FUNDAMENTAL STUDY ON FEEDBACK CONTROL SYSTEM REGARDING SLOSHING IN STORAGE TANK

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SUMMARY

Fundamental experiments were conducted on a system for reducing input and damping of sloshing by detecting wave heights of sloshing oscillations occurring in a storage tank and feeding back to a mechanical wave dissipating apparatus. Studies were made on two kinds of water tanks, and along with confirming the applicability of the theoretical equation concerning dissipation control by feedback, it was verified that effective wave dissipation could be done regarding first-order sloshing.

INTRODUCTION

As is clear from the fact that overflow occurred at crude oil tanks in Niigata Prefecture 300 km distant from the epicenter of the Nihonkai-chubu Earthquake (1983, M = 7.7), damping of sloshing oscillations is extremely weak, and there are cases when unexpected responses are seen depending on the characteristics of the input seismic waves. On the other hand, from the characteristic that damping is weak, it is clear that amplitude can be effectively reduced if damping is increased by some method, and there are many cases of studies focusing on this point having been made. However, most of these studies have attempted to increase damping by passive devices, and there have been only a limited number of cases where control of sloshing by apparatus with active functions has been tried.

The authors took up the idea of an apparatus for decreasing sloshing both by reducing input energy and by damping response through changes in the shapes of side walls and bottom plates of storage tanks in accordance with wave height of sloshing. The experiments were of two kinds: Theory Confirmation Experiments using a small square water tank and Effect Confirmation Experiments using a circular water tank of diameter 4 m.

THEORY CONFIRMATION EXPERIMENTS

Apparatus A conceptual drawing of the testing apparatus is shown in Fig. 1. The water tank was of rectangular shape 1 m in height, 1.2 m in length, and 0.8 m in width. The side walls of this water tank were made by stacking lightweight square tubes with flat bearings between the tubes for a construction that allows sliding of side walls in the horizontal direction while maintaining a plane shape. Especially, in this experiment, a leaf spring was attached to each square tube with one end connected to an inverted-type cantilever with the side walls of the water tank deformed in shear by rotation of the cantilever. A hydraulic actuator was attached to the cantilever, making it possible for
deformation to be controlled in accordance with wave height of sloshing. The water tank was installed on a shaking table, and it was made possible for any combination of excitation from the bottom and excitation by actuator from the top to be made.

Fig. 2 is a block diagram of the feedback system composed. The phase correction circuit is for adjusting the output of the wave gauge to a phase optimum for wave dissipation considering the phase characteristics of the actuator. The bandpass filter cuts drift of signals along with which it cuts off the high-frequency component which is the cause of higher-harmonic resonance. In this experiment components higher than 10 Hz were cut.

**Prediction of Effects**  With height B of wave occurring in case of input of an imaginary signal voltage A inputted in the feedback system of Fig. 2, the following equation is valid.

$$ (A - g_2 \cdot B) g_1 = B $$  

(1)

where, $g_1$ is wave-making efficiency (cm/volt) and means the wave-height value occurring in case of input of sine wave of unit amplitude to $G_1$, cutting out the $G_2$ circuit. The term $g_2$ is feedback efficiency (volt/cm) and is the output voltage of the $G_2$ circuit in relation to unit wave height. Eq. (1) is rewritten as follows:

$$ B = \frac{g_1}{1 + g_1 \cdot g_2} A $$  

(2)

On the other hand, with $B_0$ as the wave height when the circuit $G_2$ is cut out and an imaginary signal $A$ is inputted to $G_1$, the following equation will be valid as is clear from Eq. (2):

$$ B_0 = g_1 \cdot A $$  

(3)

Defining wave dissipation rate by feedback as $(B_0 - B)/B_0$, the following equation is obtained:

$$ \frac{B_0 - B}{B_0} = \frac{g_1 \cdot g_2}{1 + g_1 \cdot g_2} $$  

(4)

Now, the result of wave-height measurement on input of sine-wave voltage to $G_1$ on bypassing $G_2$ in a condition of the shaking table stopped to obtain $g_1$ in concrete terms is shown in Fig. 3. A prominent peak of primary resonance with sloshing can be seen at 0.8 Hz. With this peak as the object of wave dissipation, the value of $g_1$ will be as follows:

$$ g_1 = 100 \text{ cm/volt at 0.8 Hz} $$  

(5)

Meanwhile, feedback efficiency $g_2$ varies depending on the sensitivity of the wave gauge, the amplification rates of the phase correction circuit and the bandpass filter, parts making up $G_2$, and the amplification rate of the pre-amp and the attenuator. With the attenuator value $x$ only varied and on measuring the amplification rates specific to the individual instruments, the following equation is obtained:

$$ g_2 = 0.53 x \text{ volt/cm at 0.8 Hz} $$  

(6)

By substituting Eqs. (5) and (6) in Eq. (4), it will be possible to predict the damping effect.

**Results of Experiments**  The experiments were carried out for the cases of inputting sine waves of 0.8 Hz and seismic waves to the shaking table. These experiments were conducted for cases of feedback included and of actuator stopped, and comparisons were made. Table 1 compares the experimental results and the wave dissipation rates obtained by Eq. (6). With regard to sine wave input, prediction and experiment agree
well to confirm the applicability of Eq. (4). In case of seismic wave input, a value close to prediction is obtained when components of 0.8 Hz are focused on, but with components of all frequencies, the value is larger than the predicted value, indicating that there is a frequency component for which wave dissipation does not occur. According to Eq. (4) the wave dissipation rate should be increased further by attenuator value \( x \) being made larger, but in actuality, oscillation occurred and the value given in Table 1 was the limit.

**EFFECT CONFIRMATION EXPERIMENTS**

**Apparatus** A conceptual drawing of the testing apparatus is shown in Fig. 4. The water tank was a flat-bottomed cylinder 1.5 m in height and 4 m in diameter. The side wall of the tank could be considered as being amply rigid against sloshing. This tank was set on the shaking table cushioned on six laminated rubber pads. An actuator for feedback control was installed between the bottom plate of the tank and the shaking table. The feedback control circuit was the same as in the theory confirmation experiments, and as shown in Fig. 2. However, feedback signals higher than 10 Hz were cut off by the bandpass filter.

**Prediction of Wave Dissipation Effect** Similarly to the case of the theory confirmation experiments, the results of experiments conducted to obtain the wave-making efficiency \( g_1 \) are as shown in Fig. 5. A prominent peak of primary resonance with sloshing is seen in the vicinity of 0.43 Hz. The value of \( g_1 \) at that time is as follows:

\[
g_1 = 124 \text{ cm} / \text{volt at } 0.43 \text{ Hz} \quad \text{-------------------(7)}
\]

On the other hand, the feedback efficiency \( g_2 \) will be the following value unlike Eq. (6) because of the frequency dependencies of the various instruments.

\[
g_2 = 0.30 \times \text{ volt/cm at } 0.43 \text{ Hz} \quad \text{-------------------(8)}
\]

By substituting Eqs. (7) and (8) in Eq. (4) the wave dissipation rate is given by the equation below.

\[
\frac{B_0 - B}{B_0} = \frac{37 \times x}{1 + 37 \times x} \quad \text{-------------------(9)}
\]

**Experimental Results** Similarly to the theory confirmation experiments, experiments were carried out for the cases of inputting sine waves of 0.43 Hz to the shaking table, and of inputting seismic waves. Experiments were conducted for cases of feedback circuit working and of anchoring the water tank to the shaking table with the actuator stopped, and the wave dissipation rate was determined by comparing the sloshing wave heights occurring.

Table 2 gives comparisons of the experimental results and the wave dissipation rate obtained by Eq. (9). Similarly to Table 1, with regard to the sine wave input and the 0.43 Hz component in case of seismic wave input, the value is 80 percent of predicted. The wave seen in the vicinity of 0.8 Hz in Fig. 5 is a wave height approximately 30 percent of 0.43 Hz, and it is considered that the wave dissipation rate became lower than the predicted value due to the influence of this wave. On looking at total frequency components of seismic wave input, a wave dissipation rate of 50 percent was attained. Fig. 6 shows comparisons of sloshing waveforms during experimenting by seismic wave input. It can be seen that damping of sloshing after input of seismic wave has been finished is gradual when there is no feedback, but damping occurs rapidly when feedback is made.
CONCLUSIONS

From the results of experiments on two kinds of water tanks, i) it is possible for primary sloshing vibrations to be reduced by feeding back wave height and deforming or moving the side walls of the storage tank and ii) Eq. (4) can be applied for predicting the rate of that wave dissipation.

Topics for further developments are a practical mechanism which can be applied to an actual tank and a counter measure of higher-harmonic resonance occurring in case feedback efficiency is increased.

ACKNOWLEDGEMENT

The authors wish to express their sincere gratitude to Mr. A. Namiki of Takeru Consultants for his assistance in carrying out experiments and analyses.

![Diagram of Test Apparatus](image)

Fig. 1 Conceptual Drawing of Test Apparatus (Rectangular Water Tank)

![Diagram of Feedback Circuit](image)

Fig. 2 Feedback Circuit
Table 1  Wave Dissipation Effect  
(Theory Confirmation Experiments)

\[
\frac{(B_0 - B)}{B_0} \text{ (in percent)}
\]

<table>
<thead>
<tr>
<th>Attenuator (X)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted Value</td>
<td>84</td>
<td>91</td>
<td>94</td>
</tr>
<tr>
<td>Sine Wave Excitation</td>
<td>86</td>
<td>93</td>
<td>96</td>
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<tr>
<td>Seismic Wave Excitation Overall Value</td>
<td>81</td>
<td></td>
<td></td>
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<tr>
<td>Seismic Wave Excitation 0.8 Hz Component</td>
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<td></td>
<td>92</td>
</tr>
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</table>

Fig. 3  Resonance curve  
(Theory Confirmation Experiments)

Fig. 4  Conceptual Drawing of Test Apparatus  
(Cylindrical water tank)
Table 2  Wave Dissipation Effect  
(Effect Confirmation Experiments)

\[
\frac{(B_0 - B)}{B_0} \text{ (in - percents)}
\]

<table>
<thead>
<tr>
<th>Attenuator ((X))</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
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<tbody>
<tr>
<td>Predicted Value</td>
<td>88</td>
<td>94</td>
<td>95</td>
</tr>
<tr>
<td>Sine Wave Excitation</td>
<td>46</td>
<td>72</td>
<td>79</td>
</tr>
<tr>
<td>Seismic Wave Excitation Overall Value</td>
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</tr>
<tr>
<td>Seismic Wave Excitation 0.43 Hz Component</td>
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</table>

Fig. 5  Resonance curve  
(Effect Confirmation Experiments)

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Fig. 6  Comparisons of Sloshing Waveforms  
in Experiments Inputting Seismic Waves (El-Centro 1940 NS)