & 0 \\ 0 & 0 & \omega \end{bmatrix} \begin{Bmatrix} \dot{q}_x \\ r\,\dot{q}_\theta \\ \dot{q}_y \end{Bmatrix} + \begin{Bmatrix} Q_x \\ Q_\theta \\ Q_y \end{Bmatrix} = -m \begin{Bmatrix} \ddot{u}_{gx}(t) \\ 0 \\ \ddot{u}_{gy}(t) \end{Bmatrix} \tag{3}$$

where r is the radius of gyration of the rigid deck about CM, ω and ω_θ are the uncoupled lateral and torsional frequencies, respectively; Q_x and Q_y are the restoring shear forces and Q_θ is the restoring torque; ξ is the fractional critical damping, taken to be equal to 0.5% critical in this paper. The restoring forces and the generalized displacements are related by the incremental relationship

$$\begin{Bmatrix} \Delta Q_x \\ \Delta Q_\theta \\ \Delta Q_y \end{Bmatrix} = [K_t] \begin{Bmatrix} \Delta q_x \\ \Delta r q_\theta \\ \Delta q_y \end{Bmatrix} \tag{3a}$$

The tangential stiffness matrix $[K_t]$ is given by

$$[K_t] = \begin{bmatrix} \sum_i k_i(t) & -\sum_i k_i(t)\dfrac{y_i}{r} & 0 \\ -\sum_i k_i(t)\dfrac{y_i}{r} & \sum_i k_i(t)\left[\left(\dfrac{y_i}{r}\right)^2 + \left(\dfrac{x_i}{r}\right)^2\right] & 0 \\ 0 & 0 & \sum_i k_i(t) \end{bmatrix} \tag{4}$$

where $k_i(t) = k_i$ for columns in the elastic state or $k_i(t) = 0$ for columns in the plastic state. The interaction effect on yielding of the columns due to applied forces in the x and y directions is neglected in this study. Normalizing equation (3) with respect to total yield strength in the lateral direction, F_v, and the torsional yield strength F_θ gives

$$\begin{Bmatrix} u_x \\ u_\theta \\ u_y \end{Bmatrix} + 2\,\xi\omega \begin{bmatrix} 1 & 0 & 0 \\ 0 & \Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_x \\ u_\theta \\ u_y \end{Bmatrix} + \omega^2 \begin{Bmatrix} Q_x/F_v \\ \Omega^2 Q_\theta/F_\theta \\ Q_y/F_v \end{Bmatrix} = -(m \cdot S_a^*/F_v) \cdot \omega^2 \begin{Bmatrix} \ddot{u}_{gx}(t)/S_a^* \\ 0 \\ \ddot{u}_{gy}(t)/S_a^* \end{Bmatrix} \qquad (5)$$

where $\Omega = \omega_\theta/\omega$. In this paper, the spacing between column a is chosen to be 0.58D so that Ω equals unity. The nondimensional displacements are defined by

$$\left(u_x,\ u_\theta,\ u_y \right) = \left(\frac{q_x}{\delta_v},\ \frac{r \cdot q_\theta}{\delta_\theta},\ \frac{q_y}{\delta_v} \right) \qquad (6)$$

where δ_v and δ_θ are the translational and torsional yield deformations of the system respectively. The ground motion excitation is normalized with a spectral acceleration parameter S^*_a. The ratio of S^*_a and the peak ground acceleration follows the pattern of the smooth elastic design spectrum as proposed Newmark and Hall [7] and is shown in Fig. (2). The yield strength F_v of the system is also period dependent, with a period variation identical to S^*_a. Thus the ratio $R = m \cdot S^*_a/F_v$, appearing in the excitation term in Eq. (5), is period independent. It can be interpreted as the ratio of the elastic strength demand to the structure design strength capacity. When $R = 1$, structures subjected to a ground motion having spectral shape similar to S^*_a will just reach yielding. Large values of R correspond to the cases of structures whose design strength are lower than the elastic strength demand and hence such systems will be excited into the inelastic range when subjected to their design earthquake ground motions. To illustrate the significance of the plastic eccentricity in the inelastic response of torsionally unbalanced structures the following two structural configurations are considered.

1. <u>SP Model</u>

The properties of this model are adjusted such that it has both nonuniform distributions of stiffness and strength. The stiffness values of columns 3 and 4 are arranged to be larger than those of columns 1 and 2 so the stiffness eccentricity $e_s = 0.2$ D. Yield strengths of the columns are taken to be proportional to their stiffness values. Hence, the center of strength is offset from CM by the same distance as CS. Therefore, this model has $e_p = 0.2$ D.

2. <u>S. Model</u>

In this model, the stiffness distribution is the same as that of the SP Model, however, columns are assumed to have identical yield strength so that the center of strength CP coincides with CM resulting in zero strength eccentricity.

The inelastic responses of the two models are compared in Figures (3 to 5). In each case, other system parameters are $R = 5$, the lateral period T varies from 0.1 to 2.2 seconds and the two components of the 1940 El Centro and the 1952 Taft earthquakes records are used as input. Let $U_{\theta m}$ and U_{xm} be defined as the maximum absolute values of rotational and translational deformations in the x direction respectively, at CM. Comparing the broken and solid lines in Fig. (3) shows that the rotational deformation $U_{\theta m}$ is generally reduced in the S Model. Fig. (4) shows that the translational component U_{xm} is not systematically affected by changing the value of the plastic eccentricity between the two models. The ductility demand on the most stressed column, number 1, is shown in Fig. (5). The ductility demand is defined by

$$\mu_x = \max \left| u_y(t) - [u_\theta(t)/r](a/2) \right|$$

The influential effect that the plastic eccentricity has on the ductility demand is ev ident.

In order to evaluate the sensitivity of the inelastic torsional responses to structural eccentricity e_s and also strength eccentricity e_p, a parametric study is carried out based on the four column single mass model discussed earlier. Computation was carried out on two series of models. In the first series, all models have identical structual eccentricity $e_s = 0.2$ D, but with different values of strength eccentricities, e_p. In the second series, all models have the same strength eccentricity $e_p = 0.2$ D, but having different values of structural eccentricity e_s varying from 0 to 0.2 D. A reduction factor R = 5 is used in obtaining the yield strength of the models, and the 1952 Taft records, and the 1940 El Centro records are used as ground motion input. The responses using Taft records as input are presented in Figs. (6 and 7). Shown in Fig. (6) is the maximum rotation of the deck as a function of the lateral period of the structure. The spread of response curves in Fig. (6a) corresponding to different values of e_p, shows that rotational response of the structure is sensitive to variations in the magnitude of the strength eccentricity. This sensitivity is particularly pronounced for the short period structure. On the other hand, Fig. (6b) shows that the rotational response is insensitive to variations in the value of the stiffness eccentricity. All curves have steep slopes in the short period range with substantial values of deformations. This indicates that regardless of the magnitude of the stiffness eccentricity, the torsional response is controlled mainly by the strength eccentricity when the system is excited well into the inelastic range. The above observation is true also for ductility demand responses on the critical column as shown in Figs. (7a and 7b). For comparison purposes, curves corresponding to the symmetric case, i.e., $e_s = 0$ and $e_p = 0$, are also included in Fig. 7. The responses that correspond to the 1940 El Centro records as input show similar trends [8].

CONCLUSION AND DISCUSSION

The relatively poor correlation between the structural eccentricity and the inelastic torsional responses for some asymmetrical structual systems lead to the current proposal to use the strength eccentricity as an alternate asymmetry measure. Among other factors, the torsional response of any asymmetrical structure depends on the induced torque on the structure. In the elastic range, the traditionally used structural eccentricity is a useful system parameter to estimate the induced torque. Once a structure is excited into the inelastic range, the center of resistance (defined as the point the resultant resisting force passes through) no longer remains constant due to the yielding, loading and unloading of the different resisting elements. As a result, it is difficult to find a single system parameter capable of representing the process. However, if one is interested in the situation when a structure is excited well into the inelastic range, it is shown in this paper that the proposed parameter of strength eccentricity will be a useful parameter to correlate the peak response parameters of design interest.

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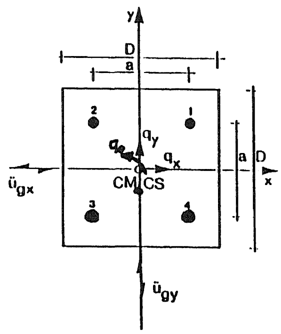


FIG.(1) SINGLE MASS MODEL

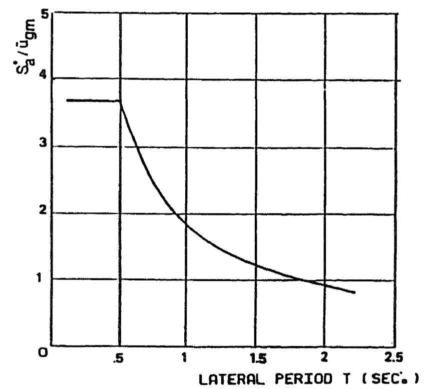


FIG.(2) SPECTRAL SHAPE OF S_a^*

Fig.(3)

Rotation of Models

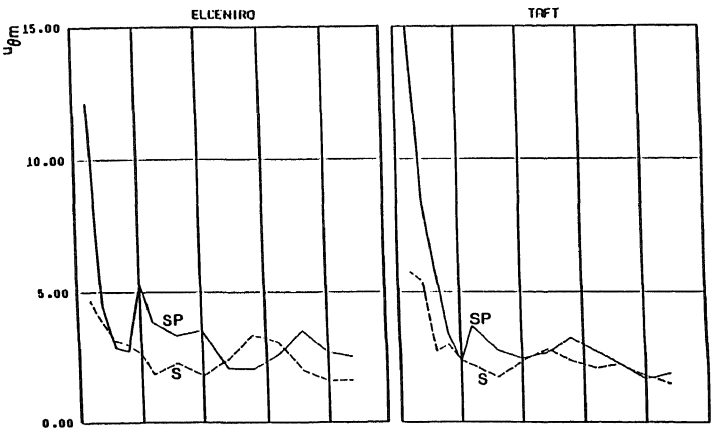
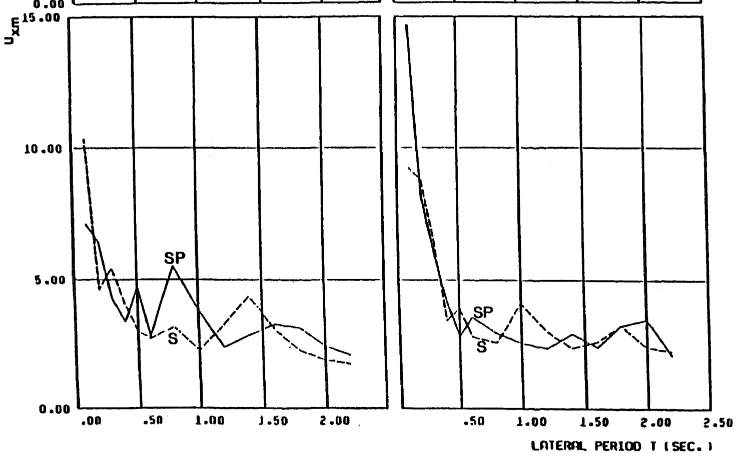


Fig.(4)

Displacement of Models



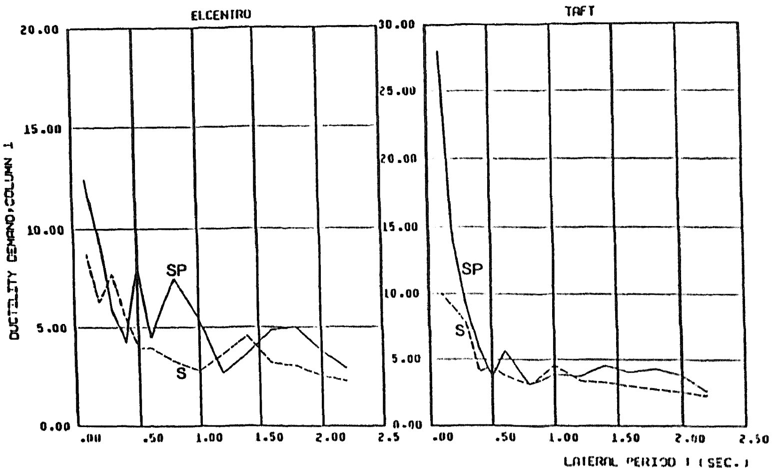


FIG.(5) DUCTILITY DEMAND ON COLUMN 1

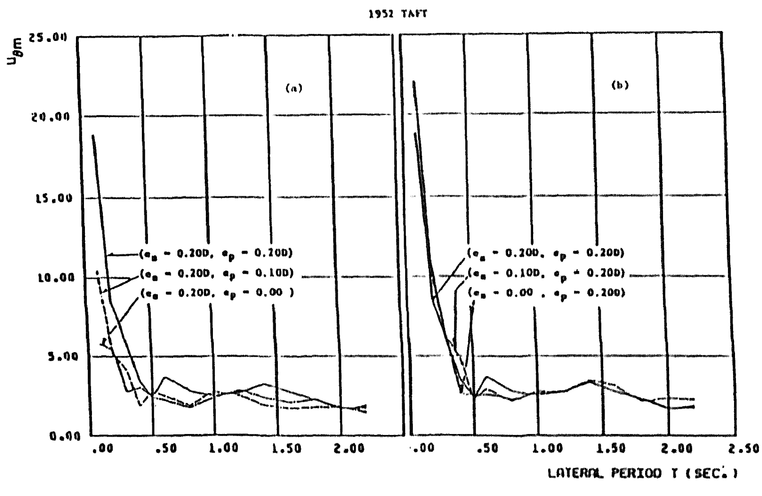


FIG. (6) MAXIMUM ROTATION OF MODEL.

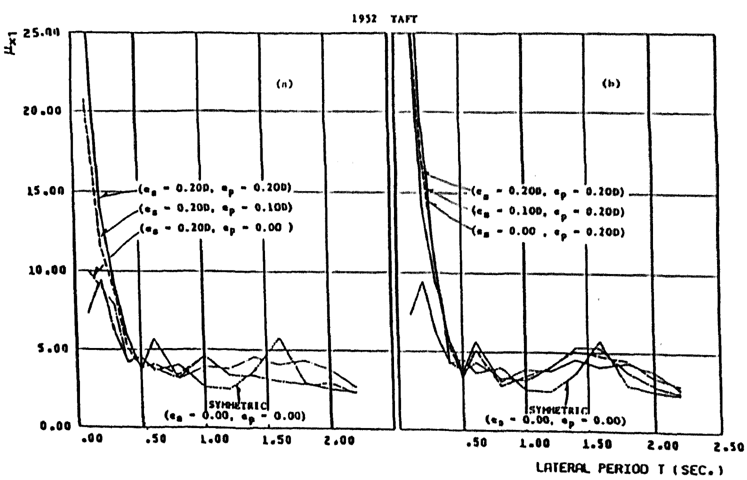


FIG. (7) DUCTILITY DEMAND ON COLUMN 1