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TRADEOFFS BETWEEN ACTIVE CONTROL AND IDENTIFICATION OF STRUCTURAL SYTEMS

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ABSTRACT

This paper explores the tradeoffs between structural identification and structural control in a quantitative manner. Through the intemediary concept of an optimal input, it is shown that significant tradeoffs between identification and control in structural systems may result. A numerical example is presented showing the methodology.

INTRODUCTION

A necessary prelude to the effective control of a structure, is a knowledge of its characteristics and properties. In other words, one needs to have information about the structural system so that adequate control algorithms can be devised. This has led to a considerable interest in the identification of structures subjected to dynamic loads For structural systems that are described by parametric models, this involves knowledge of the nature of the governing differential equations and knowledge of the values of the parameters that are involved, or at least knowledge of the bounds within which the parameters lie. Clearly, the better the system is identified (the smaller the bounding intervals within which the parameters are known to lie), the more finely tuned the controller can be made, so that for a given amount of available control energy, the control would be more efficient. The less knowledge we have about the structural system the more robust the controller needs to be, and, in general, the less efficient the control. Thus heuristically speaking there exists a duality between the concepts of identification and control, because 1) robust controllers may require reduced efforts at identification (for purposes of control), and, 2) increased efforts at identification may require less robust and more efficient controllers. However the tradeoffs between identification and control, from a practical standpoint, are still usually difficult to assess and little work has been reported to date in this area of cost-benefit analysis between these two dual concepts.

In this paper we formulate the trade-off problem between identification and control, and study in a quantitative manner their duality through the use of the intermediary concept of an optimal input. Thus the paper attempts to answer the following question: given that the optimal input time function is to have a certain prescribed energy, how does it change in character and in its effectiveness as one changes the objective criterion from one that emphasizes control to one that emphasizes identification? Some simple numerical examples are provided to indicate the quantitative nature of the results and provide a feel for them. The results in these examples show that significant trade-offs exist between identification and control and that for the same amount of energy in the input signal, the emphasis on control could lead to very high covariances of the parameter estimates. Similarly inputs that are optimal for identification could yield responses whose mean squared values may be several times those obtained for inputs that yield optimal control.

PROBLEM FORMULATION

Consider a dynamic system modelled by the first order set of differential equations

$$\dot{x}(t) = F_1\, x(t) + G\, f(t) \tag{1}$$

$$z(t) = H_1\, x(t) + v(t) \tag{2}$$

where x is an nX1 state vector, f is an mX1 control vector, z is an rX1 measurement vector and the nX1 initial condition vector, x_0, is given. We shall assume that the measurement noise is representable as a zero mean Gaussian White Noise process so that

$$E[\, v(t)\,] = 0, \text{ and,} \tag{3}$$

$$E[\, v(t)\, v(\tau)\,] = R_1\, \delta\,(\, t - \tau\,)\,. \tag{4}$$

Let the vector of unknown parameters in the system modelled by equations (1) and (2) be given by the pX1 vector θ. Let us assume that the identification is carried out with an efficient unbiased estimator so that the covariance of the estimate of θ namely, $\hat{\theta}$ is provided by the inverse of the Fisher Information Matrix. Hence,

$$\mathrm{Cov}[\,\hat{\theta}\,] = M^{-1}. \tag{5}$$

The matrices F_1 and G_1 are taken to be functions of, in general, the parameter vector θ. The optimal input for identification of the parameter vector θ is then sought such that a suitable norm related to the matrix M is maximized or minimized. In this paper, for expository purposes, we shall use the criterion for obtaining the optimal inputs for identification as the maximization of the Trace$\{W^{1/2}\, M\, W^{1/2}\}$ where W is a suitable positive definite weighting matrix. Thus the criterion for obtaining the optimal input for parameter identification is taken to be

$$J_I = \int_0^T \mathrm{Trace}\,\{\psi_p^T H_1^T R_1^{-1} H_1\, \psi_p\}\, dt \tag{6}$$

where

$$\psi_p = X_p\, W^{1/2}, \tag{7}$$

and the matrix X_p is given by,

$$[X_p]_{i,j} = \frac{\partial x_i}{\partial \theta_j} \tag{8}$$

In addition to the objective function generated by our need for identification, the objective function required to be maximized for control is,

$$J_C = -\int_0^T \{\, x^T Q_1\, x\,\}\, dt\,. \tag{9}$$

Here Q_1 is a symmetric positive definite, nXn weighting matrix. This then yields the composite objective function which is required to be maximized as

$$J = -(\alpha/2) \int_0^T \{x^T Q_1 x\}\, dt + (\beta/2) \int_0^T \mathrm{Trace}\,\{\, \psi_p^T\, H_1^T R_1^{-1} H_1\, \psi_p\,\}\,, \tag{10}$$

where α nd β are positive scalars. Clearly, when $\alpha >> \beta$, finding f(t) to maximize J is tantamount to finding the optimal control for the system (1)-(2), while when $\beta >> \alpha$, the f(t) that maximizes J is simply the optimal input for identification of the pX1 parameter vector θ. In particular, when $\alpha = 0$, and $\beta =1$, the optimal input for 'best' identification is obtained; when $\alpha = 1$, and $\beta = 0$, the optimal input for 'best' control is obtained. Denoting the nX1 vector

$$x_{\theta_i} = \frac{\partial x}{\partial \theta_i} \tag{11}$$

and assuming that the matrix W is diagonal, so that,

$$W = \mathrm{Diag}(w_1, w_2, \ldots, w_p) \tag{12}$$

we can generate an augmented n(p + 1) vector,

$$y(t) = [\ \{ x(t) \}^T, \{w_1^{1/2} \partial x(t)/\partial\theta_1 \}^T, \ldots, \{ w_p^{1/2} \partial x(t)/\partial\theta_p \}^T\ ] \tag{13a}$$

$$y(0) = [\ x_0^T, 0, 0, \ldots, 0\ ] \tag{13b}$$

which is then governed by the differential equation

$$\dot{y} = Fy + Gf, \quad y^T(0) = \{ x_0^T, 0 \} \tag{14}$$

where F is the suitably defined n(p + 1) X n(p + 1) matrix and G is an n(p+1) X m matrix.
The objective function (10) can now be rewritten, after some algebra, as

$$J = -(\alpha/2) \int_0^T y^T Q y^T \, dt + (\beta/2) \int_0^T y^T H^T R^{-1} H y \, dt \tag{15}$$

where, the matrices Q, H, and R^{-1} are the following block diagonal matrices : $Q = \mathrm{Diag}\{ Q_1, O, O, \ldots, O, O\}$, $H = \mathrm{Diag}\{O, H_1, H_1, H_1, \ldots, H_1\}$, and, $R^{-1} = \mathrm{Diag}\{ R_1^{-1}, R_1^{-1}, \ldots, R_1^{-1}\}$.
Thus the objective function needs to be maximized under the constraint equations (14) and the energy constraint

$$\int_0^T f^T f \, dt = E, \tag{16}$$

where the parameter E is given a priori..

DETERMINATION OF OPTIMAL INPUTS FOR SIMULTANEOUS CONTROL AND IDENTIFICATION

Using the Lagrange multipliers $\lambda(t)$ and $\mu(t)$ we therefore obtain the augmented objective function to be

$$\underline{J} = -\frac{\alpha}{2} \int_0^T \{y^T Q y\} \, dt + \frac{\beta}{2} \int_0^T \{y^T H^T R^{-1} H y\} \, dt + \int_0^T \lambda^T(t) \{ \dot{y} - Fy - Gf\} \, dt$$
$$- \int_0^T \frac{\eta(t)}{2} \{ f^T f - \frac{E}{T} \} \, dt + \frac{1}{2} \int_0^T \mu(t) . \dot{\eta}(t) \, dt \ . \tag{17}$$

Taking the first variation of equation (17) and setting it to zero for an extremal, we obtain for $\delta \underline{J} = 0$, the following set of equations for the variables y(t) and the multipliers $\lambda(t)$ and $\mu(t)$

$$\dot{y}(t) - Fy = \frac{1}{\eta} V\lambda(t),\ y(0)=y_0;\ \dot{\mu}(t) = -f^T f = -\frac{1}{\eta^2}\lambda^T(t)\,V\lambda(t),\ \mu(0)=0, \mu(T) = -E \quad (18a)$$

$$\dot{\lambda}(t) + F^T\lambda(t) = -\alpha Qy + \beta\{H^T RH^{-1}\}y,\ \lambda(T) = 0;\ \dot{\eta}(t) = 0 \quad (18b)$$

where $V = (GG^T)$. We note that the equation set (18) constitutes a nonlinear two point boundary value problem containing 2[np + n+ 1] first-order differential equations. The optimal input vector, f(t), is obtained through the solution of this two point boundary value problem using the relation:

$$f(t) = -\frac{1}{\eta} G^T \lambda(t) \quad (19)$$

This two-point boundary value problem can be numerically solved in various ways. In this paper, the equation set (18) is solved using the multiple shooting technique with Newton iteration.

ILLUSTRATIVE EXAMPLES

Consider the system modelled by a single degree-of-freedom oscillator described by the differential equations

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t);\quad x_1(0) = a_0,\ x_2(0) = b_0 \quad (20)$$

where f(t) is the optimal input to be applied. Denoting $x = [x_1\ \ x_2]^T$, $x_k = \partial x/\partial k$, and $x_d = \partial x/\partial c$, the objective function is taken to be

$$J(T) = -a\,J_c(T) + b\,J_k(T) + d\,J_d(T);\ a,b,d > 0 \quad (21)$$

where,

$$J_c = \int_0^T x_1^2\,dt\ ;\ J_k = \int_0^T x_k^2\,dt;\ \ J_d = \int_0^T x_d^2\,dt \quad (22)$$

Since the vector G = [0 1 0 0 0 0], the vector $\{V\lambda/\eta\}$ has only one non-zero component namely, $\lambda_2(t)/\eta$, where $\lambda_2(t)$ is the second element of the vector $\lambda(t)$. Figure 1 shows some of the numerical results for the following parameters values (which we shall assume are taken in consistent units):

$$b=0,\ T=5,\ E = 2.25;\ \ k=50,\ c=2;\ \text{and,}\ \ x_1(0) = 0,\ x_2(0) = 10. \quad (23)$$

The aim is to study the trade-off between 1) identifying the damping parameter, c, in the best possible way, and 2) controlling the system so that its mean square response over the time period T is a minimum, given that an input (forcing function) of 5 units duration with an energy of up to 2.25 units is to be used. The responses of the system together with the optimal inputs as obtained from equation (18) are shown for the two extreme cases: 1) a=0, d=1, corresponding to the optimal input required for identification of the damping parameter c, and, 2) a=1, d=0, corresponding to the optimal input required for minimizing the response. As seen from Figure 1(b), the optimal inputs required for 'best' identification and for 'best' control (the term 'best' is used in in terms of the cost function (21) utilized) are widely different form each other. In fact they are seen to be, for the entire duration over which they last, almost exactly out of phase. Differences in the response of the system to combined influence of the initial velocity and the forcing functions obtained for the two cases are shown in Figure 1(a). Figure 1(c) shows the Fisher Information Matrices for damping, which in this case are scalars, namely $J_d(t)$, for the abovementioned two extreme cases, as a function of time, t. The difference between these at T= 5 is about 55%. Alternatively put, the input forcing function, which controls the system response maximally, causes a response which is only about 55% as informative about the system parameter c as that caused by a forcing function that is designed to maximally provide information about the parameter, c. The manner in which the integral of the response quantity squared, $J_c(t)$, changes with time for the two cases mentioned above is shown in Figure 1(d). As seen, at T = 5, the optimal control input is about 35% more effective in reducing the mean square response than the input which optimally determines the parameter c.

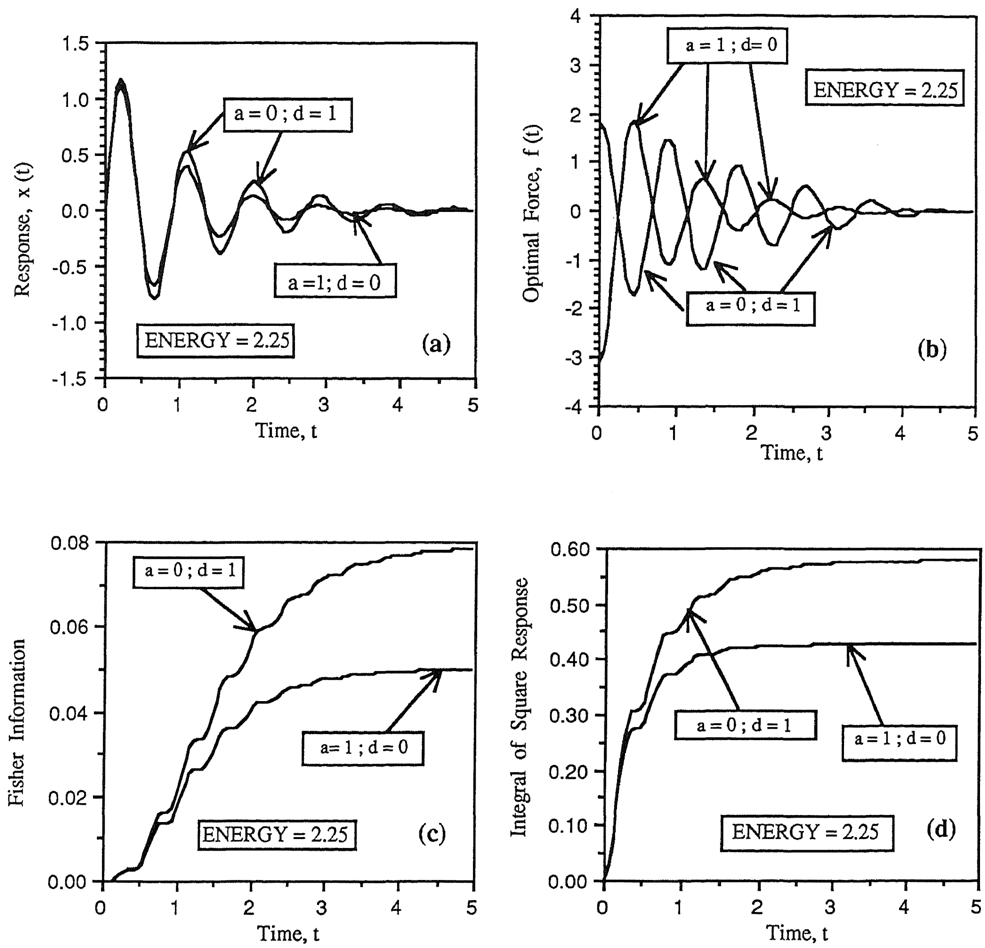

Figure 1

The optimal input when a=0 corresponds to the that required for 'best' estimation of the parameter c in equation (20). The quantity η is simultaneously solved for, in the set (18), thereby eliminating the need to find its value by trial and error. Had this not been done a very high computational expense would have been incurred to ensure that the energy constraint is satisfied. Noting that the inverse of $J_d(T)$, for an efficient unbiased estimator, is the covariance of the estimate of the parameter c, Figures 2(a) and 2(b) provide the trade off between control and identification. We use the parameter

$\gamma = [\{a\, J_c(T)\}/\{d\, J_d(T)\}]$ to study the effectiveness of control versus identification. As seen in Figure 2(a), in going from $\gamma = 0$ to $\gamma = 4$, $J_c(T)$, the mean square response, **falls off** by about 35%; similarly the covariance of the estimate of c **increases** by about 55% as γ varies over the same interval. For larger values of the input energy, Figure 2(b) shows that significant reductions in J_c and significant increases in the covariance of the parameter estimates can occur. Covariance estimates can deteriorate by 450% and control efficiencies by 300%.

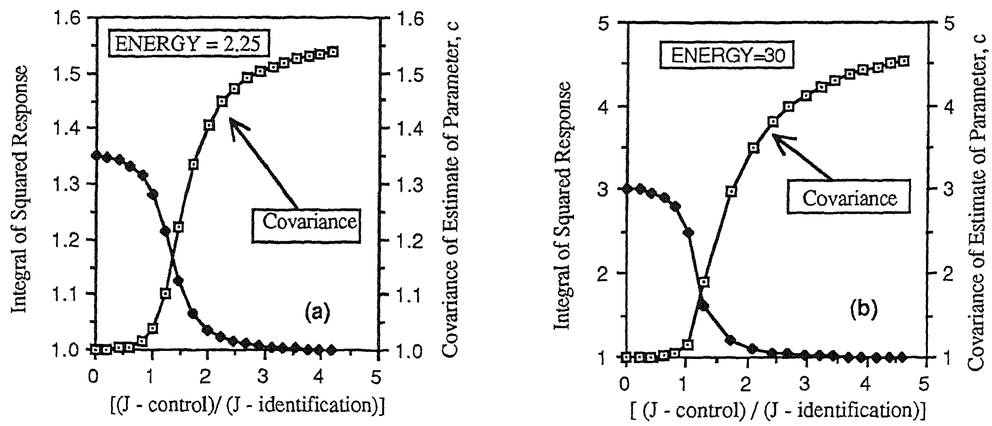

Figure 2

CONCLUSIONS AND DISCUSSION

In this paper we have formulated the problem of the tradeoff between optimal control and optimal identification through the intermediary concept of an optimal input. It is shown that the duality between identification and control can be quantified by looking at and determining optimal inputs, which have a certain amount of energy, and which maximize the objective function. A crucial element in the analysis is the determination of the optimal input which satisfies in an automatic way the nonlinear energy constraint. A numerical example, which deals with control and identification of the parameters of a single degree-of-freedom oscillator, is used to illustrate the concepts involved. It is shown that improved control leads to serious deterioration in the covariance of the parameter estimation and vice-versa. In general, as the energy of the input increases the tradeoffs between identification and control are shown to become more and more intense.

REFERENCES

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