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ACCIDENTAL TORSION IN YIELDING SYMMETRIC STRUCTURES

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SUMMARY

Building codes recognize a variety of sources of accidental eccentricity in buildings subjected to seismic excitation. This study examines the adequacy of the 1980 and 1985 provisions for accidental eccentricity of the National Building Code of Canada to account for torsion induced in nominally symmetric structures due to the unintended variation in the strength of elasto-plastic lateral load resisting elements. Examined is an idealized single-story structure for a range of variation in individual element strength, as well as the influence of torsional to translational frequency ratio Ω_o and lateral period of vibration T_o.

INTRODUCTION

Building codes for earthquake resistant design generally specify the design eccentricity as $e_d = e_{d1} \pm e_{d2}$ where e_{d1} is the dynamic eccentricity and e_{d2} is the additional or accidental eccentricity. Whereas e_{d1} is defined by the known distribution of mass and the structural layout of the building, the accidental term e_{d2} is intended to account for such factors as unforeseen variation in relative stiffnesses, uncertainty in the distribution of mass, possible torsional ground motion and the effects of inelastic or plastic action. In the National Building Code of Canada (NBCC, Ref. 1) the accidental eccentricity e_{d2} was increased in 1985 from 0.05 D_n to 0.10 D_n, where D_n is the dimension of the building perpendicular to the direction of excitation.

This paper examines the adequacy of the above code accidental eccentricities to account for torsion introduced by unintended variation of strengths of otherwise identical structural elements. Thus, the accidental torsion examined is that due to inelastic or plastic action.

ACCIDENTAL STRENGTH VARIATION

Individual members The strength level of individual load resisting elements varies due to the inherently random nature of the material itself as well as tolerances and geometric errors during the construction process. For normal loadings the Limit States Design procedure of the Canadian Standards Association (Ref. 2) is based on the design point of resistance R' which is related to mean strength $\bar{R}$ by

$$R' = (1 - \lambda)\bar{R} \tag{1}$$

where λ is the accidental strength variation parameter (Ref. 3) expressed as

$$\lambda = \frac{\beta\ V_R}{\{1 + [\frac{V_S(1 - 0.75\ \beta\ V_R)}{V_R(1 + 0.75\ \beta\ V_S)}]^2\}^{1/2}} \tag{2}$$

in which β = safety index; V_R = coefficient of variation of resistance R; and V_S = coefficient of variation of load S.

For steel beams reported magnitudes (Ref. 4) are $\beta = 3.0$ and $V_R = 0.13$. With $V_S = 0.2$ (Ref. 2) the accidental strength parameter becomes $\lambda = 0.3$. For reinforced concrete beams on the other hand, the typical values $\beta = 4.2$ (Ref. 5) and $V_R = 0.14$ (Ref. 6) predict $\lambda = 0.5$ for the same magnitude of V_S. Thus, for normal loading design, accidental strength parameter λ has large magnitudes, although it should be recognized that the probabilities of occurrence are small.

Complete structures In buildings as a whole, a variety of factors may introduce accidental variation in the strength of lateral load resisting elements. One such source arises if the structure comprises a mixed or hybrid system of elements. For example, with a shear core on one side of the building and a different assemblage acting on the opposite side, the design criterion may consist of balancing the lateral stiffnesses in order to avoid an elastically eccentric structure. In such cases, the stiffnesses may be made more or less identical but, unless additional special care is also taken, the lateral strength levels could be left substantially different, thus implying a potentially sizeable magnitude for the equivalent accidental strength parameter λ.

STRUCTURAL MODEL WITH ACCIDENTAL ECCENTRICITY

The idealized structural model adopted for investigation consists of a rigid rectangular deck of total mass m, supported by two elasto-plastic lateral load resisting elements as shown in Fig. 1, thus representing a one-story nominally symmetric structure. The deck has dimensions $D_n = 3\rho$ and $D = \sqrt{3}\ \rho$, where D = dimension parallel to the direction of earthquake excitation and ρ = mass radius of gyration about the mass centre CM. With ground excitation confined to the y-direction, accidental eccentricity gives rise to response exhibiting two degrees-of-freedom, namely translation in the y-direction and rotation θ.

Within the elastic range the structure is assumed to be symmetric, with the two lateral resisting elements possessing equal stiffness k. Thus, centre of mass CM and centre of resistance CR are coincident for elastic behaviour. However, element 1 is assumed to possess yield strength $R_{y,1} = R$, while element 2 has strength $R_{y,2} = (1 - \lambda)\ R$ (see Fig. 1). Based on the previously discussed variation of strength for individual steel and reinforced concrete members, it is assumed that accidental strength parameter λ varies over the range given by $0 \leqslant \lambda \leqslant 0.4$. In terms of probability of occurrence, a strength difference between two individual structural members of $\lambda = 0.2$ has a probability of 6 per cent for $V_R = 0.14$. The larger values of λ are envisioned to apply in situations where buildings possess lateral load resisting assemblages of differing strength, for which a probability of occurrence is not available.

Although behaviour is examined for structures which are symmetric within the elastic range, for evaluation of the adequacy of code accidental torsion provisions the results are normalized with respect to coupled response for eccentricity $e = e_{d2}$. Thus, if actual (although nominal) eccentricity exists, the elastic lateral-torsional coupling depends on the eccentricity e between the centre of mass CM and the centre of resistance CR, as well as on the uncoupled torsional to lateral frequency ratio Ω_o. The latter is defined by

$$\Omega_o^2 = \omega_{\theta 0}^2/\omega_y^2 = (K_{\theta 0}/m\rho^2)/(K_y/m) \qquad (3)$$

where $K_{\theta 0}$ = torsional rigidity about CR and K_y = total y-direction lateral stiffness. The above definition of frequency ratio Ω_o is adopted rather than one based on K_θ related to CM because it is independent of the eccentricity e. Eccentricity e itself is normalized with respect to ρ about CM; namely, normalized eccentricity is given by $e^* = e/\rho$.

To evaluate the anticipated lateral-torsional dynamic response of the model, the concept of static plastic eccentricity e_p is introduced. The latter represents the eccentricity of the plastic centroid PC from CR, i.e. the location at which load is applied statically to cause only translational response. Equilibrium with both elements 1 and 2 yielding statically gives

$$e^*_p = \lambda\, \Omega_o/(2 - \lambda) \qquad (4)$$

Thus e^*_p represents the normalized static accidental eccentricity resulting from unequal yield strengths of elements 1 and 2. As Eq. (4) indicates, this parameter is a function of strength variation parameter λ and frequency ratio Ω_o. Fig. 2 shows the influence of parameters Ω_o and λ on the expected magnitude of e^*_p. For the assumed peak value of $\lambda = 0.4$, maximum expected $e^*_p = 0.3$.

OUTLINE OF PARAMETRIC STUDY

The model structure was subjected to a parametric study consisting of the time-history analyses for earthquake records with time step $\Delta t = 0.01$ sec employing the computer program DRAIN-2D. Five per cent viscous damping was assumed for the two response modes. The input parameters consisted of the following: (1) three earthquake records - El Centro 1940 NS, Olympia 1949 N80E and Taft 1952 N69W; (2) six translational periods - T_o = 0.25, 0.50, 0.75, 1.0, 1.5 and 2.0 sec; and (3) three torsional to translational frequency ratios - Ω_o^2 = 0.5, 1.0 and 1.5. Response of these 54 cases was examined in the form of average (AVG), average + 1.0 σ (AVG + 1.0 σ) and extreme values, where σ denotes standard deviation. In addition, the results were normalized with respect to the response associated with $\lambda = 0$ and specified eccentricities $e^* = e/\rho$ = 0, 0.15 and 0.30. Eccentricity $e^* = 0.15$ represents the accidental eccentricity of NBCC 1980 for the present model ($e_{d2} = 0.05\ D_n$), whereas $e^* = 0.30$ denotes the increased accidental eccentricity provision of NBCC 1985 ($e_{d2} = 0.10\ D_n$). The level of earthquake intensity was selected to correspond to a structural load reduction factor Q = 4. Thus, the strength levels of elements 1 and 2 were established from maximum element force for elastic symmetric response R_{e1} according to

$$R_{y,1} = R_{e1}/Q \quad ; \quad R_{y,2} = (1 - \lambda)\ R_{e1}/Q$$

RESULTS AND DISCUSSION

Adequacy of code provisions Fig. 3 shows the effect of strength variation parameter λ on the inelastic maximum edge displacement $y_{i,max}$ normalized with respect to $y^*_{i,max}$, where $y^*_{i,max}$ represents maximum edge displacement for the corresponding structure with $\lambda = 0$ and specified accidental eccentricities e^* = 0, 0.15 or 0.30. This normalization was adopted since it allows evaluation of the adequacy of the code minimum torsional requirements.

With the AVG + 1.0 σ response treated as the measure for design level response, Fig. 3(a) shows that maximum λ induces approximately 85 per cent increase in response over that for symmetric behaviour. Whereas extreme values of response may fall well above response associated with the code eccentricities (especially for large λ), the proposed AVG + 1.0 σ level of response correlates reasonably well with the magnitude of response associated with the code eccentricities (see Figs. 3(b) and 3(c)).

Fig. 4 summarizes the evaluation of the adequacy of the code provisions. Fig. 4(a) shows that $e^* = 0.15$ of NBCC 1980 adequately represents the torsional effect of strength variation for λ up to approximately 0.13, but underestimates the response by 35 per cent for $\lambda = 0.4$. On the other hand, $e^* = 0.30$ predicts response ratio $y_{i,max}/y^*_{i,max} \leqslant 1.06$ for the entire range of λ. Thus, the NBCC 1985 accidental eccentricity provision $e_{d2} = 0.10\ D_n$ appears to represent well the effective eccentricity required to account for a wide range of unintended variation of lateral element strength.

A similar observation is indicated by Fig. 4(b), presented this time in terms of accidental plastic eccentricity e^*_p. This form of denoting strength variation within a structural system is more general in that this parameter can be evaluated for a multi-element system, whereas λ is limited to the two-element system of Fig. 1. Whereas $e_{d2} = 0.05\ D_n$ is seen to be adequate for $e^*_p \leqslant 0.07$, $e_{d2} = 0.10\ D_n$ adequately accounts for the effect of accidental plastic eccentricity over the range $0 \leqslant e^*_p \leqslant 0.20$.

Influence of Ω_o This parameter represents the relative torsional stiffness of the structure and has been shown in previous studies (Ref. 7) to affect the coupled lateral-torsional response for elastic behaviour if eccentricity is small. For the inelastic response of this study, Fig. 5(a) shows that Ω_o exerts relatively weak influence on amplification of eccentric response over that for a symmetric structure. However, when examining the adequacy of the minimum code eccentricity provisions in Figs. 5(b) and 5(c), one notes that Ω_o also remains unimportant, except for torsionally flexible structures. The data for $\Omega_o^2 = 0.5$ of Fig. 5(c) indicate that $e_{d2} = 0.10\ D_n$ of NBCC 1985 becomes inadequate for $e^*_p > 0.15$. Thus, only torsionally flexible structures are prone to experience significant deviation above the present code provision if the static plastic eccentricity is relatively large.

Influence of period T_o Examination of the statistical response curves as a function of individual lateral periods of vibration T_o showed no consistent trend in the $y_{i,max}/y^*_{i,max}$ ratio. However, by grouping the six periods into the following three categories the influence of T_o was more readily identified: (1) long period (T_o = 1.5, 2.0 sec); (2) intermediate period (T_o = 0.75, 1.0 sec); and (3) short period (T_o = 0.25, 0.50 sec). The AVG + 1.0 σ response curves of Fig. 6 show the expected effect of the three categories of periods T_o. Fig. 6(a) shows that long period structures experience greater amplification over symmetric response due to accidental eccentricity. However, Figs. 6(b) and 6(c) indicate that, compared to response associated with minimum code accidental eccentricity, lateral period of vibration T_o is not a critical parameter.

CONCLUSIONS

Based on the results presented in this paper, the variation in the strength of lateral load resisting elements may result in considerable accidental torsion. Compared to symmetric response, this may result in sizeable amplification factors for maximum edge displacement. For a structure relying on two lateral load resisting elements the AVG + 1.0 σ amplification is a factor of 1.8 for magnitude of strength variation given by $\lambda = 0.40$. In terms of the adequacy of code accidental eccentricity provisions for multi-element systems, minimum eccentricity of $0.05\ D_n$ becomes inadequate when the static plastic eccentricity $e_p \geqslant 0.07\ \rho$, whereas $0.10\ D_n$ is able to account for $e_p \leqslant 0.20\ \rho$ (ρ = mass radius of gyration).

In addition to variation in strength, elements exhibiting different types of hysteretic behaviour on opposite sides of a building also introduce accidental torsion. The extent of this phenomenon is currently being investigated.

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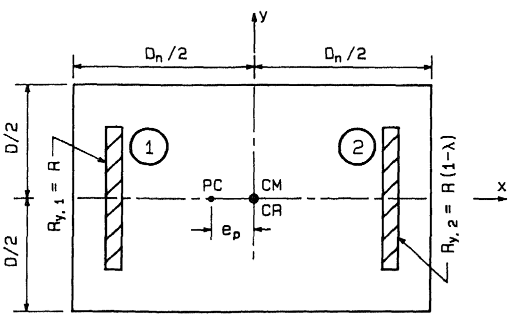

Fig. 1 Structural model with accidental eccentricity due to strength variation λ

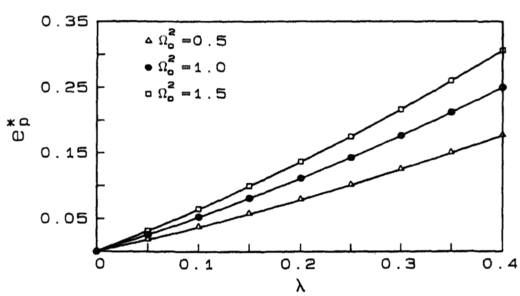

Fig. 2 Plastic eccentricity e_p^* as function of accidental strength variation parameter λ

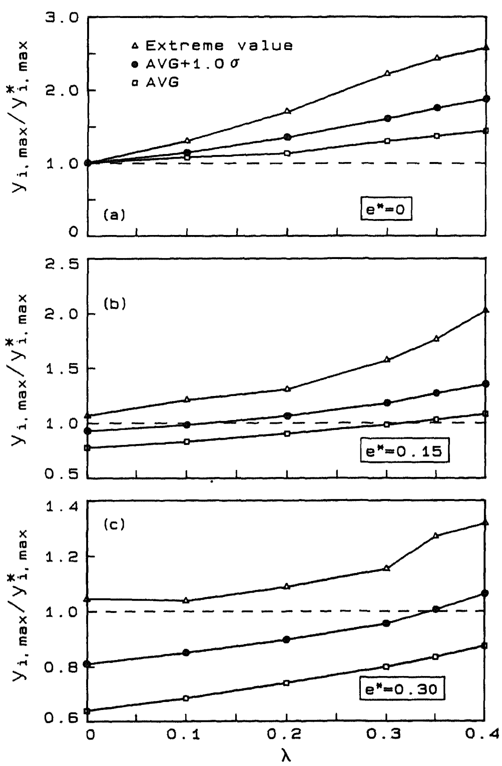

Fig. 3 Effect of strength variation λ-comparison with response for: (a) symmetric; (b) NBCC 1980 eccentricity; (c) NBCC 1985 eccentricity

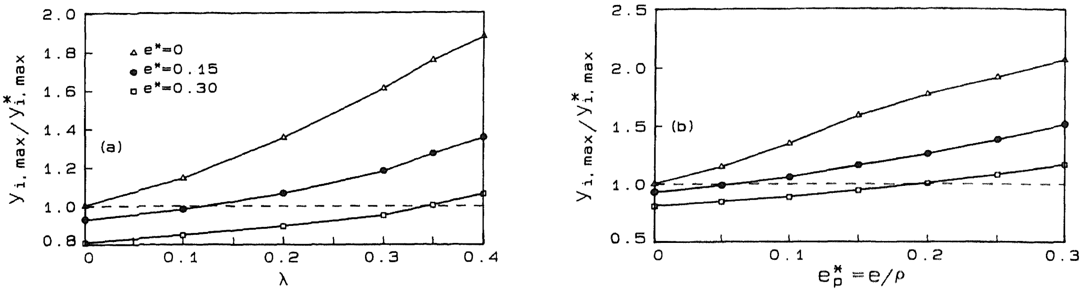

Fig. 4 Adequacy of code provisions (AVG+1.0σ response) for: (a) strength variation λ; (b) plastic eccentricity e_p^*

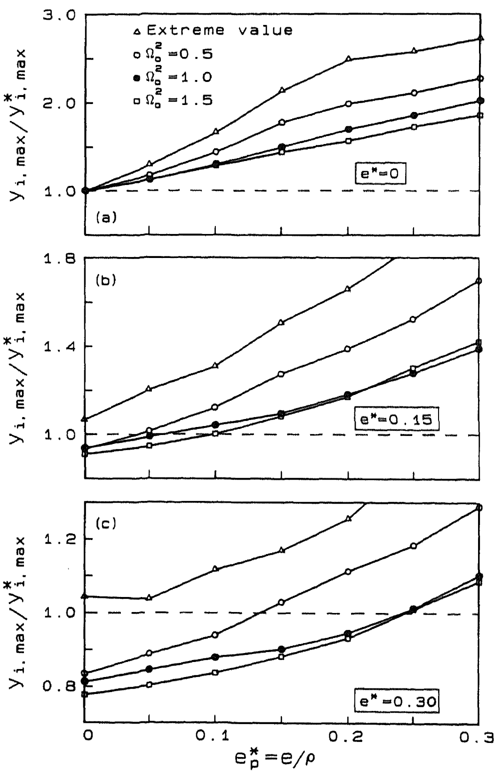

Fig. 5 Effect of frequency ratio Ω_o -comparison with response (AVG+1.0σ) for: (a) symmetric; (b) NBCC 1980 eccentricity; (c) NBCC 1985 eccentricity

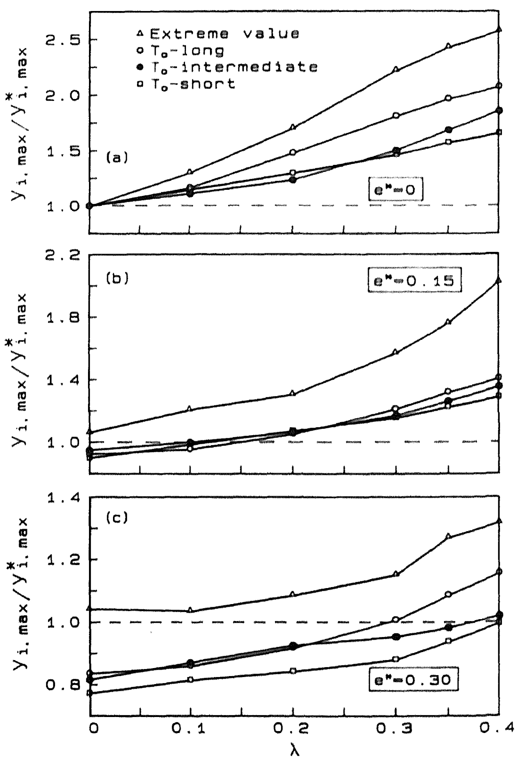

Fig. 6 Effect of period T_o-comparison with response (AVG+1.0σ) for: (a) symmetric; (b) NBCC 1980 eccentricity; (c) NBCC 1985 eccentricity