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LATERAL-TORSIONAL RESPONSE OF BUILDINGS DURING EARTHQUAKES

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SUMMARY

Lateral-torisonal response of an elastic system having stiffness eccentricity is investigated based upon a random vibration approach. The power spectral density function of ground motions assumed was obtained from averaged frequency characteristics of acceleration records of several past strong earthquakes. The expected maximum values of lateral-torsional response of a single story one-way torsionally coupled elastic system are stochastically obtained, where major parameters considered in an analytical model are those eccentricityfactor e_y/r and frequency ratio $\zeta_{x\theta}$ ($=\omega_x/\omega_\theta$; ω_x and ω_θ are uncoupled natural frequencies in each direction).

INTRODUCTION

Many of buildings which have unbalanced arrangement of structural resisting elements were damaged during past strong earthquakes. In this paper, general torsional response behaviour of an elastic system having stiffness eccentricity is investigated based upon a random vibration approach.

STOCHASTIC MODEL OF TORSIONALLY COUPLED SYSTEM

Equations of Motion The equations of motion of an undamped N-story lumped mass system at the center of gravity (designated by symbol C_{iM} in Fig. 1) of each floor deck are represented by ;

$$\underline{\ddot{u}}+\underline{\Lambda}\,\underline{u}=-\underline{\ddot{u}}_g \tag{1}$$

where,

$$\underline{u}=\{u_{1x}\ \ r_1u_{1\theta}\ \ u_{1y}\cdots u_{ix}\ \ r_iu_{i\theta}\ \ u_{iy}\cdots u_{Nx}\ \ r_Nu_{N\theta}\ \ u_{Ny}\}^T \tag{2}$$

$$\underline{\ddot{u}}_g=\{\ddot{u}_{gx}\ \ 0\ \ \ddot{u}_{gy}\cdots\ddot{u}_{gx}\ \ 0\ \ \ddot{u}_{gy}\cdots\ddot{u}_{gx}\ \ 0\ \ \ddot{u}_{gy}\}^T \tag{3}$$

The variables in the above equations denote ;

$u_{ix}, u_{i\theta}, u_{iy}$: displacement components in the x-, θ - and y-directions, respectively, of the center of mass of the i-th story deck

$\ddot{u}_{gx}, \ddot{u}_{gy}$: the x- and y-directional components of ground acceleration

r_i : the radius of gyration of the i-th story deck about a vertical axis through the center of mass at the i-th story and is given by ;

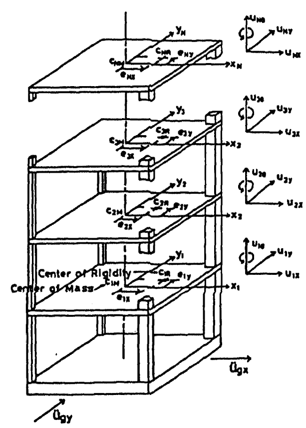

Fig. 1 Structure Model

$$r_i=\sqrt{I_i/m_i} \tag{4}$$

in which,

m_i, I_i : the i-th story mass and inertia of rotation about a vertical axis through the the center of mass, respectively

The matrix $\underline{A}$ in Eq. 1 denotes a normalized stiffness matrix of the system and is represented by ;

$$\underline{A}=\begin{bmatrix} \underline{b}_1 & \underline{c}_1 & & & & & 0 \\ \underline{a}_2 & \ddots & \ddots & & & & \\ & \ddots & \underline{a}_i & \underline{b}_i & \underline{c}_i & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & \ddots & \ddots & \underline{c}_{N-1} \\ & & & & & \underline{a}_N & \underline{b}_N \end{bmatrix} \tag{5}$$

where, the submatrices, $\underline{a}_i$, $\underline{b}_i$ and $\underline{c}_i$ are represented by ;

$$\underline{a}_i=\omega_{ix}^2\begin{bmatrix} -1 & \frac{e_{iy}}{r_i}\beta_i & 0 \\ \frac{e_{iy}}{r_i} & -\frac{\beta_i}{\zeta_{ix\theta}^2} & -\frac{e_{ix}}{r_i}\cdot\frac{1}{\zeta_{ixy}^2} \\ 0 & -\frac{e_{ix}}{r_i}\cdot\frac{\beta_i}{\zeta_{ixy}^2} & -\frac{1}{\zeta_{ixy}^2} \end{bmatrix} \tag{6-1}$$

$$\underline{b}_i=\omega_{ix}^2\begin{bmatrix} 1 & -\frac{e_{ix}}{r_i} & 0 \\ & \frac{1}{\zeta_{ix\theta}^2} & \frac{e_{ix}}{r_i}\cdot\frac{1}{\zeta_{ixy}^2} \\ \text{SYM.} & & \frac{1}{\zeta_{ixy}^2} \end{bmatrix}+\alpha_{i+1}\gamma_{i+1}\omega_{ix}^2\cdot\begin{bmatrix} 1 & -\frac{e_{i+1y}}{r_{i+1}}\beta_{i+1} & 0 \\ & \frac{\beta_{i+1}^2}{\zeta_{i+1x\theta}^2} & \frac{e_{i+1x}}{r_{i+1}}\cdot\frac{\beta_{i+1}}{\zeta_{i+1xy}^2} \\ \text{SYM.} & & \frac{1}{\zeta_{i+1xy}^2} \end{bmatrix} \tag{6-2}$$

$$\underline{c}_i=\alpha_{i+1}\gamma_{i+1}\omega_{ix}^2\cdot\begin{bmatrix} -1 & \frac{e_{i+1y}}{r_{i+1}} & 0 \\ \frac{e_{i+1y}}{r_{i+1}}\beta_{i+1} & -\frac{\beta_{i+1}}{\zeta_{i+1x\theta}^2} & -\frac{e_{i+1x}}{r_{i+1}}\cdot\frac{\beta_{i+1}}{\zeta_{i+1xy}^2} \\ 0 & -\frac{e_{i+1x}}{r_{i+1}}\cdot\frac{1}{\zeta_{i+1xy}^2} & -\frac{1}{\zeta_{i+1xy}^2} \end{bmatrix} \tag{6-3}$$

In the above equations, all the centers of mass at each floor are assumed to be on a vertical line. The variables in Eqs. 6-1 to 6-3 are ;

$$\omega_{ix}^2=\sum_j k_{ijx}/m_i,\ \ \omega_{iy}^2=\sum_j k_{ijy}/m_i,\ \ \omega_{i\theta}^2=(\sum_j k_{ijx}\cdot y_{ij}^2+\sum_j k_{ijy}\cdot x_{ij}^2)/I_i \tag{7-1}$$

$$e_{ix}=\sum_j k_{ijy}\cdot x_{ij}/\sum_j k_{ijy},\ \ e_{iy}=\sum_j k_{ijx}\cdot y_{ij}/\sum_j k_{ijx} \tag{7-2}$$

$$\zeta_{ixy}=\omega_{ix}/\omega_{iy},\ \ \zeta_{ix\theta}=\omega_{ix}/\omega_{i\theta} \tag{7-3}$$

$$\alpha_{i+1}=m_{i+1}/m_i,\ \ \beta_{i+1}=r_{i+1}/r_i,\ \ \gamma_{i+1}=\omega_{i+1x}^2/\omega_{ix}^2 \tag{7-4}$$

where,

k_{ijx}, k_{ijy} : spring constants in the x- and y-directions, of the j-th resisting element at the i-th story

x_{ij}, y_{ij} : the coordinates in the x- and y-directions, respectively, of the j-th resisting element at the i-th story

<u>Power Spectral Density Function of Response</u> The displacement $\underline{u}$ can be expressed by ;

$$\underline{u}=\underline{\phi}\,\underline{\eta} \tag{8}$$

where ϕ and η are the normal modes and coordinates.

Equation 1 is transformed into the following modal expression by substituting Eq. 8 and taking modal damping into account ;

$$\ddot{\eta}_r+2h_r\omega_r\dot{\eta}_r+\omega_r^2\eta_r=-(\beta_{r1}\cdot\ddot{u}_{gx}+\beta_{r3}\cdot\ddot{u}_{gy}),\quad r=1,2,\cdots,3N \tag{9}$$

where,

$$\lceil\omega_r^2\rfloor=\underline{\phi}^T\underline{\Lambda}\,\underline{\phi} \tag{10}$$

$$\underline{\phi}^T\underline{\phi}=\underline{E}\ \text{(Unit Matrix)} \tag{11}$$

and

$$\beta_{r1}=\sum_i \phi_{3i-2,r}\ ,\quad \beta_{r3}=\sum_i \phi_{3i,r} \tag{12}$$

The power spectral density function of the k-th component of a response vector, $S_k(\omega)$, is generally represented by (Ref. 1) ;

$$S_k(\omega)=[H_{k1}(\omega)H_{k2}(\omega)H_{k3}(\omega)]\begin{bmatrix} G_{11}(\omega) & 0 & G_{13}(\omega)\\ 0 & 0 & 0\\ G_{31}(\omega) & 0 & G_{33}(\omega)\end{bmatrix}\cdot\begin{bmatrix} H^*_{k1}(\omega)\\ H^*_{k2}(\omega)\\ H^*_{k3}(\omega)\end{bmatrix} \tag{13}$$

where, $H_{kl}(\omega)$ denotes a transfer function of the k-th component of a displacement response vector when subjected to the l-th component of ground accelerations (l=1, 2, and 3) and $G_{l_1l_2}(\omega)$ the cross spectral density function between the l_1- and l_2-components of ground accelerations. In Eq. 13, the rotation-component of the ground accelerations is assumed to be zero and the symbol * denotes a conjugate complex number.

Assuming that phase spectram of the cross spectral density function, $G_{13}(\omega)$, is random and neglecting the phase lag, the function can be given by (Ref. 2) ;

$$G_{13}(\omega)\fallingdotseq|G_{13}(\omega)|=\varepsilon_{13}(\omega)\sqrt{G_{11}(\omega)\cdot G_{33}(\omega)} \tag{14}$$

where, $\varepsilon_{13}(\omega)$ is a coherence function.

Substituting Eq. 14 and the transfer function of the single degree of freedom system into Eq. 13, one can finally obtain ;

$$S_k(\omega)=\sum_{l=1,3}G_{ll}(\omega)\cdot\left[\left\{\sum_{r=1}^{3N}\beta_{rl}\cdot\phi_{kr}\cdot f_r^{\mathrm{I}}(\omega)\right\}^2+\left\{\sum_{r=1}^{3N}\beta_{rl}\cdot\phi_{kr}\cdot f_r^{\mathrm{II}}(\omega)\right\}^2\right]+2\varepsilon_{13}\sqrt{G_{11}(\omega)\cdot G_{33}(\omega)}\left[\sum_{r=1}^{3N}\beta_{r1}\cdot\phi_{kr}\cdot f_r^{\mathrm{I}}(\omega)\cdot\sum_{r=1}^{3N}\beta_{r3}\cdot\phi_{kr}\cdot f_r^{\mathrm{I}}(\omega)+\sum_{r=1}^{3N}\beta_{r1}\cdot\phi_{kr}\cdot f_r^{\mathrm{II}}(\omega)\cdot\sum_{r=1}^{3N}\beta_{r3}\cdot\phi_{kr}\cdot f_r^{\mathrm{II}}(\omega)\right] \tag{15}$$

where,

$$\left.\begin{aligned} f_r^{\mathrm{I}}(\omega)&=\frac{1-\left(\dfrac{\omega}{\omega_r}\right)^2}{\left\{1-\left(\dfrac{\omega}{\omega_r}\right)^2\right\}^2+4h_r^2\left(\dfrac{\omega}{\omega_r}\right)^2}\cdot\frac{1}{\omega_r^2}\\ f_r^{\mathrm{II}}(\omega)&=\frac{2h_r\left(\dfrac{\omega}{\omega_r}\right)}{\left\{1-\left(\dfrac{\omega}{\omega_r}\right)^2\right\}^2+4h_r^2\left(\dfrac{\omega}{\omega_r}\right)^2}\cdot\frac{1}{\omega_r^2}\\ r&=1,2,\cdots,3N\end{aligned}\right| \tag{16}$$

The power spectral density function of ground accelerations, $G_{ii}(\omega)$, in Eq. 15 can be represented by ;

$$G_{ii}(\omega)=\frac{1+4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left\{1-\left(\frac{\omega}{\omega_g}\right)^2\right\}^2+4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}\cdot G_0 \qquad (17)$$

The averaged frequency characteristic of accelerations of several past strong earthquakes gives the following values about the parameters in Eq. 17 (Ref. 2 & 3) ; those are $\omega_g=4.8\pi\ \mathrm{sec}^{-1}$, $\xi_g=0.6$ and $G_0=120\mathrm{gal}^2\cdot\mathrm{sec/rad}$. The corresponding function is shown in Fig. 2.

Assuming that a response function is stationary Gaussian process with zero mean and applying the Rice's relation to predict the extreme values, the maximum response can be stochastically estimated by (Ref. 3) ;

$$|u_k|_{max}\simeq\sqrt{2\,\sigma_{u_k}^2\log_e\left(\frac{T_d}{\pi}\cdot\frac{\sigma_{\dot{u}_k}}{\sigma_{u_k}}\right)} \qquad (18)$$

where,

$$\sigma_{u_k}^2=\int_{-\infty}^{\infty}S_k(\omega)d\omega\ ,\ \sigma_{\dot{u}_k}^2=\int_{-\infty}^{\infty}\omega^2\cdot S_k(\omega)d\omega \qquad (19)$$

and T_d denotes time duration in second.

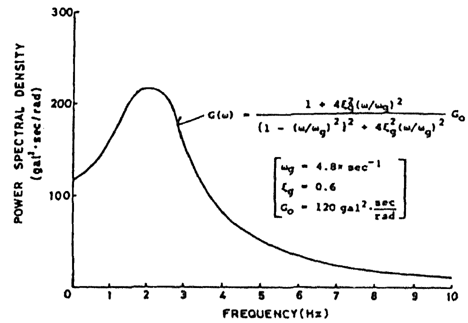

Fig. 2 Power Spectral Density Function of Ground Motions

RESPONSE OF ONE-WAY TORSIONALLY COUPLED SINGLE STORY SYSTEM

Natural Frequency Normalized coupling frequencies of a one-way torsionally coupled single story system, $\tilde{n}_1(=n_1/\omega_x)$ and $\tilde{n}_2(=n_2/\omega_x)$, are obtained by solving Eq. 20.

$$\left.\begin{aligned}\tilde{n}_1^2+\tilde{n}_2^2&=1+1/\zeta_{x\theta}^2\\ \tilde{n}_1^2\cdot\tilde{n}_2^2&=-(e_y/r)^2+1/\zeta_{x\theta}^2\end{aligned}\right\} \qquad (20)$$

where,

$$(e_y/r)\cdot\zeta_{x\theta}<1 \qquad (21)$$

The frequencies, $\tilde{n}_1$ and $\tilde{n}_2$, are varied with changing eccentricity e_y/r and frequency ratio as shown in Fig. 3.

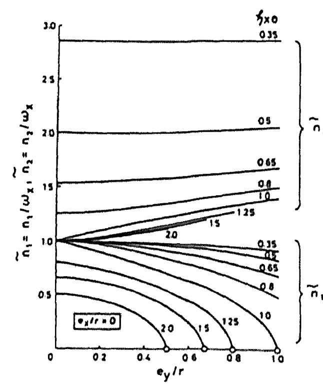

Fig. 3 Natural Frequencies of One-Way Torsionally Coupled System

Torsional Response Based upon a stochastic model described above, response of the coupled system is obtained and its tendency is discussed in the following.

Torsional response factor, ru_θ/u_x, is defined and dealt with to evaluate torsional response characteristics of the coupled system. Figures 4 and 5 show torsional response factor of a system in which uncoupled natural period in the x-direction, T_x, is 0.4sec and modal damping in each mode, h_i, is 5%. The torsional response factor, ru_θ/u_x, is greatly varied with the changing values of e_y/r and $\zeta_{x\theta}$ as shown in these figures.

From these two figures, the following features are found:

1. Torsional response factor linearly increases with increasing eccentricity e_y/r , when $\zeta_{x\theta} \leqq 0.5$.
2. Torsional response factor reaches the maximum at some value of eccentricity and, after then, does not increase with increasing eccentricity, when $\zeta_{x\theta} \geqq 1.0$.
3. Torsional response factor increases with increasing frequency ratio $\zeta_{x\theta}$ and reaches the convexity at aproximately $\zeta_{x\theta}=1$. This convexity is notable for smaller eccentricity.

The chain lines in Figs. 4 and 5 show static torsional displacement, $(ru_\theta/u_x)_S$, of the corresponding system when applying static force in the x- direction through the center of mass. The static torsional displacement is presented by ;

$$(ru_\theta/u_x)_s=(e_y/r)\cdot\zeta_{x\theta}^2 \qquad (22)$$

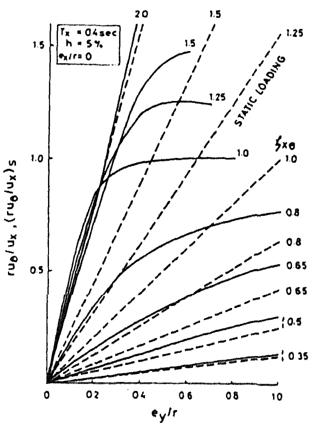

Fig. 4 Torsional Response Factor vs. Eccentricity

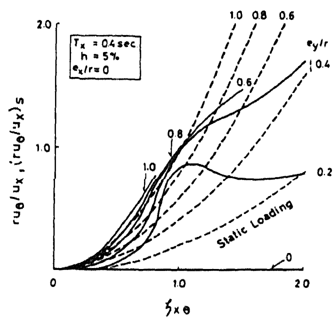

Fig. 5 Torsional Response Factor vs. Frequency Ratio

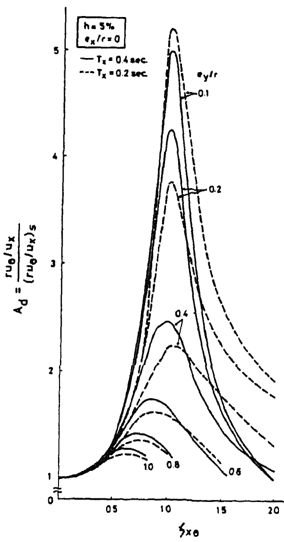

Fig. 6 Dynamic Torsional Amplification Factor

A dynamic torsional amplification factor, A_d, which is defined as (dynamic) torsional response factor devided by the static torsional displacement, is shown in Fig. 6. In this figure, the amplification factor obtained for a system which uncoupled natural period in the x-direction, T_x, is 0.2sec. is also shown. The amplification factor characteristic is very similar to a resonance curve of a SDOF system. It gives a predominant peak at $\zeta_{x\theta} \fallingdotseq 1$ and the peak is sharper for smaller eccentricity.

Eccentricity and Damage due to Torsional Response

Figure 7 shows contour lines of torsional response factor, ru_θ/u_x, on an $e_y/r-\zeta_{x\theta}$ plane. This figure indicates that the torsional response factor is more sensitive to frequency ratio, $\zeta_{x\theta}$, compared with eccentricity, e_y/r, for smaller value of the frequency ratio, say less than 1.0, and in contrast, more sensitive to e_y/r than $\zeta_{x\theta}$ for relatively larger value of $\zeta_{x\theta}$. It should be noted that most of all buildings having ordinary geometrical arrangement in structural elements are, roughly speaking, in the range of 0.7～1.2 in frequency ratio $\zeta_{x\theta}$.

The eccentricity characteristics of R/C buildings considered to be damaged mainly due to contribution of excessive torsional response during past strong earthquakes are listed in Table 1 and are plotted in Fig. 7. The Hachinohe Library building, single story, has large magnitude of the eccentricity constant,

Table 1 Torsional Characteristics of Damaged Buildings

Building	No. of Stories	Story No.	Eccentricity e_y/r	Frequency Ratio $\zeta_{x\theta}$	Eccentricity Const. $\theta_x=(e_y/r)\cdot\zeta_{x\theta}$	Remarks
Hachinohe Library Bldg.	1	1	0.65	1.33	0.86	Heavily Damaged during the 1968 Tokachi-Oki Earthquake
Mutsu City Office Bldg.	3	3 2 1	0.29 0.25 0.13	0.98 0.94 0.74	0.28 0.24 0.10	3rd Story Heavily Damaged during the 1968 Tokachi-Oki Earthquake
Kurayoshi-Higashi City Office Bldg.	3	3 2 1	0.239 0.507 0.613	0.93 1.13 0.87	0.22 0.57 0.53	2nd Story Damaged during the 1983 Tottori Earthquake

θ_x $(=e_y/r\cdot\zeta_{x\theta})$ =0.86. The thick line in Fig. 7 is the uppermost bound given by Eq. 21 and thus, this building is located near the bound. Other two buildings are three stories and the characteristic values were obtained for each story. According to the figure, it is recognized that the damages due to torsional vibration during earthquakes are produced at the story in which the factor, ru_θ/u_x, is greater than 0.8, and also, at the story in which the highest value of the factor is expected in all of the stories of a building. An analysis of damages needs to be discussed not only elastic stiffness distribution, but also inelastic characteristics of a building, however, the tendency described above strongly suggests importance of two variables, e_y/r and $\zeta_{x\theta}$, on torsional response behavior.

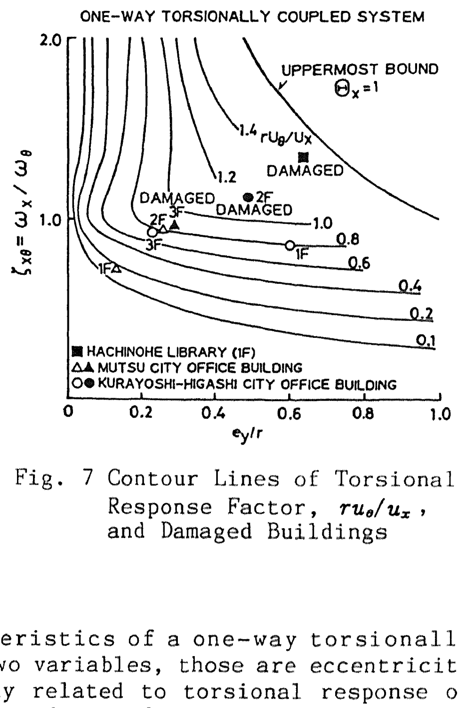

Fig. 7 Contour Lines of Torsional Response Factor, ru_θ/u_x, and Damaged Buildings

CONCLUSION

The lateral-torsional response characteristics of a one-way torsionally coupled elastic system was made clear. The two variables, those are eccentricity e_y/r and frequency ratio $\zeta_{x\theta}$, are most closely related to torsional response of structures subjected to ground motions. The tendency of the torsional response obtained from the analysis is quite reasonable to explain the damage features.

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