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METHOD OF SHEAR FORCE COEFFICIENT FOR TRANSLATION-TORSION COUPLED EARTHQUAKE RESPONSE OF ECCENTRIC STRUCTURE

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SUMMARY

In the first place, the concept and the definition of eccentric structure are given in this paper. Then, it points out that "dynamic eccentricity method" has made progress than "static method", but it still bears serious shortcomings. Therefore, method of shear force coefficient for translation—torsion coupled earthquake response of eccentric structure is presented. It has following advantages than "dynamic eccentricity method", Its physical concept is clear; The analysis processes have adquate theoretical basis; The calculation parameters are simple and clear; The formula is convenient in engineering calculations; Moreover the applied range is clear, too.

ECCENTRIC STRUCTURE AND ITS CHARACTERISTICS

Mass center not coinciding with rigidity center in each story or between stories is called eccentric structures. For example, unsymmetrical arrangement of members resisting lateral force; Unsymmetrical plane or elevational figure; although mass center coinciding with rigidity center in each story, yet these centers do not coincide between each story, etc. Its characteristics are, The earthquake response is coupled and complex three-dimensional vibration; There are a lot of such structures in engineering; Eccentricity effect always makes the earthquake calamity more serious. Therefore, many earthquake countries pays attention to research earthquake response of eccentric structure. Analysis shows that eathquake response exists many differences for mass eccentricity, rigidity eccentricity as well as both mass and rigidity eccentricity. Therefore, they are separated into three categories in which mass eccentricity occurs most in engineering and its earthguake response is rather small, usually.

STATIC METHOD AND DYNAMIC ECCENTRICITY METHOD

Due to complexity of earthquake response of eccentric structure, so static methed was first adopted. Its calculation formula is as follows (Ref.1)

$$Qri = \frac{Kri}{Kr} \cdot Qr + Kri \frac{Qr \cdot e_s}{K\phi} \cdot Yi \tag{1}$$

where, Kr and Kri are translation rigidity of the r story and the i member of resistance lateral force for r story, respectively. $K\phi$ is torsional rigidity of the story for rigidity center. Yi is distance of the i resistance lateral force member to rigidity center. e_s is static eccentricity. Qr is translation shear force of the story. Obviously, calculation on earthquake translation response is dynamic, but torsion response is static. Because it is so rough that it isn't adopted at present.

In the sixties, dynamic eccentricity method was put forward. Enlargement of static eccentricity is further considered by this methed. Its formula is as follows (Refs. 2. 3),

$$e_d = 1.5e_s \pm 0.05B \tag{2}$$

where, B is building length vertical to earthquake direction. At present, this method is adopted in many countryies. But it has following shortcomings, (a)The physical concept of dynamic eccentricity is not clear. In fact, it is a calculation eccentricity. (b) Since this method is derived by analyzing single story eccentric structures, so that, on appling the method to multistory eccentric structures it will have no adequate theoretical basis. (c)The scope of application of this method has not been given. In view of the above-mentioned facts, this paper further proposes a method of shear force coefficient for translation-torsion coupled earthquake response of eccentric structures. It is also named method of earthquake torsion effect coefficient.

METHOD OF EARTHQUAKE SHEAR FORCE COEFFICIENT

Earthquake Shear Force Coefficient let us define earthquake shear force coefficient, i.e. earthquake torsional effect coefficient as follows,

$$\alpha_d = \frac{\Delta ri}{\Delta rio} = \frac{Qri}{Qrio} \tag{3}$$

where, Δri, Qri——maximum relative displacement between each story and maximum earthquake shear force in consideration of torsion response for the i frame of the r story, respectively;

Δrio, Qrio—maximum relative displacment between each story and maximum earthquake shear force for the i frame of the r story, respectively, when torsion response is not considered.

Its physical concept is very clear. It is a multiple of enlargement or reduction of translational earthquake shear force when torsion response is considered.

Mechanical Model and Analysis Method Eccentric structure of shearing shape play a leading role, the rigidity floor, axial deformation of column are not considered, and thus it can be reduced to mechanical model of deformation compatibility in two-dimensional space. Its dynamic equation is

$$[M]\{\ddot{d}\}+(a[M]+b[K])\{\dot{d}\}+[K]\{d\}=-[M]\{E\}\ddot{X}g \tag{4}$$

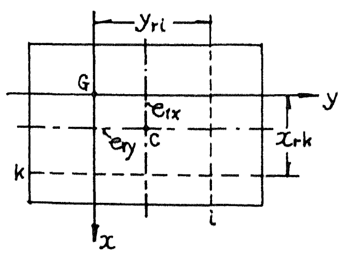
where,

$$[M]=\begin{bmatrix}[m] & & 0\\ & [m] & \\ 0 & & [J]\end{bmatrix},\quad [K]=\begin{bmatrix}[Kxx] & 0 & [Kx\phi]\\ 0 & [Kyy] & [Ky\phi]\\ [K\phi x] & [K\phi y] & [K\phi\phi]\end{bmatrix},$$

$$[Kxx]=\sum_{i=1}^{m}[Kxi],\qquad [Kyy]=\sum_{k=1}^{L}[Ky_k],$$

$$[Kx\phi]=-\sum_{i=1}^{m}[Kxi][Yi]=[K\phi x]^T=-\sum_{i=1}^{m}([Yi][Kxi])^T,$$

$$[Ky\phi]=\sum_{k=1}^{L}[Ky_k][X_k]=[K\phi y]^T=\sum_{k=1}^{L}([X_k][Ky_k])^T,$$



r(1…n), i(1…m), k(1…l)

Fig. 1 The rth floor

$$[K\phi\phi]=\sum_{i=1}^{m}[Yi][Kxi][Yi]+\sum_{k=1}^{L}[X_k][Ky_k][X_k],\ [Yi]=diag(Y_1i\cdots Yri\cdots Yni),\ [X_k]=diag(Xi_k\cdots Xr_k\cdots Xn_k)$$

vector of earthquake action factor is $\{E\}=\{E_H\}+\{E_V\}$. $\{E_H\}$ and $\{E_V\}$ are vector of seismic horizontal and vertical action factor, respectively. According to mode the displacement vector is decomposed as,

$$\{d\}=[P]\{q\} \tag{5}$$

Let us take into account Eq.(5) and then the Eq.(4) multiplied by $[P]^T$

$$[P]^T[M][P]\{\ddot{q}\}+a[P]^T[M][P]\{\dot{q}\}+b[P]^T[K][P]\{\dot{q}\}+[P]^T[K][P]\{q\}=-[P]^T[M]\{E\}\ddot{X}g \tag{6}$$

where, $[P]^T[M][P]==[Mg]$ is generalized mass matrix. $[P]^T[K][P]==[Kg]$ is generalized rigid matrix. Since, equation accords orthogonality, $[Mg]==diag(M_1\cdots M_j\cdots M_3n)$, $[Kg]==diag(K_1\cdots K_j\cdots K_3n)$. So That we can derive

$$\ddot{q}_j+(a+b\,\omega^2_j)\dot{q}_j+\omega^2_j q_j=-\gamma_j\cdot\ddot{X}g \qquad (j=1,2,\cdots,3n)\ (7)$$

where

$$\gamma_j=\frac{\{P_j\}^T[M]\{E\}}{\{P_j\}^T[M]\{P_j\}} \qquad (j=1,2,\cdots,3n)\ (8)$$

solution of Eq.(7) is

$$q_j=\gamma_{jx}\,\delta_{jx}+\gamma_{jy}\,\delta_{jy}+\gamma_{jz}\,\delta_{jz} \qquad (j=1,2,\cdots,3n)\ (9)$$

where, γ_{jx}, γ_{jy}, γ_{jz} are participation factor of the j mode when importing $\ddot{X}g$, $\ddot{Y}g$, $\ddot{Z}g$, respectively. δ_{jx}, δ_{jy}, δ_{jz} are generalized displacment of the j mode when importing $\ddot{X}g$, $\ddot{Y}g$, $\ddot{Z}g$, respectively. Usually, earthquake resistant code provides that the horizontal seismic is imported along two principal axes direction of building when the analysis is run. That is $\ddot{Y}g=0$, $\ddot{Z}g=0$ so $\delta_{jy}=\delta_{jz}=0$. Under this condition Eq.(8) is

$$\gamma_{jx}=\frac{\sum_{r=1}^{n}Mr\cdot Xjr}{\sum_{r=1}^{n}(MrX_j^2r+MrY_j^2r+Jr\Phi_j^2r)} \quad \text{or} \quad \gamma_{jy}=\frac{\sum_{r=1}^{n}Mr\cdot Yjr}{\sum_{r=1}^{n}(MrX_j^2r+MrY_j^2r+Jr\Phi_j^2r)} \qquad (j=1,2,\cdots,3n)\ (10)$$

where X_jr, Y_jr, ϕ_jr——displacement components of the mode. Then, in order to improve calculation accuracy, on the basis of characteristic of translation-torsion coupled earthquake response, the seismic relative displacement between each story for the i member of resistance lateral force of the r story is written

$$U_jri=X_jri\cdot q_j=[(X_jr-Yri\cdot\phi_jr)-(X_{j,r-1}-Y_{r-1,i}\cdot\phi_{j,r-1})]\cdot q_j \qquad (j=1,2,\cdots,3n)\ (11)$$

Then, appling earthquake response spectrum, the maximum seismic relative displacement between each story is

$$\triangle_jri=X_jri\cdot\gamma_j\frac{Sa(\omega_j)}{\omega^2_j} \qquad (j=1,2,\cdots,3n)\ (12)$$

Let us take into consideration concentration of frequency of translation-torsion coupled earthquake response, adopting Eq.(13) or CQC formula, we can process modal combination

$$\triangle ri=\left[\sum_{j=1}^{3n}\triangle^2_jri+\sum_{\substack{n=1\\n\neq m}}^{3n}\sum_{m=1}^{3m}\frac{\triangle nri\triangle mri}{1+\varepsilon^2_{nm}}\right]^{1/2}, \qquad \varepsilon^2_{nm}=\frac{1-\xi^2}{\xi^2}\left(\frac{\omega n-\omega m}{\omega n+\omega m}\right) \qquad (13)$$

or

$$\triangle ri=\left[\sum_{j=1}^{3n}\sum_{s=1}^{3n}\rho_{js}\triangle_j\triangle s\right]^{1/2}, \qquad \rho_{js}=\frac{8\xi^2(1+\lambda)\lambda^{3/2}}{(1-\lambda^2)^2+4\xi^2\lambda(1+\lambda)^2}, \qquad \lambda=\frac{\omega s}{\omega_j} \qquad (14)$$

When the seismic action being along the y direction, the above method does apply well. As stated above, the earthquake responses consideration of torsion response for each frame of any stories are obtained.

Mass Center, Rigid Center, Parameters Variation Range of Eccentric Structure The mass center of each story and the total mass center of that part of structure above the r story are calculated as follows:

$$Xcr=\frac{\sum_i Mri\cdot Xri}{\sum_i Mri}, \quad Ycr=\frac{\sum_i Mri\cdot Yri}{\sum_i Mri}, \quad Xr=\frac{\sum_{r=r}^{n}\sum_i Mri\cdot Xri}{\sum_{r=r}^{n}\sum_i Mri}, \quad Yr=\frac{\sum_{r=r}^{n}\sum_i Mri\cdot Yri}{\sum_{r=r}^{n}\sum_i Mri} \qquad (15)$$

We define the rigidity center of each story is resultant force through point of elastic restoration force which acts on the story, when the r story produces one unit translational displacement only. The calculating formula is

$$e_{rx}=\frac{\sum_k Kr_k\cdot Xr_k}{\sum_k Kr_k}, \qquad e_{ry}=\frac{\sum_i Kri\cdot Yri}{\sum_i Kri} \qquad (16)$$

where, Kr_i, Kr_k—elastic acting forces of the r story for the i member of resistance lateral force on the x direction and the k member of resistance lateral force on the y direction, respectively, when the r story produces one unit translational displacement only.

Xr_i, Yr_k—the i member of resistance translation force on the x direction and the k member of resistance lateral force on the y direction to the distance of mass center of the r story, respectively.

On the basis of data from experiment and practical measurement we get parametric variational range of eccentric structure(Refs. 6, 7):

$$\xi = \frac{\omega_\phi}{\omega_x} = \left(\frac{K\phi\phi \cdot M}{Kxx \cdot J}\right)^{1/2} \approx 1.0\sim(1.8\sim1.9), \quad \eta = \frac{e_s}{\rho} = \frac{e_s}{(J/M)^{1/2}} \approx 0.1\sim(0.4\sim0.8) \quad (17)$$

where ω_x and ω_ϕ are translational and torsional circular frequence of correspondent uncoupled structure. Thus, when we study rules of earthquake response on eccentricity structure, the current difficult for arbitrary adjustment of mechanical parameters can be avoided. And therefore, it is very helpful that we discover and summarize rules of earthquake response on eccentric structure.

The Rules of Earthquake Response on Eccentric Structure We select representative eccentric structure on frame and frame-shear wall which are taken for calculating example. On the basis of above analysis method, we perform quite a lot of calculation. And therefore, the main rules are obtained as follows:

1. For earthquake response of eccentric structure, all eccentricities which occur on the same sides is bigger than that of eccentricities which occur on the defferent sides. Therefore, it is the most severe danger to eccentric structure.

2. Eccentricity at the upper part of stucture for the affect of earthquake response is bigger than the lower.

3. If there is bigger eccentricty above the structure and there are also bigger eccentricities of the same direction at other stories, earthquake torsional response becomes very serious, we should avoid the occurance of the bigger eccentricity above the structure.

4. $K\phi\phi/Kxx$ is important parameter which affects earthquake torsional response. When it reduces to a certain number, the torsion mode play a leading role in the first mode. that is, $\omega_\phi < \omega_x$ or $\omega_\phi < \omega_y$. The above stated torsional response is quite bigger than the other ones. Thus it can be seen that earthquake response of eccentric structure is very complex, yet there are rules of earthquake response of eccentric structure on certain categories. In figure 2, earthquake torsion affect coifficients of various mass eccentricities are given. It shows that there exist main rules as stated above.

Calculation Formula of The Method of Earthquake Shear Force Coefficient We analyze quite a lot of data with statistics and select main parameters which have important influence on earthquake response. Non-dimensional quantities consists of these parameters. They are called "eccentric effect parameter". Then, we find out the relation between the parameters versus earthquake response. Wherefrom, the brief calculation formula used in engineering is obtained. Seismic shear forces of marginal members of resistance lateral force of the r story are

$$Qr_b = \alpha_d \cdot Qr_b o \quad (18)$$

$$\alpha_d = 4.50Ce + 0.65 \qquad (0.10 < Ce < 0.30) \quad (19)$$

$$Ce = \frac{e_r \cdot Y_m}{K\phi\phi / Kxx} \quad (20)$$

where, α_d is earthquake shear force coefficient; Ce is eccentric effect parameter; e_r is eccentricity of total center of mass above the r story to center of rigidity of the r story; Y_m is horizontal distance of marginal members to total center of mass above the story; Kxx is translational rigidity of the r story; $K\phi\phi$ is torsion rigidity to center of mass of the r story. The other ones may be given by linear insert.

The applied conditions of above formula are: the mass eccentric structure must pertain to the type for which the shear shape predominates; for rigid floors; for fundamental uniform distribution of vertical rigidity. If $Ce < 0.10$, then $\alpha_d < 1.1$, we do not consider the affect of earthquake torsion response. When $Ce > 0.30$, then $\alpha_d > 2.0$, the above formula does not apply. At this state, we may make use of method of response spectrum with the mode analysis as well as method of step-by-step integration to process calculation. When $0.10 \leqslant Ce \leqslant 0.30$, then $1.0 \leqslant \alpha_d \leqslant 2.0$, this range includes the most general eccentric structure used in engineering. Statistical analysis shows that its standard deriation is $\sigma=0.15$. It means that the possibility of the error of α_d exceeding 0.15 is about 13.7%. It is quite accurate for usually earthquake resistant design.

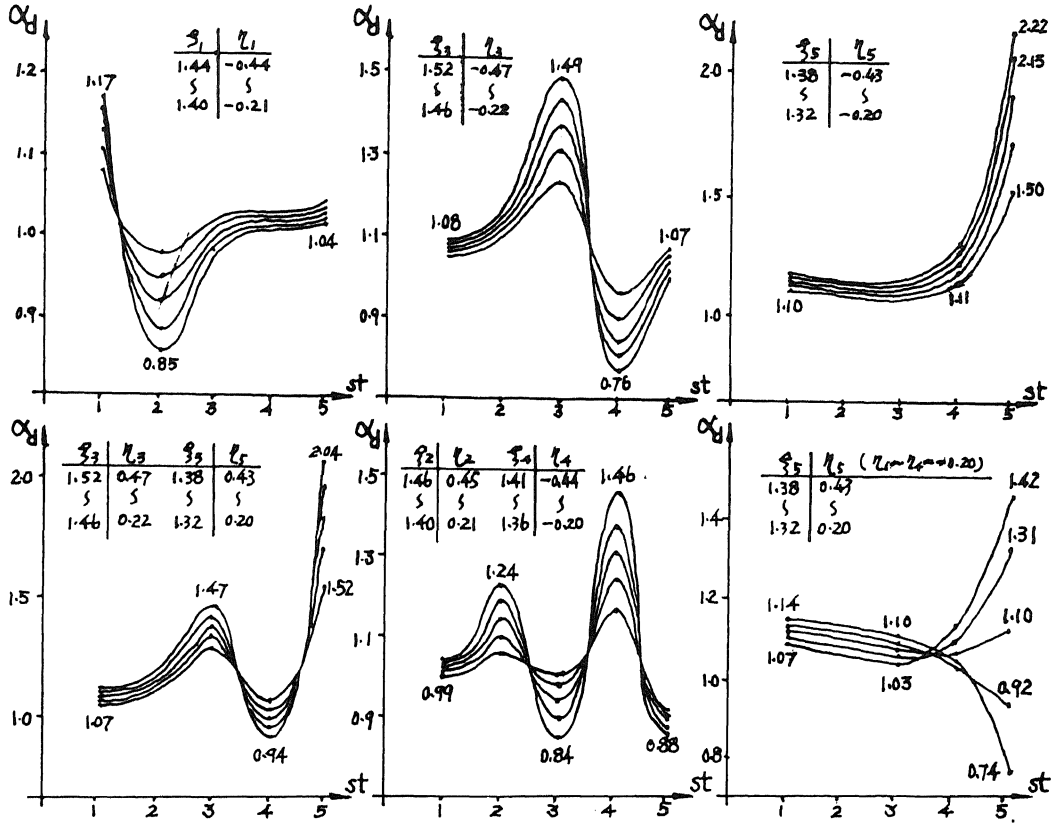


Fig. 2 αd Curve

SEVERAL VIEWS AND CONCLUSIONS

1. Eccentric structure means that the mass center is not coinciding with the rigid center in each story or neither does between stories. Rigid center is defined as the point through which the resultant of elastic restoration forces which acts on the story, when the r story has one unit translational displacement only. For the condition of translation vibration only, then mass center coincides with center of inertial force, and so does rigid center with center of restoration force namely, these two center are coincident and fixed points. but they are not center of inertial force and center of retoration force during the coupled translation-torsion vibration. Therefore, rigid center is not instantaneous rotary center. The above definitions are only for the sake of easy understanding translation torsion coupled concept and for convenient in brief calculation. It is important that we should clarify these concepts and definition for studing earthquake response of eccentric structure.

2.Because parameters are not independent each other in this problem, so it is complex and very difficult to find rules. In the past, people often regulate the non-dimensional parameters respectively from the view point of mechanics, to obtain several quite complex relation curves on natural frequence or earthquake response versus non-dimensional parameters. Since the practice variation range of parameters on eccentric structure for each type is seldom known, hence the approach of seeking out regulation from above curves is very difficult. Therefore, at present attempting to obtain general rule of earthquake respouse of eccentric structure of various types as shown is impossible. However, the analysis shows that it does exist rules for eccentric structure of various types. To point out the above statements clearly is important for seismic resistant research and design.

3.In order to establish formula of simplified, eccentric effect parameter requires both including the main parameters of eccentric structure and convenient to calculate. It is non-dimensional quantity. There is obvious regular relation between the earthquake response and the eccentric effect parameter. Therefore, in eccentric effect parameter, we have taken affect of e_s, Y_m, $K_{\phi\phi}/K_{xx}$ into consideration for earthquake response.

4.The method of earthquake shear force coeficient and its calculation formula are presented in this paper. It make obvious progress than the method of dynamic eccentricity, Its physical concept is clear; The analysis programs have adequate theoretical basis; Both simple and clear calculation parameters as well as formula are convient for use in engineering calculation and its applied range is clear. Therefore, for seismic resistant design of eccentric structure like this category it provides one brief and effective analysis method.

5.Further more problems requiring study are as follows, Characteristics and rules of earthquake response of eccentric structure on various types; influence of nonrigid floor for the earthquake response of eccentric structure; analysis of earthquake response under the condition of multi-dimensional seismic import; experiment research of restoration force characteristic when the member in question is subjected to complex force; elastic-plastic earthquake response, etc.

REFERENCE

1. OIAO, Tianmin., Yang, Guitong., Analysis of Earthquake Response for Single Storey Eccentric Buildings, J. of Taiyuan Univ. of Technology, No.4, (1984).
2. Rosenbulth, E., Aseismic Provisions for The Federal District, Mexico, Proc. 2nd WCEE, (1960).
3. Bustamante, J. I. and Rosenbluth, E., Building Code Provisions on Torsional Oscillation, Proc, 2nd WCEE, (1960).
4. Newmark, N. M. and Rosenbluth, E., Fundamentals of Earthquake Engineering,(1971).
5. Wilson, E. L., Derkiureghian, A., Boyo, E. P., A Replacement for The SRSS Method in Seismic Analysis, Earth. Eng. Struct. Dynamic, 9, (1981).
6. Research Report Volume of Calculation Method of Earthquake Loads for The Single Story Eccentric Frame Structure, Taiyuan Univ. of Technology, (1979).
7. Kan, C. L., and Chopra, A. K., Elastic Earthquake Analysis of Torsionally Coupled Multistory Buildings, Earth. Eng. Struct. Dynamic, Vol. 5, No.4, (1977).