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PREDICTION OF THE INELASTIC RESPONSE OF TORSIONALLY COUPLED SYSTEMS SUBJECTED TO EARTHQUAKE EXCITATION

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SUMMARY

A methodology is presented to allow designers to predict the ductility demand of elements in simple torsionally coupled systems by using elastic dynamic analyses and other readily available design tools. The method is validated by numerous non-linear inelastic analyses, and is not affected by variations in the parameters traditionally thought to influence the response of torsionally coupled systems.

INTRODUCTION

Although torsional movements are thought to be responsible for the failure of many structures during major earthquakes, a simple method to estimate the inelastic response of torsionally coupled systems does not yet exist. There is still no consensus on how inelastic response of initially eccentric systems is affected by various parameters. One of the major problems in the study of torsionally coupled systems seems to be the difficulty in finding a reliable comparative torsion-free "benchmark" system whose response would not be sensitive to any of the parameters thought to influence the inelastic torsional response, as well as on the difficulty in setting an unbiased liaison between the true system and its "benchmark". This knowledge is most needed as torsional coupling is practically unavoidable in both new and retrofitted structures: eccentricities can be either initially present, or will develop following non-simultaneous yielding of elements in initially symmetric structures. Although both cases have been studied by the authors, only the results for initially eccentric structures will be presented herein.

EQUATION OF MOTION AND ELEMENT MODEL

For this study, a monosymmetric single-story system with rigid floor-diaphragm is considered (Fig. 1). The equation of motions for this two-degrees-of-freedom system were derived around the center of mass as follow:

$$\begin{pmatrix} m & 0 \\ 0 & mr^2 \end{pmatrix}\begin{pmatrix} \ddot{v}_X \\ \ddot{v}_\theta \end{pmatrix} + \begin{pmatrix} K_X & -K_X e \\ -K_X e & K_\theta \end{pmatrix}\begin{pmatrix} v_X \\ v_\theta \end{pmatrix} = -\begin{pmatrix} m\ddot{v}_{gX} \\ 0 \end{pmatrix} \qquad \text{with} \quad \Omega = \omega_\theta/\omega_X = T_X/T_\theta$$

$$\begin{pmatrix} \ddot{v}_X \\ r\ddot{v}_\theta \end{pmatrix} + \omega_X^2\begin{pmatrix} 1 & -e/r \\ -e/r & \Omega^2 \end{pmatrix}\begin{pmatrix} v_X \\ rv_\theta \end{pmatrix} = \begin{pmatrix} -\ddot{v}_g \\ 0 \end{pmatrix} \qquad \omega_X^2 = K_X/m, \quad \omega_\theta^2 = K_\theta/mr^2$$

where K_X and K_θ are the system's stiffnesses for the two degrees-of-freedom of interest (translational along X and torsional around θ), ω_X and ω_θ are the translational and torsional uncoupled frequencies, and Ω is the ratio of those uncoupled frequencies (which varies depending on the reference point around which the equations of motions are derived). The reader not familiar with those equations should refer to Ref. 1 for detailed explanations.

For this study, a bi-linear inelastic element model with strain-hardening was chosen, but the methodology presented hereafter has been found to work equally well with other types of non-linear element models. Strain-hardening was set to 0.5% (E_{SH} = 0.005 E), making the element model practically elasto-perfectly plastic. Elements of the torsionally coupled system were modeled to share the same yield displacements (Fig. 1). The damping was chosen to be of the Rayleigh type, arbitrarily set at 2% of the critical damping for each of the true frequencies of any given system analyzed.

NON-LINEAR ANALYSES OF TORSIONALLY COUPLED SYSTEMS

Parametric Study The intent of this parametric study is to establish the relationship between equivalent single-degree-of-freedom (SDOF) systems' ductility and torsionally coupled systems' element ductilities, more specifically to investigate the effect of various parameters on torsionally coupled elements response when equivalent SDOF systems are calibrated to target ductilities. The study was performed for ten values of uncoupled period T_X, six values of the ratio of uncoupled frequencies Ω, two target ductility levels μ, and two normalized eccentricities (e/r). The particular parametric values selected are shown on Fig. 2 and 3. The liaison between the true systems and the benchmark systems was accomplished as described below:

1) Equivalent SDOF systems were defined to have a period equal to the first period of their corresponding torsionally coupled system when $\Omega \geq 1.0$, and equal to the second period when $\Omega < 1.0$. This decision was dictated by observations on the nature and variations of the true periods, components of the corresponding mode shapes and edge displacement modal participation factors, as a function of Ω, as well as by other considerations. Furthermore, these SDOF systems were designed such that they shared the same inelastic element model and same yield displacement δ_y as the elements of the torsionally coupled systems.

2) Using the program NONSPEC (Ref. 2), the proper strength factors were calculated for each SDOF system in order to attain target ductilities of 4 and 8. For simplicity, the earthquake levels were scaled to produce the necessary strength factors. For this study, ductility demand is defined as the maximum displacement, in absolute value, divided by the yield displacement. These steps were to insure that the SDOF systems were insensitive to variations in ground motion intensity.

3) The program for non-linear structural analysis ANSR-1 (Ref. 3) was used to verify (and improve if needed) the accuracy of the target ductility demands predicted by NONSPEC. All final ductilities for the SDOF systems analyzed were within 10% or less of their targeted ductilities.

4) The same equivalent SDOF systems were then re-analyzed elastically, using the respective earthquake excitation levels that produced the desired target ductilities in item 2 above.

5) The torsionally coupled systems were first analyzed elastically for an arbitrary level of excitation. Then, for each individual parametric case, a new earthquake scaling to be applied to the torsionally coupled systems was calculated such that the torsionally coupled system's weak (more flexible) element maximum elastic response would equal the one of the equivalent SDOF system.

6) Using the new earthquake scalings found in the previous step, the inelastic response of the torsionally coupled systems were calculated, and the ductility demands were calculated for each element.

7) The ductility factors calculated for each individual torsionally coupled case analyzed above were then divided by the ductility factors obtained from their respective equivalent SDOF system, to obtain a ratio of the ductilities (indicated "Ductility Ratios" on Fig. 2 and 3) that is independent of the selected target ductilities.

8) To provide results mostly independent of the particular characteristics of single earthquakes, the above 7 procedures were repeated for 5 different earthquake records (El-Centro 1940, Olympia 1949, Parkfield 1966, Paicoma Dam 1971, and Taft 1952), and the mean (and mean-plus-one-standard-deviation, although not presented here) ductility ratios were calculated.

Observations of the Results By observation of the ductility ratios results (Fig. 2), one may notice that for the Ω=1.0 case, all weak element ductility ratios are equal to 1.0 (i.e. weak element ductility demands are equal to the equivalent SDOF system ductility demands). The analytical demonstration that equal displacements must be observed for the case Ω=1.0, if the above procedure is followed, is presented in Reference 1.

It is apparent from Fig. 2 that the element ductility ratios obtained by the method outlined above are independent of the uncoupled periods (T_X), normalized eccentricities (e/r), target ductility μ, and ratios of uncoupled frequencies (Ω). This means that the method is mainly stable in providing a reliable estimate of the torsionally coupled system's element ductilities based on the concept of an equivalent SDOF system.

The ductility ratios, as measured by the mean of response for five earthquakes, remain mainly close to unity, only exceeding a value of 2.0 for the weak element when T_X = 0.4, Ω=0.8 and (e/r)=0.3. This is a direct consequence of the unique extreme ductility ratio that occurred for the Pacoima Dam earthquake for this particular combination of parameters. Should the Pacoima Dam contribution at this particular point be removed, the mean for the weak element response would drop to 1.39 for target ductility of 4, and 1.05 for target ductility of 8. Other than this particular point of unusually high sensitivity, the mean weak element response exceeds 1.5 only in five occasions (the maximum value being 1.7). Considering the nature of ductility measurements in earthquake engineering, and the accuracy desired in ductility predictions, it can be said that, element ductility ratios of 1.25 or less are not considered significant, ductility ratios from 1.25 to 1.5 are considered of moderate importance, and ratios above 1.5 are judged to be of major importance. Following this arbitrary convention, the predicted increase in ductility from this method are shown to be mostly of moderate importance, which is very satisfying. A conservative strategy would be to plan for a weak element ductility ratio of 1.5, and a strong element ductility ratio of 1.0 (although when Ω=1, a weak element ductility of 1 can be used, of course).

PREDICTION OF TORSIONAL RESPONSE

Obviously, the concept of an equivalent SDOF system can be potentially very useful in design. Although there apparently is no easy way to obtain an EXACT match of the weak element displacement with a meaningful equivalent SDOF system for all values of Ω, it has been shown in the preceding section that the proposed method can provide a relatively accurate prediction of the initially eccentric system's element ductility demand. It will now be illustrated how the equivalent SDOF system procedure can be used, in a design approach, to predict the inelastic response of torsionally coupled system

A design engineer using dynamic elastic analysis tools (like elastic response spectrum method or time history analysis) may easily calculate the elastically predicted response for the weak element. It is supposed, for simplicity, that the calculation is performed for a single ground motion; let this calculated elastic response be called R_W.

The elastic response of the equivalent SDOF system can be read from an SDOF elastic response spectrum (readily available for most earthquake records); let this SDOF response be called R_{SDOF}. In order to match the elastic response of the weak element and of the equivalent SDOF system, the earthquake applied to the equivalent SDOF system should be scaled by R_W/R_{SDOF}.

It is now possible to obtain a prediction of the ductility demand on the equivalent SDOF system subjected to this corrected earthquake level by consulting inelastic response spectra (Ref. 2). These spectra are relatively straight forward to calculate using standard numerical analysis procedures, and need only be constructed once for each combination of earthquake, damping and element model. Single earthquake or multiple earthquake spectra can also be constructed. It is understood that the element model for the SDOF must match the one for the elements of the torsionally coupled system.

It is then straight forward to calculate the strength factor, defined as:

$$\eta = R_Y / m\, a_{MAX}$$

where a_{MAX} is the maximum earthquake ground acceleration, and read the ductility demand for this equivalent SDOF system off the inelastic response spectrum. If $\Omega=1$, this equivalent SDOF system's ductility demand can be assumed equal to both the weak and strong element ductilities of the torsionally coupled system; otherwise, a conservative weak element ductility should be estimated as being possibly 50% larger.

Example An initially eccentric two-element structure having the response parameters $T_X=0.2$ sec., $\Omega=2$ and $e/r=0.1$ is analyzed. For this system, the two true periods are $T_1=0.20$ sec. and $T_2=0.10$ sec. The element model is bi-linear with 0.5% strain hardening, and damping is 2% of critical. The yield displacement of this system is $\delta y=0.12$ inch. The 1940 El Centro earthquake (N-S component) was scaled to a peak acceleration of 0.46g and an elastic dynamic time-history analysis was performed; the resulting edge displacement for the weak element was 0.50 inches.

Then using an elastic response spectrum for this earthquake component (available from the Caltech Strong Motion Database, Volume III) which had an actual recorded peak acceleration of 0.348g, the pseudo-displacement for the equivalent SDOF system (with period $T_{SDOF}=0.20$ sec.) was found to be Sd=0.36 inch. In order to match the weak element elastic displacement with the equivalent SDOF system, the earthquake used in the equivalent SDOF concept must be scaled as:

$$\frac{\text{Weak element displacement}}{\text{Sd from equivalent SDOF}} = \frac{0.50}{0.36} = 1.39$$

Therefore, the peak acceleration for this equivalent SDOF increases by 1.39 and becomes 0.483g (187 in/s^2), and using the same yield displacement of 0.12 inch for the equivalent SDOF system, the strength level can be estimated by:

$$\eta = \frac{Ry}{m\, a_{MAX}} = \frac{K \quad \delta y}{m\, a_{MAX}} = \omega_{SDOF}^2 \frac{\delta y}{a_{MAX}} = 987 \frac{(0.12)}{187} = 0.64$$

Finally, reading from the constant ductility response spectra of Fig. 4 (which has been derived for two ductility levels, 2% damping and bi-linear model with 0.5% strain hardening), one can see that for a period of 0.20 sec. and a strength ratio of 0.64, the ductility demand on the equivalent SDOF system is approximately 4. Since Ω is not equal to unity, it is appropriate to increase by 50% the predicted weak element value, and use directly the obtained SDOF ductility as an estimate of the strong element value. The estimated weak element ductility is then 6, and the estimated strong element ductility remains 4. This is adequate, as the calculated strong and weak element ductilities for the initially eccentric system are respectively 5.2 and 3.0 (Fig. 3). In order to illustrate the methodology, only one earthquake excitation has been used. In a true design procedure, it is essential that many earthquake records be included.

CONCLUSIONS

Many initially eccentric systems were analyzed and compared with equivalent SDOF systems in order to investigate the effect of various parameters on their element responses. A methodology has been proposed to perform a meaningful liaison between the equivalent SDOF system and corresponding initially eccentric system, and, for bi-linear inelastic element model, was found to provide a reliable way to predict the inelastic response of structural elements in a two-element system. It was found that the ratio of ductilities obtained using the proposed method were unaffected by changes in the level of excitation (target ductility level), ratio of uncoupled frequencies Ω, uncoupled period T_X, and normalized eccentricities (e/r).

In the case of Ω=1.0, the equivalent SDOF response will perfectly match the weak element response of the torsionally coupled system, provided the inelastic element models are similar, that is, yield displacements, damping and strain hardening values are similar in the case of bi-linear models (the formal analytical proof of this could not be presented because of space limitations).

For other values of Ω, it was shown that the ductility ratios obtained by the proposed equivalent SDOF method, and following the methodology explained in the previous section, are often close to unity in the case of mean response from five earthquake excitations, with a conservative design value to be taken as 1.5. It is understood that response under a single earthquake excitation may strongly differ from the one predicted using the mean response from five earthquake, the same way this can also be expected in the case of symmetric structures.

An easy design procedure, relying mainly on elastic analysis and readily available design tools, has been proposed, and can be used to obtain good estimates of element ductilities for simple torsionally coupled systems.

REFERENCES

1. Bruneau, M., Mahin, S.A., "Inelastic Seismic Response of Structures with Mass and/or Stiffness Eccentricities in Plan", EERC Report No. 87-12, University of California, Berkeley, September 1987.

2. Mahin, S.A., Lin, J., "Construction of Inelastic Response Spectra for Single-Degree-of-Freedom Systems", EERC Report No. 83-17, University of California, Berkeley, June 1983.

3. Mondkar, D.P., Powell, G.H., "ANSR-1 - General Purpose program for analysis of nonlinear structural response:, EERC Report No. 75-37, University of California, Berkeley, December 1975.

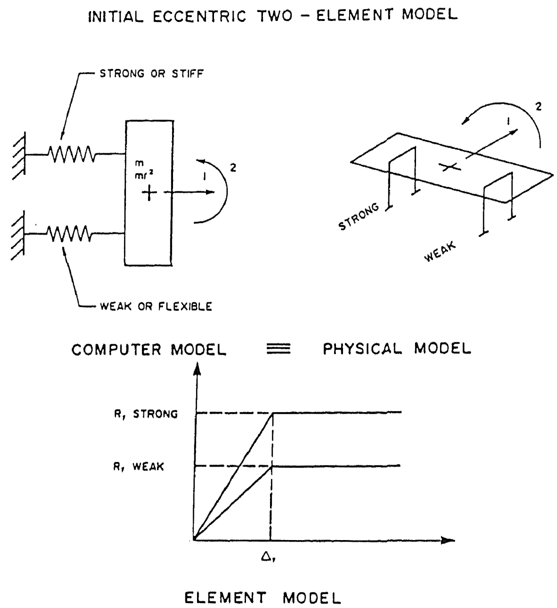

Figure 1 Element Model Used in this Study

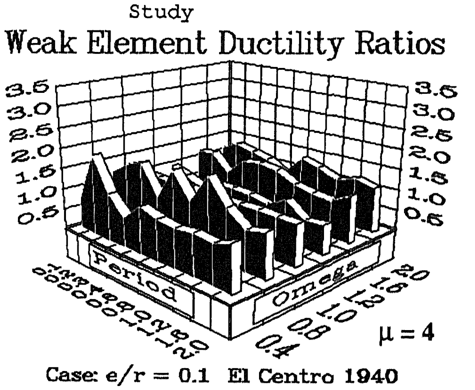

Figure 3 Weak Element Ductility Ratios for Target Ductility of 4 and El Centro Earthquake Record

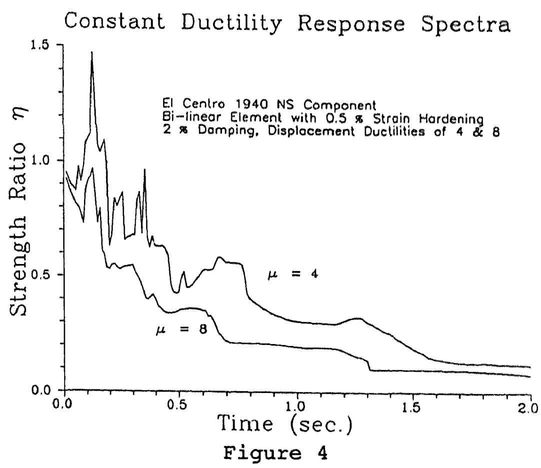

Figure 4

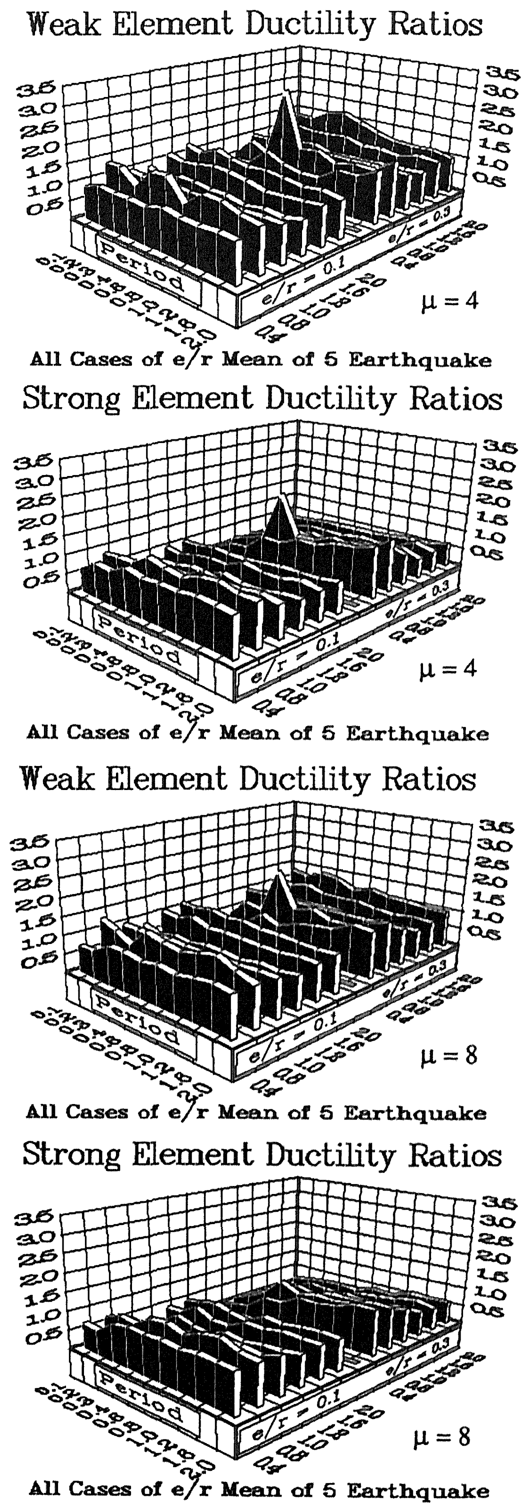

Figure 2 Target Ductility of 4 (Top Two) and 8 (Bottom Two)