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STOCHASTIC VIBRATION CONTROL OF TALL BUILDING

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SUMMARY

The state equations of tall buildings with TMD and Tendon control devices under the action of random excitations are derived in this paper. By using modal matrix, separation theorem and optimal control theory, the optimal close-loop critical modes control roles and the response covariances of controlled critical modes are obtained, and also the response covariances of residual modes are found, the critical-mode control is superior insofar as the amount of on-line computations is concerned.

INTRODUCTION

The responses of tall building structures under the action of environmental loads, such as winds, earthquakes, and waves, which are random in nature, and the applied control forces are random. Structural vibration control theory and its applications to building structures have been developed in recent years, also, various methods of designing control devices to alleviate tall building vibrations have been found by researchers.

If the objective function is quadratic, then the optimal close-loop control system requires the solution of a matrix Riccati equation. The solution of a matrix equation is almost prohibitive for a structure with many degrees of freedom, such as a tall building. Therefore, a good method must be found, in this paper, using modern control theory, the critical modes are controlled in a close-loop control form, the remaining uncontrolled modes may be excited by the control forces, and also by environmental loads. Optimal critical-mode control analysis is carried out to determine the stochastic response and control forces and the uncontrolled mode response, the remaining uncontrolled modes are disadvantageous to the stochastic response of tall building structures with a control system.

FORMULATION

Equations of Motion The structural model chosen for the present study is an n-story shown in Fig. 1. Consider an n-story building in which an active mass damper on the top (nth) floor and some tendon control devices in the building are installed. Let x_i be the displacement of the ith floor. Then, the equations of motion for floors are given by

$$m_i\ddot{x}_i + c_i(\dot{x}_i-\dot{x}_{i-1}) + k_i(x_i-x_{i-1}) - c_{i+1}(\dot{x}_{i+1}-\dot{x}_i) - k_{i+1}(x_{i+1}-x_i) = P_i \quad (1)$$

$i=1,2,\ldots,l,\ l+m+1,\ldots,n-1;\ m+1 \leqslant n$

and $m_i\ddot{X}_i + C_i(\dot{X}_i-\dot{X}_{i-1}) + K_i(X_i-X_{i-1}) - C_{i+1}(\dot{X}_{i+1}-\dot{X}_i)$

$$-K_{i+1}(X_{i+1}-X_i) = P_i-U_i \qquad i=l+1,\ldots,l+m \tag{2}$$

and $$M_n\ddot{X}_n + K_n(X_n-X_{n-1}) + C_n(\dot{X}_n-\dot{X}_{n-1}) + K_d(X_d-X_n) + C_d(\dot{X}_d-\dot{X}_n) = P_n-U_d \tag{3}$$

and $$M_d\ddot{X}_d + K_d(X_d-X_n) + C_d(\dot{X}_d-\dot{X}_n) = U_d \tag{4}$$

Let x_o = the earthquake displacement; y_j = the relative displacement of the jth floor with respect to the j-1th floor; $x_i = x_o+\sum_{j=1}^{i} y_i$. Then Eqs.(1-4) can be cast into a matrix equation as follows:

$$m\ddot{y} + C\dot{y} + Ky = P + Bu + F\ddot{x}_o \tag{5}$$

in which $y = (y_1,y_2,\ldots,y_{n+1})^T$; $F = -(m_1,m_2,\ldots,m_n,m_d)^T$;

$P = (P_1,P_2,\ldots,P_n,0)^T$; $u = (u_1,u_2,\ldots,u_m,u_d)^T$;

$$m = \begin{bmatrix} m_1 & 0 & \ldots & 0 & 0 \\ m_2 & m_1 & \ldots & 0 & 0 \\ \cdots\cdots \\ m_n & m_n & \ldots & m_n & 0 \\ m_d & m_d & \ldots & m_d & m_d \end{bmatrix} \qquad c = \begin{bmatrix} c_1-c_2 & 0 & \ldots & 0 & 0 \\ 0 & c_2-c_3 & \ldots & 0 & 0 \\ \cdots\cdots \\ 0 & 0 & & c_n & c_d \\ 0 & 0 & & 0 & c_d \end{bmatrix}$$

The matrix K in Eq.5 can be obtained from the matrix c by replacing c_i($i = 1,2,\ldots,n$) and c_d by K_i($i=1,2,\ldots,n$) and K_d, respectively.

$$B = \begin{bmatrix} [0]_{Lx(m+1)} \\ -1 & 0 & \ldots & 0 & 0 \\ 0 & -1 & & 0 & 0 \\ \cdots\cdots \\ 0 & 0 & \ldots & -1 & 0 \\ 0 & 0 & & 0 & 0 \\ \cdots\cdots \\ 0 & 0 & \ldots & -1 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ \cdots\cdots \\ 0 & 0 & \ldots & 0 & -1 \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ mx(m+1) \\ \\ \\ (n+1)(m+1) \end{matrix}$$

Eq.5 can be converted into a first-order matrix equation with a dimension of 2n+2.

$$\dot{Z} = AZ + EU + D \tag{6}$$

in which $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$; $E = \begin{bmatrix} 0_{(n+1)x(m+1)} & \vdots & M^{-1}B \end{bmatrix}$;

$$D = \begin{bmatrix} 0_{(n+1)x1} \\ M^{-1}P+M^{-1}F\ \ddot{X}_o \end{bmatrix} ; \qquad Z = \begin{Bmatrix} Y \\ \dot{Y} \end{Bmatrix}$$

Modal Decomposition— The eigenalues of A are n+1 pairs of distinctive complex conjugates. Let $x_j = \mu_j \pm i\Omega_j$ (j=1,2,...,n+1) be the jth pair of eigenvalues of A with the corresponding pair of eigenvectors $a_j \pm ib_j$, in which $i = \sqrt{-1}$, μ_j and Ω_j are real values; and a_j and b_j are real vectors, the real matrix T constructed from $T=[a_1, b_1, \ldots, a_j, b_j, \ldots, a_{n+1}, b_{n+1}]$ will transform matrix A into a canonical form $[\Lambda]$, i.e. $[\Lambda] = T^{-1}AT$, in which $[\Lambda_j] = \begin{bmatrix} u_j & \Omega_j \\ \Omega_j & u_j \end{bmatrix}$. With the aid of the transformation $Z=T\{V\}$, Eq.6 is uncoupled into n+1 pairs as follows:

$$\{\dot{V}\} = [\Lambda]\{V\} + EU + D_1 \tag{7}$$

in which $E_1=T^{-1}E$; $D_1=T^{-1}D$, let p be the number of lowest modes to be controlled. Then Eq.7 can be partitioned into critical modes and residud modes as follows:

$$\{\dot{V}_c\} = [\Lambda_c]\{V_c\} + E_{1c}U + D_{1c} \tag{8}$$

$$\{\dot{V}_r\} = [\Lambda_r]\{V_r\} + E_{1r}U + D_{1r} \tag{9}$$

in which $\{V\} = \begin{bmatrix} \{V_c\} \\ \{V_r\} \end{bmatrix}$; $[\Lambda] = \begin{bmatrix} [\Lambda_c] & [0] \\ [0] & [\Lambda_r] \end{bmatrix}$;

$D_1 = \begin{bmatrix} D_{1c} \\ D_{1r} \end{bmatrix}$; $E_1 = \begin{bmatrix} E_{1c} \\ E_{1r} \end{bmatrix}$; $\{V_c\}$, D_{1c} and E_{1c} are 2p vectors; $[\Lambda_c]$ is 2p by 2p matrix. A performance index, J, in the commonly used quadratic form is considered: $J=\frac{1}{2}E\left\{\int_{t_o}^{t_f}[Z^TQZ+U^TRU]\right\}dt$. For the optimal critical mode control considered herein. $J_p=\frac{1}{2}E\left\{\int_{t_o}^{t_f}[\{V_c\}^TQ_c\{V_c\}+U^TRU]dt\right\}$.

OPTIMAL CONTROL AND THE RESPONSE OF A CONTROLLED SYSTEM

The following assumptions are mode to simplify the analysis:

$$\begin{aligned} &E[D(t)] = E[G(t)] = E[Z(t_o)] = 0 \\ &E[Z(t_o)Z^T(t_o)] = P_o \ ; \quad E[D(t)D^T(\tau)] = N(t)\delta(\tau-t); \\ &E[G(t)G^T(\tau)] = M(t)\delta(\tau-t); \\ &E[Z(t_o)G^T(t)] = E[Z(t_o)D^T(t)] = E[D(t)G^T(t)] = 0 \end{aligned} \tag{10}$$

with the aid of the linear transformation T Eq.10 can be converted into a newform as follows:

$$\begin{aligned} &E[G(t)] = 0;\ E[V_c(t)] = E[D_{1c}(t)] = 0;\ E[V_c(t_o)V_c^T(t_o)] = P_{co} \ ; \\ &E[D_{1c}(t)D_{1c}^T(\tau)] = N_c(t)\delta(\tau-t);\ E[G(t)G^T(\tau)] = M(t)\delta(\tau-t); \\ &E[V_c(t_o)G^T(t)] = 0;\ E[V_c(t_o)D_{1c}^T(t)] = 0;\ E[D_{1c}(t)G^T(t)] = 0 \end{aligned} \tag{11}$$

in which G(t) is a measure white noise vector in the measure equation $II=HZ+G$ or $II=H_c[V_c]+G$.

Optimal Control Optimal control can be obtained by separation theorem and optimal control theory

$$V(t) = -C(t)[\hat{V}_c] \tag{12}$$

in which $c(t)=R^{-1}E_{1c}^{T}S(t)$, in which

$$\dot{S}(t) = -S[\Lambda_c] - [\Lambda_c]S + SE_{1c}R^{-1}E_{1c}^{T}S - Q_c$$
$$S(t_f) = 0 \tag{13}$$

$$[\dot{\hat{V}}_c] = [\Lambda_c][\hat{V}_c] + E_{1c}U^*(t) + K(t)[I-H_c[\hat{V}_c]]$$
$$[\hat{V}_c(t_o)] = 0 \tag{14}$$

in which $K(t)=PH_c^TM^{-1}(t)$

$$\dot{P} = [\Lambda_c]P + P[\Lambda_c] + N_c(t) - PH_cM^{-1}(t)H_cP^T$$
$$p(t_o) = P_{co} \tag{15}$$

The Response of a Controlled System Let ε be $[\hat{V}_c]-[V_c]$, then covariance matrix of ε and $[\hat{V}_c]$ can be obtained by Eq.15 and the following equations, respectively.

$$\dot{\Gamma}_{[\hat{V}_c]} = [[\Lambda_c]-E_{1c}R^{-1}E_{1c}^{T}S]\Gamma_{[\hat{V}_c]} + \Gamma_{[\hat{V}_c]}[[\Lambda_c]-E_{1c}R^{-1}E_{1c}S] + PH_c^TM^{-1}(t)H_cP^T \tag{16}$$

in which $P(t)=E[\varepsilon\varepsilon^T]$; $\Gamma[\hat{V}_c]=E[[\hat{V}_c][\hat{V}_c]^T]$

The response covariance of controlled modes can be obtained by the following equation

$$\Gamma[\hat{V}_c] = E[[V_c][V_c]^T] = \Gamma[\hat{V}_c](t) + P(t) \tag{17}$$

The covariance of control forces is

$$\Gamma_{u^*(t)} = E[U^*U^{*T}] = R^{-1}E_{1c}^{T}S\Gamma[\hat{V}_c]S^TE_{1c}R^{-1} \tag{18}$$

The optimal objective function is

$$J = \frac{1}{2}\int_{t_o}^{t_f} T_r[SN_c + S^TE_{1c}R^{-1}E_{1c}^{T}SP]dt \tag{19}$$

the response covariance of residual modes is

$$\Gamma[V_r] = E[[V_r][V_r]^T] = \Gamma[V_{r1}] + \Gamma[V_{r2}] \tag{20}$$

in which
$$\dot{\Gamma}[V_{r1}] = [\Lambda_r]\Gamma[V_{r1}] + \Gamma[V_{r1}][\Lambda_r]^T + N_r$$
$$\dot{\Gamma}[V_{r2}] = [\Lambda_r]\Gamma[V_{r2}][\Lambda_r]^T + E_{1r}^TR^{-1}E_{1c}^TS\Gamma[\hat{V}_c]S^TE_{1c}R^{-1}E_{1r}^T \tag{21}$$

the total response covariance of a controlled structure can be obtained by

$$\Gamma_z(t) = E[ZZ^T] = T\begin{bmatrix}\Gamma[\hat{V}_c] + P(t) & 0 \\ 0 & \Gamma[V_{r1}] + \Gamma[V_{r2}]\end{bmatrix}T^T \tag{22}$$

in which we have assumed that

$$E[[V_c][V_r]^T] = 0, \quad E[[V_r][V_c]^T = 0$$

CONCLUSION

The optimal close-loop critical modes controls have been obtained, also, the response covariances of residual modes have been found, the method present herein is simple, since the number of critical modes p is usually small, the on-line computation involving the solution of 4p first-order differential equations for the determination of control forces is not excessive, the critical-mode control is superior insofar as the amount of on-line computations is concerned.

REFERENCES

1. Leipholz, H.H., edited, Structural control, North-Holland, Publishing Company, (1980).
2. Li Quiqing and Zou Zujun, Review on the Structural Vibration Control. Earthquake Engineering and Engineering Vibration, No. 1, (1987).
3. Xie Xukai, Modern Control Theory Liaoling Renmin Publishing Company, (1984).

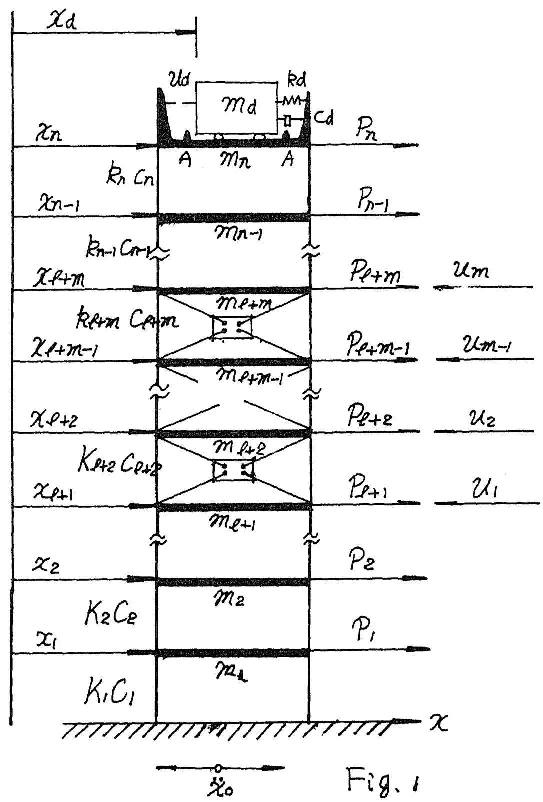

Fig. 1