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BASIC STUDY ON ACTIVE SUPPRESSION METHOD OF SLOSHING OF LIQUID IN TANKS DURING EARTHQUAKES BY AIR INJECTION

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SUMMARY

Analytical and experimental investigations were carried out to propose an suppression method of sloshing of liquid in tanks during earthquakes by air injection. Basic equation of motion of liquid in rectangular tank was derived, and it was shown that this equation has the same form as that of U-tube. The characteristics of the force induced by air injection was investigated experimentally by using U-tube. Based on these experimental results, force model for air injection was set up. Computer simulation results based on this model showed good agreement with experimental results, and several important results were obtained.

INTRODUCTION

Many report informed that liquid in large tank made sloshing vibration during earthquakes. For example, oil overflowed due to sloshing at many tanks and fire broke out at not a few tanks during Niigata Earthquake-1964, very heavy sloshing of wave height 4m occurred in a tank in Niigata area during Nihonkai-cyubu Earthquake-1983. Thus, we always have certain possibility of hazard that we will suffer big damage due to sloshing.

To prevent such hazards, techniques to prevent or to suppress sloshing during earthquakes is needed. Hayama et al proposed U-tube sloshing absorber, and made analysis on it.(Ref.1) It seems, however, it has some structural difficulty. Shibata et al proposed a method to suppress sloshing by air injection.(Ref.2) Present research lies on the same line as Ref.2.

Suppose that air is injected into sloshing liquid in tank, it exerts some force on liquid. We can make use of this force to suppress sloshing, provided that we do some appropriate active control for air injection. To realize this method we have to know the characteristics of the force induced by air injection, and how to control air injection. In this paper, basic characteristics of the force induced by air injection are mainly studied.

THEORETICAL ANALYSIS

Basic Equations for Rectangular Tank Suppose that a rectangular tank shown in Fig.1 is subjected to the ground acceleration $\alpha_g(t)$, and air is injected at point (x_a, y_a). If we set assumptions:

(a)Tank is two-dimensional and rigid

(b) Liquid in tank can be regarded as incompressible ideal fluid

(c) Force induced by injected air can be regarded as point force

the motion of liquid is governed by next equations: (Ref.2)

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{1}$$

$$\left.\frac{\partial\phi}{\partial x}\right|_{x=\pm a} = 0 \tag{2}$$

$$\left.\frac{\partial\phi}{\partial y}\right|_{y=0} = 0 \tag{3}$$

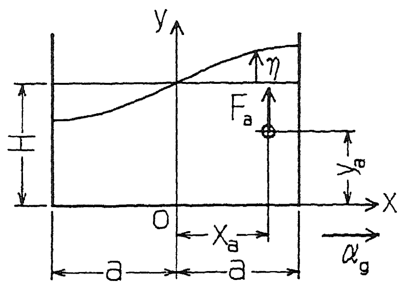


Fig.1 Rectangular tank

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial y} = -\alpha_g'(t)x + \frac{1}{\rho}F_a'(t)\delta(x-x_a);\ y = H \tag{4}$$

where, the co-ordinate system taken is a moving coordinate system with its origin at the center of the bottom of tank, and ϕ: velocity potential, 2a: width of tank, H: depth of liquid, ρ: density of liquid, g: gravitational acceleration. $F_a(t)\delta(x-x_a)$ represents the force induced by injected air at point (x_a, y_a). The notation $\delta(\cdot)$ is the Dirac delta function, so as shown in the assumption above, this force is treated as a point force. The characteristics of it will be discussed in detail later.

Obtaining the solution of ϕ, the displacement of free surface η and the dynamic pressure p_d are given by the next expressions:

$$\eta = -\frac{1}{\rho}\left\{\alpha_g(t)x + \left.\frac{\partial\phi}{\partial t}\right|_{y=H} - \frac{1}{\rho}F_a(t)\delta(x-x_a)\right\} \tag{5}$$

$$p_d = -\rho\left\{\alpha_g(t)x + \frac{\partial\phi}{\partial t} - \frac{1}{\rho}F_a(t)\delta(x-x_a)u(y-y_a)\right\} \tag{6}$$

where $u(\cdot)$ is the unit step function.

Solving the governing equations by way of modal analysis method, we get the next basic equations for response analysis:

$$\xi_i'' + 2\zeta_i\omega_i\xi_i' + \omega_i^2\xi_i = -\alpha_g(t) + \frac{\varepsilon_i^2\sin(\varepsilon_i x_a/a)}{2\rho a^2\sin(\varepsilon_i)}F_a(t) \quad (i=1,2,\cdots) \tag{7}$$

where

$$\varepsilon_i = (2i-1)\pi/2, \quad \omega_i = \sqrt{(g/a)\varepsilon_i\tanh(\varepsilon_i H/a)} \tag{8}$$

ω_i is the natural angular frequency of the i-th mode. In the above, the term $2\zeta_i\omega_i\xi_i'$ is intentionally introduced to represent the damping effect.

Obtaining the solutions of above equations, the displacement of free surface η and the dynamic pressure p_d are given by:

$$\eta = \frac{a}{g}\sum_{i=1}^{\infty}\frac{2}{\varepsilon_i^2}X_i(x)\omega_i^2\xi_i(t) \tag{9}$$

$$p_d = -\rho a\sum_{i=1}^{\infty}\frac{2}{\varepsilon_i^2}X_i(x)\left\{\alpha_g(t) + Y_i(y)\xi_i''(t) - \frac{\varepsilon_i^2\sin(\varepsilon_i x_a/a)}{2\rho a^2\sin(\varepsilon_i)}u(y-y_a)F_a(t)\right\} \tag{10}$$

where

$$X_i(x) = \sin(\varepsilon_i x/a)/\sin(\varepsilon_i), \qquad Y_i(y) = \cosh(\varepsilon_i y/a)/\cosh(\varepsilon_i H/a) \tag{11}$$

As we are mainly concerned with suppression of the first mode sloshing, the basic equation becomes:

$$\xi_1'' + 2\zeta_1\omega_1\xi_1' + \omega_1^2\xi_1 = -\alpha_g(t) + \frac{\pi^2\sin(\pi x_a/2a)}{8\rho a^2}F_a(t) \tag{12}$$

Then, the first mode sloshing displacement at the side wall is given by:

$$h_1(t) = \left.\eta_1\right|_{x=a} = \frac{8a}{\pi^2 g}\omega_1^2\xi_1(t) \tag{13}$$

Basic Equation for U-tube Next, we think of U-tube. Suppose that U-tube shown in Fig.2 is subjected to the ground acceleration $\alpha_g(t)$, and air is injected at point $x=x_a$. If we set assumptions:

(a) Liquid in the tube is incompressible ideal fluid

(b) Volume change by injected air is small enough

the basic equation for liquid motion in the tube is given by:

$$\xi'' + 2\zeta\omega\xi' + \omega^2\xi = -\alpha_g(t) + \frac{1}{2\rho AR}F_a(t) \qquad (14)$$

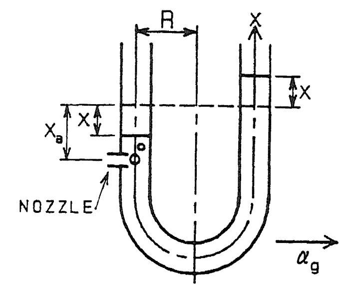


Fig.2 U-tube

where $x=(2R/\ell)\xi$: displacement of liquid, $\omega=\sqrt{(2g/\ell)}$: natural angular frequency of liquid motion, ζ: damping ratio, R: radius of curvature of the tube, ℓ: length of liquid in the tube, a: cross sectional area of the tube. This equation is the same form as Eq.(7) or (12), thus it is noted that the liquid motion in rectangular tank can be simulated by the liquid motion in U-tube.

Outline of Research In the above, the basic equations for rectangular tank was derived. It is noted from the result that if we know the force induced by air injection, $F_a(t)$, we can calculate the sloshing response by using Eq.(12). So, the most important point in this research is the identification of the force $F_a(t)$.

Thus, we made our research by following the next line:

(1) Identify the force induced by air injection, $F_a(t)$, by experiments using U-tube

(2) Set up some force model for $F_a(t)$

(3) Make computer simulations by using this force model

(4) Compare the simulation results with the experimental results and check the validity of the force model

As we are concerned with liquid motion in tank, it is best to identify the force by experiments using tank model, however, if air is injected into liquid in tank, all the modes are excited and the response get very complicated. But we know, from the analysis above, that the basic equation for liquid motion in tank and in U-tube have the same form each other and liquid motion in rectangular tank is simulated by liquid motion in U-tube. So, for simplification and to avoid complexity of mode superposition, we first investigate by using U-tube, and then apply it to tank.

The force model From the results of the experiments using U-tube below, it was found that the force induced by injected air, $F_a(t)$, is impulsive and the magnitude of impulse is proportional to the volume of air injection per unit time and the square of the depth of air injection. Thus, based on the experimental results, we set a impulsive force model as shown in Fig.3, where I_1 and I_2 is the magnitude of the impulse and given by Eq.(18a) and (18b) below and T_1 and T_2 are the time of action and were set T_1=0.1 sec, T_2=0.2 sec.

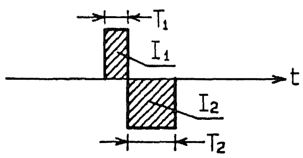


Fig.3 Force model

$$F_a(t) = \begin{cases} I_1/T_1 & (0 \leq t \leq T_1) \\ -I_2/T_2 & (T_1 \leq t \leq T_1+T_2) \\ 0 & (T_1+T_2 \leq t) \end{cases} \qquad (15)$$

Computer simulation   Based on the basic equations and the force model set up above, computer simulations were made to check the validity of the force model and to check the suppression effect of this method.

(1)Simulation to check the validity of the force model   First of all, simulation to check the validity of the force model was made. Liquid motion was simulated when model force applied on liquid at rest. Fig.4 shows an example of simulation results. Corresponding experimental result is also shown in the figure. It is known from this figure that the simulation result agrees well with experimental result. Thus, the force model is thought to be valid.

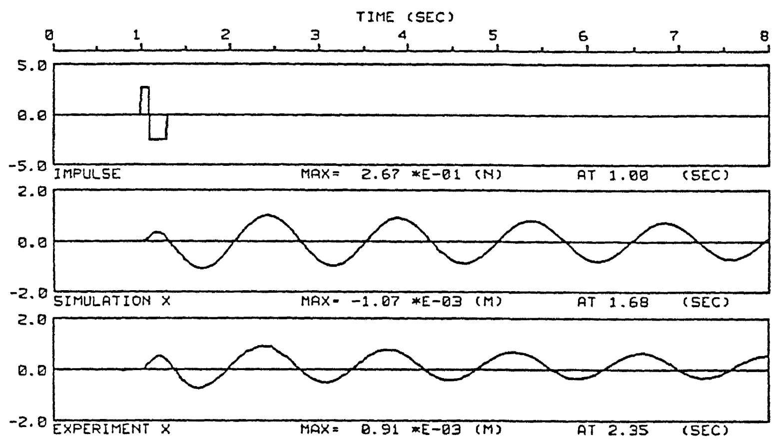


Fig. 4 An example of simulation results

(2)Simulation to check the suppression effect   Next, simulation to check the suppression effect was made. In this simulation, liquid motion to a resonant sinusoidal input was suppressed. When response grew to the steady state, air injection was started. Then, the amplitude began to change, and sufficient time later, the response got to another steady state. Suppression effect was evaluated by Amplitude Magnification Factor (AMF):

$$\mathrm{AMF} = \frac{\text{Response amplitude with air injection}}{\text{Response amplitude without air injection}} \qquad (16)$$

Simulation was carried out by changing the timing of air injection start: $\tau_s = T_s/T$, where T_s is the time when the air injection is started and T is the natural period of liquid motion. The time is taken for each cycle of response as shown in Fig.5. The time origin for each cycle is set at the time when displacement crosses zero level from negative to positive.

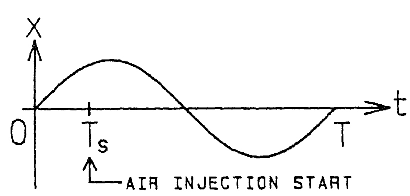


Fig.5 Timing of air injection start

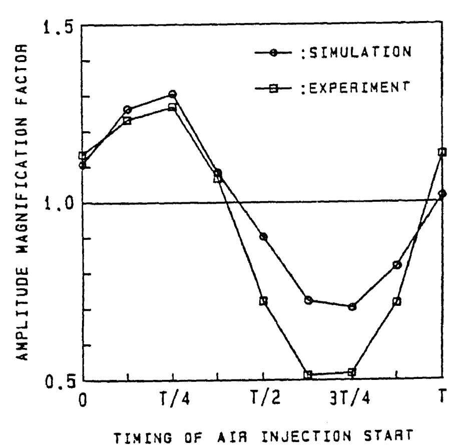


Fig.6 Suppression effect

Fig.6 shows the simulation results for suppression effect together with the experimental results. This shows that the tendency agrees well with each other and it is noted from this figure that the best suppression effect appears at τ_s=3T/4. This also shows the validity of the force model.

EXPERIMENTAL ANALYSIS

Experimental analysis was carried out to get fundamental information for setting the model of force induced by air injection and to get the amplitude magnification factor for checking the simulation results. The overview of the experimental set up is shown in Fig.7.

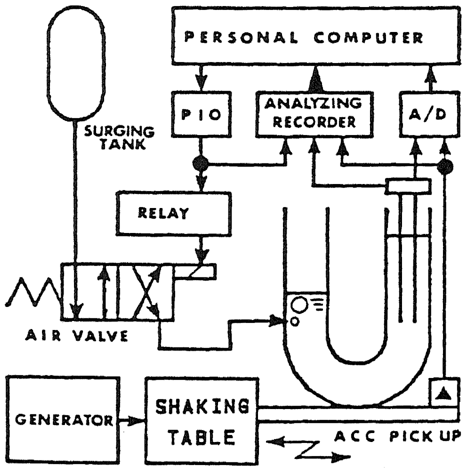


Fig.7 Experimental set up

<u>Experiment to get fundamental information to model the force induced by air injection</u> Suppose that air is injected into liquid at rest and the displacement of liquid, x, is measured, then the force induced by air injection is identified by using Eq.(14) as follows:

$$F_a(t) = \rho A \ell (x'' + 2\zeta\omega x' + \omega^2 x) \qquad (17)$$

Based on this method, experiment was carried out by changing the duration of air injection and the volume of air injection per unit time. Fig.8 shows an example of experimental results. Identified force is also shown in this figure. It is shown from this figure that the force consists of two impulses: first positive and second negative.

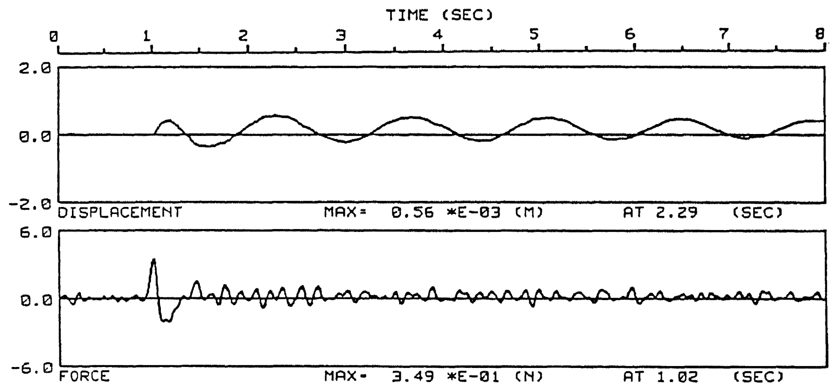


Fig.8 An example of experimental result and identified force

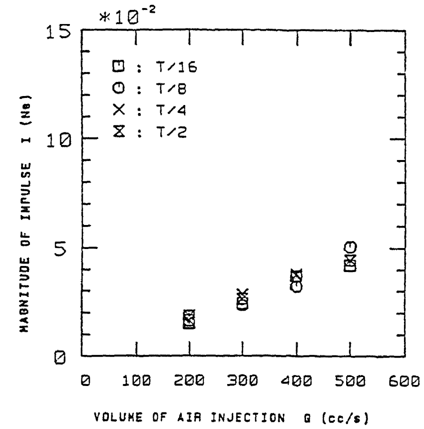
Fig.9(a) shows the relation between the magnitude of the first impulse, I_1, and the volume of air injection per second, Q, when the depth of injection, h, is constant. It is found from this figure that I_1 is approximately proportional to Q. Fig.9(b) shows the relation between I_1 and h, when Q is constant. It is found from this figure that I_1 is approximately proportional to h^2.

Thus, he magnitude of the first impulse can be fitted by:

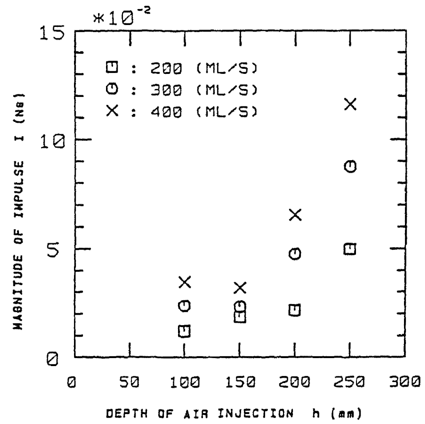
$$I_1 = C_1 Q h^2 \quad (C_1 = 4.358 * 10^3 \mathrm{kg/m^3} \text{ for this model}) \tag{18a}$$

Situation is almost same for the second impulse and:

$$I_2 = C_2 Q h^2 \quad (C_2 = 5.753 * 10^3 \mathrm{kg/m^3} \text{ for this model}) \tag{18b}$$



(a)Relation between I and Q



(b)Relation between I and h

Fig.9 Relation between impulse I and volume of air injection per second Q or depth of air injection h

<u>Experiment to check the suppression effect</u> Shaking table was excited by sinusoidal wave whose period is equal to the natural period of liquid motion. When the response of liquid motion grew to the steady state, air injection was started. Then the amplitude began to change, and sufficient time later, the response got to another steady state. The steady state amplitudes before and after air injection were measured, and the AMF (amplitude magnification factor) was evaluated. Definition of AMF and parameters changed are same as these in computer simulation. The results are shown in Fig.6 together with the simulation results. It is shown from Fig.6 that suppression effect is best when the air injection start timing τ_s=3T/4 and worse when τ_s=T/4.

CONCLUSIONS

(1)Basic equations to describe the motion of liquid in rectangular tank which is subjected to the ground motion and air injection were derived.

(2)Characteristics of the force induced by injected air were studied experimentally, and force model was set up based on the results.

(3)By using this model, computer simulations were made to check the suppression effects. The results agreed well with the experimental results.

REFERENCES

1. Hayama, S. et al, A Study on an Anti-Sloshing Device in a Liquid Container, Bull. of JSME, (C), Vol.51, No.470, October 1985, pp.2505-2511
2. Shibata, H. et al, Study of Active Control System for Suppressing Earthquake-Induced Sloshing in Liquid Tanks, Report of ERS No.III-7, March 1985, pp.45, The Institute of industrial Science, Univ. of Tokyo.