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7-11-3 OPTIMUM CONTROLLER LOCATION FOR MITIGATING EARTHQUAKE INDUCED RESPONSE OF STRUCTURES PROVIDED WITH POINT ACTUATORS

A.G.Chassiakos, S.F.Masri, G.A.Bekey, R.K.Miller
School of Engineering, University of Southern California
Los Angeles, CA. 90089-0242, USA

SUMMARY

This paper is concerned with the problem of determining the optimum controller locations in flexible (building-like) structural systems undergoing earthquake ground motion and provided with a number of vibration suppression actuators that are distributed throughout the structure. It is assumed that the controllers deliver continuous-time forces, and that the force magnitudes are determined in a "state-feedback" fashion. The feedback matrix is obtained by optimal control methods, with quadratic optimization criteria.

1. INTRODUCTION

The active control of flexible structures is one of the most important problems of current research in several engineering disciplines.

As far as Civil Engineering structures are concerned, the needs for structural integrity, enhancement of human comfort and safety, and reduction of the structure's potential damage from disturbances due to earthquakes, immediately bring active control into the picture.

The present work is motivated by the work described in Masri et al.,(Ref. 1, Ref. 2), on the pulse control of flexible structures. These references deal with structures linear with finite degrees of freedom, or nonlinear structures, using local displacement and velocity measurements. The control forces are pulses of a short duration compared to the fundamental period of the structure. The pulses are enacted at certain times during a random excitation history. The main pulse parameters are: the pulse duration, the pulse time-shape, the force magnitude and the controllers' locations. Once these parameters have been decided upon, the algorithms presented in the above references yield the pulse-firing time instants, in an "open-loop" fashion, so that the structure's response remains within predefined bounds. *Simulations and experimental results suggest that the location of the controllers plays an important role in the problem.*

Here we isolate the controllers' location problem and give a solution to it, which is both justified theoretically and validated by simulations. In order to achieve this, we *relax the assumption of using pulses* as control forces. We assume controllers which deliver *continuous-time forces*, so that the force magnitudes are determined in a "state-feedback" fashion. We develop an algorithm which solves the controllers' location problem for any linear structure with finite or infinite degrees of freedom.

Within this framework we examine two cases:

(i) a continuous Euler-Bernoulli beam with random base excitation

(ii) a chain-like flexible structure with 3 degrees of freedom whose base is subjected to random excitation.

2. PROBLEM STATEMENT

2.1 Model of a Continuous Beam with Random Base Excitation : Consider a cantilever Euler-Bernoulli beam, whose base is subjected to random acceleration along the y direction (fig. 1). Let $y(t)$ denote the distance of the base point from a global coordinate system (x, y); x the distance of a point on the beam along the x-axis; $w(x,t)$ the elastic displacement measured from the undeformed beam position; EI the beam's flexural rigidity; $\rho(x) = \rho = constant$ the beam's linear density and $f(x,t)$ the control forces.

Let the control forces $f(x,t)$ act pointwise along the length of the beam at the points $\{\xi_1, \ldots, \xi_k\}$. Then $f(x,t)$ can be approximated as the sum of delta functions: $f(x,t) = \sum_{j=1}^{k} \delta(x-\xi_j)u_j(t)$. Assumming that $w(x,t)$ is small, and using eigenfunction expansion, with $\phi_i(x)$ representing the $i-th$ eigenfunction, we get the equations of motion for each mode:

$$\begin{pmatrix} \dot{q}_i \\ \ddot{q}_i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega_i^2 & 0 \end{pmatrix} \begin{pmatrix} q_i \\ \dot{q}_i \end{pmatrix} + \begin{pmatrix} 0 \\ b_i \end{pmatrix} v(t) + \begin{pmatrix} 0 & \ldots & 0 \\ B_{i1} & \ldots & B_{ik} \end{pmatrix} \begin{pmatrix} u_i(t) \\ \ldots \\ u_k(t) \end{pmatrix} \tag{2.1}$$

where $v(t) = \ddot{y}(t), \quad b_i = -\int_0^l \phi_i(x)\,dx\,, \quad B_{ij} = \frac{1}{\rho}\phi_i(\xi_j)$

Concatenating eqs.(2.1) for every mode i and assuming a small damping ζ_i for each one, we obtain an infinite dimensional linear system, which if truncated at the first N modes yields:

$$\dot{q} = Aq + Bu + bv \quad \text{with} \quad A = \text{blockdiag}\begin{pmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{pmatrix}, \; i = 1, \ldots, N \tag{2.2}$$

and with similar definitions for B, b. In (2.2), v is the driving random input and u is the vector of control forces.

2.2 Model of an N DOF Structure with Random Base Excitation: Consider an N degree of freedom (NDOF) chain-like structure (Fig. 2). The equations of motion for this system are:

$$M\ddot{y} + C\dot{y} + Ky = -M\ddot{s} + f \tag{2.3}$$

The random base acceleration $\ddot{s}(t)$ plays the role of $v(t)$ appearing in eq.(2.2) and is assumed to have the same statistical properties as $v(t)$. Assuming proportional damping, (2.3) can be decoupled and written as a first order equation:

$$\dot{q} = Aq + b_N\ddot{s} + Bu, \qquad q = [q_1, \dot{q}_1, \ldots, q_N, \dot{q}_N]^T \tag{2.4}$$

where $\dot{y} = \Phi q$, Φ is the decoupling matrix and the definitions of A, b_N, B are obvious. It is seen that eq.(2.4) has the same structure as (2.2) of the previous section. Hence a common control law will be developed for both systems (2.2) and (2.4). This control law is presented in the next section.

2.3 Control Law: In the development of the common control law for both systems (2.2) and (2.4) we assume full information on the state q, which is used for the feedback loop construction. The random excitation $v(t)$ of (2.4) (respectively the random base acceleration $\ddot{s}(t)$ of (2.5)) is taken as a white noise vector process of intensity V:

$$E\{v(t)\,v(t+\tau)\} = V\delta(\tau) \geq 0.$$

The system is taken initially at rest, $q(0) = 0$. We consider a quadratic performance criterion of the form:

$$J = E\left\{\int_0^\infty (q^T R_1 q + \frac{1}{\rho_2}u^T u)\,dt\right\} \tag{2.8}$$

In (2.8), R_1 is the state weighting matrix and ρ_2 is the "cost" of the control energy. Now suppose that the controllers' locations have already been selected. Using the well established stochastic regulator theory with quadratic criteria, we have at optimality (Ref. 3):

$$u = -Fq, \quad F = \rho_2 B^T P \tag{2.9}$$

and P is the solution of the matrix Riccati equation: $R_1 - \rho_2 PBB^T P + A^T P + PA = 0$. The values of $u(t)$ in (2.9) will minimize criterion (2.8).

3. LOCATIONS OF ACTUATORS

The control law (2.9) was obtained considering the locations ξ_i given. In this section we present a procedure which will yield the optimal locations of the controllers. Here the term "optimal locations" is used in the sense that these locations will minimize the value of the criterion J, among all the possible actuator locations.

To achieve this, we divide the continuous beam into M intervals (not necessarily of the same length). Let $\{m_1, m_2, \ldots, m_M\}$ be the set of points at which the beam is divided. In principle the number M can be very large, so that every point of the beam can be represented. (In the case of an N DOF structure, $M = N$).

By using the optimal control law (2.9), the minimum value of the criterion J turns out to be (Ref. 3): $J^o = Tr(PV)$, where Tr is the matrix Trace operator. Now let

$$\hat{B} = \begin{bmatrix} 0 & \cdots & 0 \\ \phi_1(m_1) & \cdots & \phi_1(m_M) \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \phi_N(m_1) & \cdots & \phi_N(m_M) \end{bmatrix}_{(2N \times M)}$$

and the characteristic function χ_i, defined as

$$\chi_i \triangleq \chi(m_i) = \begin{cases} 0, & \text{if no actuator is placed at } m_i \\ 1, & \text{if an actuator is placed at } m_i \end{cases}$$

Following the development presented in (Ref. 4), we summarize the actuator location algorithm:

The characteristic functions χ_i^* of the optimal actuator locations satisfy the inequality

$$\sum_{i=1}^{M} \chi_i^* \, \Omega_{ii}^* \geq \sum_{i=1}^{M} \chi_i \, \Omega_{ii}^*, \qquad \text{for all admissible } \chi_i \tag{3.1}$$

where $L \triangleq \text{diag}\{\chi_i\}$, Ω_{ii}^* are the diagonal elements of the matrix $\Omega^* = \hat{B}^T P^* \Psi^{*T} P^* \hat{B}$ and the matrices P^*, Ψ^* are the solutions for P, Ψ of the canonical equations

$$R_1 - \rho_2 P\hat{B}\hat{B}^T P + A^T P + PA = 0 \tag{3.2}$$

$$V^T - \rho_2 \, \hat{B}L\hat{B}P^T\Psi + \rho_2 \, \Psi P^T \hat{B}L\hat{B}^T + A\Psi + \Psi A^T = 0 \tag{3.3}$$

The admissible χ_i's are those satisfying $\sum_{i=1}^{M} \chi_i = k$, where k is the total number of controllers used.

4 SIMULATION RESULTS

4.1 Simulation of a Continuous Beam with Random Base Excitation: The results presented in this section are from Chassiakos,(Ref. 4). The model development is given in Section 2.2. For this simulation full state information was assumed. The values of the system parameters are taken from Sakawa et al.,(Ref. 5), in which an experimental verification of these values was also performed: $EI = 2.04 \ Nm^2$, $\rho = 4.05 \times 10^{-1} \ kg/m^2$, $l = 1.05 \ m$. In addition a small internal damping $\delta_I = 5.87 \times 10^{-4}$ was assumed, so that the damping coefficient for each mode was calculated as $\zeta_i = \delta_I \omega_i$. The results presented here, cover four simulation runs. A four mode model (i.e., 8 states) was used. The random base excitation has duration 10 sec and is considered a zero-mean white noise process with standard deviation $\sigma = 2$ (Fig. 10).

The time responses of the beam's tip, with and without control are shown in Figs. 3–6.

Between runs, several parameters were varied, namely:

- The number of controllers
- The location of the controllers
- The control weighting coefficient, ρ_2.

Table 1 summarizes the parameters used for each run.

Run	*No of act.*	*Act. loc.*	ρ_2	*Value of criterion*
1a	1	0.2	1	7.6673
1b	1	1.0	1	0.6935
1c	1	1.0	20	0.2153
2a	2	0.9, 1.0	20	0.1840

Table 1: Continuous Beam

4.2 Discussion: In runs 1a, 1b, 1c we consider one controller at various locations, whereas run 2a deals with two controllers.

- Run 1a (Fig. 3) is performed with the controller arbitrarily placed at $\xi_1 = 0.2$.
- The location algorithm, yields $\xi_1 = 1.0$ as the optimal location (Fig. 4). Comparison of Figs. 3 and 4 shows clearly that the location obtained through the algorithm, gives much better responses.
- In run 1c (Fig. 5) the value of ρ_2 was changed from 1 to 20, and this reduces the response amplitudes even further, as expected from the stochastic regulator theory.
- In run 2a (Fig. 6) two controllers are used. The location algorithm yields the optimal values $(\xi_1,\ \xi_2) = (0.9,\ 1.0)$. The controllers now affect the second mode amplitudes as well, and we obtain a smaller value of the criterion and very small displacements.

4.3 Simulation of a 3 DOF System with Random Base Excitation: The results of this section are taken from Chassiakos, (Ref. 4). The model development is given in 2.2. For the simulation purposes the following numerical values were used:

$$m = \text{diag}\{2,2,2\} \quad \text{lbsec}^2/\text{ft}$$

$$K = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{pmatrix} \times 10^3 \ \text{lb/ft}$$

$$C = \alpha K, \quad \alpha = 0.0014 \quad \text{sec}$$

Run	*No of act.*	*Act. loc.*	ρ_2	*Value of criterion*
1a	1	1	20	16.5979
1c	1	3	20	8.7701
2c	2	(2,3)	20	7.0786

Table 2: A 3 DOF System

Table 2 summarizes the simulation parameters used for each run.

In run 1a (Fig. 7), a controller was arbitrarily placed at the 1st station. The algorithm correctly obtained the optimal location as being the 3rd station (run 1c), and the value of the criterion was reduced by 50% compared to run 1a.

Simulation run 2c, deals with two controllers. The placement algorithm when applied to an initially arbitrary location (e.g., location (1,2)), yielded the optimal placement (locations (2,3) in this case). The criterion is further reduced to $J = 7.0786$. However the reduction from $J = 8.7701$ in run (1c) to $J = 7.0786$ in run (2c), does not justify the use of one additional controller. Nevertheless the dramatic reduction of $J = 16.5979$ in run (1a) to $J = 8.7701$ in run (1c), shows the power of the optimal location algorithm.

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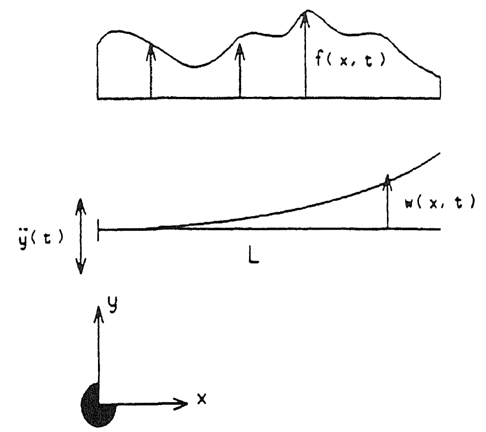

Fig. 1

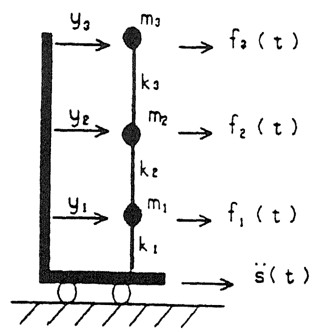

Fig. 2

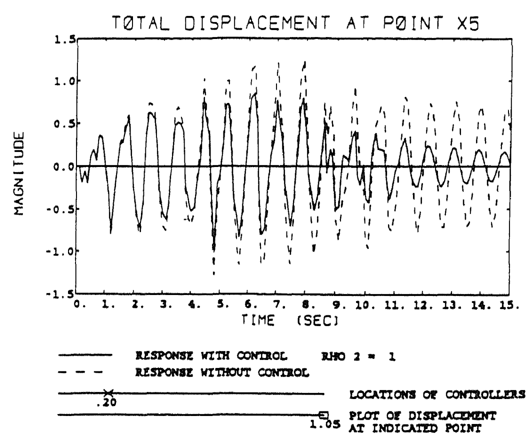

Fig. 3

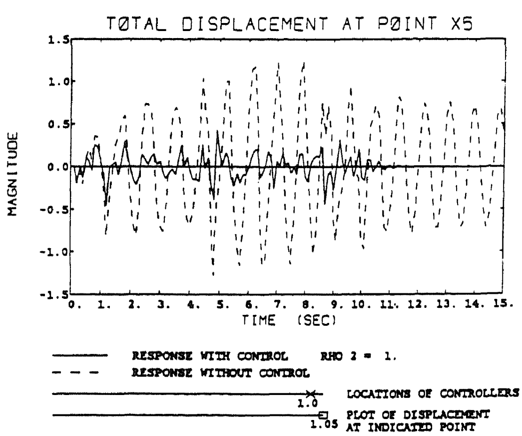

Fig. 4

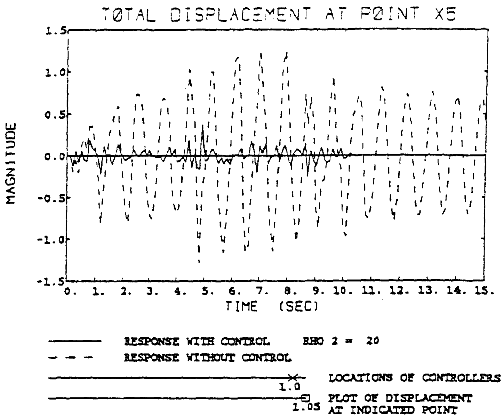

Fig. 5

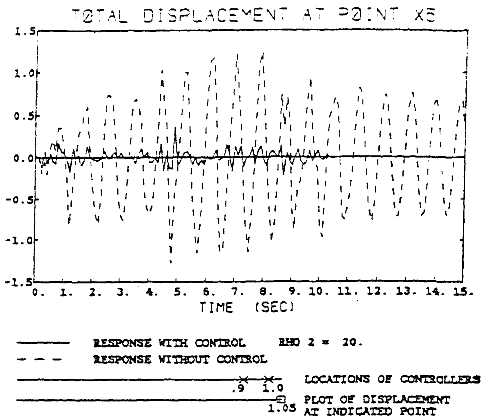

Fig. 6

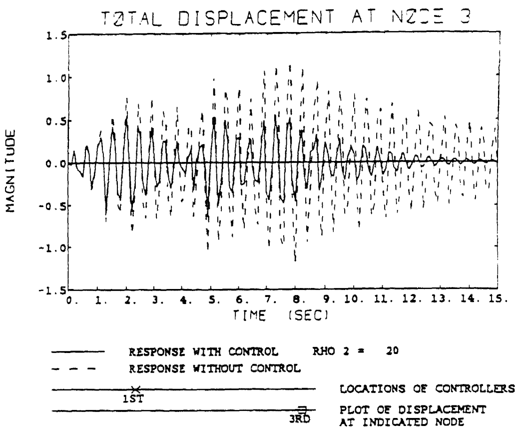

Fig. 7

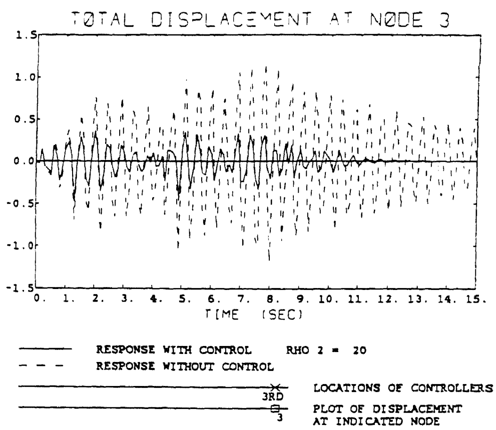

Fig. 8

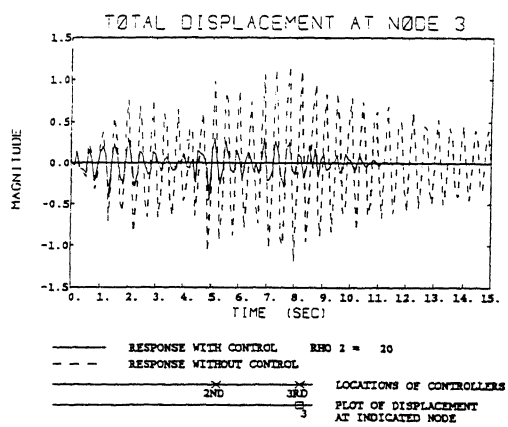

Fig. 9

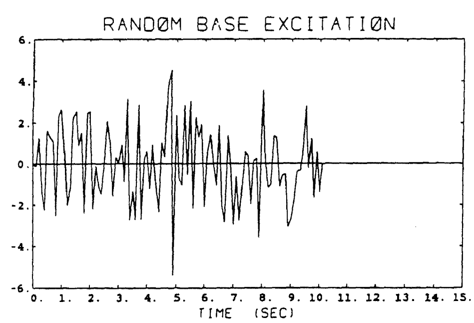

Fig. 10