Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-2-5

AN EQUIVALENT TORSIONAL COMPONENT OF GROUND MOTION FOR EARTHQUAKE DESIGN OF BUILDINGS

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SUMMARY

An equivalent torsional excitation component for earthquake design of buildings is proposed. It is inversely proportional to a wave propagation velocity characteristic of the local soil, which should be evaluated as a frequency dependent parameter to insure equivalence with eccentricities specified by current codes. The spectral analysis under the combined action of the translational and the rotational component is discussed, and a procedure which requires only of the displacement spectrum for computing the modal maxima is presented.

INTRODUCTION

The excitation of torsional modes due to earthquake motion introduces design requirements that for long have been thought to increase significantly in the inelastic range. Though the results of some research (Refs. 1,2) tend to point out that that intuitive assumption may not be completely true, torsion continues to be a menacing proposition when excursions into the inelastic range are to be considered, particularly so, for buildings with little torsional redundancy. For this reason, it would be desirable to have greater safety for forces originated in torsion than for those due to the translational part of the response, in much the same way as in reinforced concrete design a flexural failure is preferred to a shear failure. The required comparisons of forces "considering torsion" with those "not considering torsion" of current codes and specifications are indirect attempts to achieve this purpose. The authors have shown (Ref. 3) that it is not possible to define, for that purpose, origins of eccentricity having a physical meaning in a dynamic problem, except for the very restrictive class of buildings they have called "compensable". A quasi-compensability can be defined, but the effort needed to limit the range of validity of such an approximation has proved to be so time consuming, and the ranges themselves so narrow (Ref. 4), that a scheme for dealing with torsion that does not require the definition of origins of eccentricity has been considered preferable.

An alternative that avoids the definition of eccentricity, and allows to deal with torsion independently if different safety factors are desired for it, consists in introducing torsion under the form of a rotational component of the base excitation. A secondary advantage of such a scheme, which of course, has to be three-dimensional, is that it can be used with spectral superposition formulae that consider the effect of modal coupling (Ref. 5), thus eliminating the need of introducing, as some codes do, "dynamic amplification factors" to account for tuning effects. The rationale backing such a procedure lies in visualizing kinematic soil-structure interaction as a wave propagation problem in the way Newmark did in a classical paper (Ref. 6) in which he showed that this interaction is equivalent to a torsional component of the ground motion. He further showed that the equivalent rotational component can be interpreted, for one-story symmetrical buildings, as structure dependent eccentricities of the order of magnitude of those of seismic codes.

AN EQUIVALENT TORSIONAL COMPONENT

The kinematic interaction in a surface structure with rigid base can be approximately determined on the basis of geometric considerations. In fact, the local displacement components of the ground involve a point rotation about a vertical axis given by

$$\theta = \frac{1}{2}\left(\frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}\right) \tag{1}$$

which can be related to time derivatives following the scheme originally presented by Newmark (Ref. 6), based on the analysis of wave propagation in the vicinity of the point. The displacement components can be represented as the sum of a number of wave forms travelling in both surface directions at their own respective apparent velocity. Such a representation, i.e.,

$$u_g = \Sigma f_i(y - c_{yi}t) + \Sigma F_j(x - C_{xj}t) \tag{2a}$$
$$v_g = \Sigma g_i(x - c_{xi}t) + \Sigma G_j(y - C_{yj}t) \tag{2b}$$

is exact for body waves, and should be acceptable for surface waves in distances of the order of magnitude of the plan dimensions commonly found in buildings. Lower case letters are associated with shear waves, while upper case letters are related to compressional waves. A further simplification can be introduced if the problem is focused from a design point of view, recognizing that what has to be considered is an earthquake in the most unfavorable direction. It is then reasonable to assume that the waves travelling in that direction, say direction y, must be substantially larger than those propagating in the orthogonal direction, x. As a consequence, the expressions in (2) can be rewritten as

$$u_g = \Sigma f_i(y - c_{yi}t) \qquad v_g = \Sigma G_j(y - C_{yj}t) \tag{3}$$

from whence the space derivatives of the ground motion components are

$$\frac{\partial u_g}{\partial y} = \Sigma \frac{df_i(\eta)}{d\eta} \qquad \frac{\partial v_g}{\partial x} = 0 \tag{4}$$

and the x direction ground velocity is

$$\dot{u}_g = -\Sigma c_{yi} \frac{df_i(\eta)}{d\eta} \tag{5}$$

If the value of all the wave velocities in equation (5) were identical, comparison with the expressions given in equation (4) would lead to the result

$$\frac{\partial u_g}{\partial y} = -\frac{1}{c}\dot{u}_g \tag{6}$$

in which c is the common velocity. Since this in general will not be the case, equation (6) can only be regarded as an approximation in which c is the average of the individual apparent velocities. Assuming this approximation to hold true, substitution into equation (1) yields the estimation of the point rotation

$$\theta = -\frac{1}{2c}\dot{u}_g \tag{7}$$

which can be interpreted as a solid rotation of the rigid base, provided the plan dimensions of the foundations are not large.

This expression differs from Newmark's in a one-half factor, and can account for the overestimation that he later found in his formula for torsional effects (Ref. 7). This difference is due to the fact that in his derivation, Newmark argued that the two space derivatives in the definition given by (1) are bound to be almost equal in value, with opposite signs, disregarding that such an assumption implies a vanishing shear deformation of the ground surface. The value that should be given to c is open to discussion. It is certainly not the velocity of shear waves in the local foundation soil, as it must include the contribution of refracted waves of much larger apparent velocities. It must be dependent though, on local soil properties, perhaps with due consideration to fault distance that may alter the average slope of incoming waves. Velocities of up to ten times the shear wave velocity in the local soil have been suggested (Ref. 7) to take into account the sloping of refracted waves. At the present stage, a recommendation as to the values to be taken can only be related to present codes, as the torsional requirements for symmetric buildings with a low coupling of rotational modes, and founded in firm soil, should not be increased.

TWO-COMPONENT OR COMPLETE EXCITATION

Newmarks's original paper and the work of people who have followed his scheme (Ref. 8), are intended to be used in symmetrical buildings, in which an x-direction excitation will affect the x-direction modes only, and a torsional excitation will only affect the torsional modes. In such cases, the use of a torsional spectrum, defined as if the rotational component were an independent excitation, is clearly understandable. However, in a general case in which coupling allows at most to classify modes as predominantly x or y-direction modes, or predominantly torsional modes, all the modes of the building will be excited by either type of ground motion, so that the usefulness of the torsional spectra cannot be taken for granted. The answer to this problem requires the analysis of the structure under the combined action of the two excitation components, for instance, the x-direction ground displacement together with the rotational component proportional to the x-direction ground velocity. The equations of motion of an n-story building under this "complete excitation" can be written in matrix form as

$$[M]\{\ddot{s}\} + [C]\{\dot{s}\} + [K]\{s\} = -\ddot{u}_g[M]\{r\} - \frac{1}{2c}\dddot{u}_g[M]\{R\} \tag{8}$$

where the displacement vector $\{s\}$ has as components the displacements in the x and y-directions of the story centers of mass and the rotations of the story diaphragms, the matrices [M], [C] and [K] are the mass, damping and stiffness matrices corresponding to those degrees of freedom, and the vectors $\{r\}$ and $\{R\}$ are the influence coefficient vectors associated with the x-direction and rotational excitation, respectively.

Notwithstanding the presence of the time derivative of the ground acceleration, that has to be evaluated through numerical differentiation, direct step by step integration of these equations has been found to offer no difficulty, at least within the range of periods of ordinary buildings. However, for modal analysis, a scheme that does not require providing the time derivative can be easily devised. Such a scheme is useful in itself for time history studies, but its main significance lies in that it allows a workable spectral superposition procedure to be defined.

Once modal uncoupling has been achieved, the modal equation associated with a mode of frequency ω and modeshape $\{\phi\}$, will have the form

$$\ddot{Y} + 2\zeta\omega\dot{Y} + \omega^2 Y = -\ell\ddot{u}_g - L\dddot{u}_g \tag{9}$$

in which the two modal earthquake-excitation factors, associated respectively to the ground displacement and to the ground velocity component of the excitation, are given by

$$\ell = \frac{\{\phi\}^t[M]\{r\}}{\{\phi\}^t[M]\{\phi\}} \qquad L = \frac{1}{2c}\frac{\{\phi\}^t[M]\{R\}}{\{\phi\}^t[M]\{\phi\}} \tag{10}$$

The difficulty introduced by the second component of the excitation is but apparent. In fact, if the solution of

$$\ddot{y} + 2\zeta\omega\,\dot{y} + \omega^2 y = -\,\ddot{u}_g \tag{11}$$

is known, it will also satisfy

$$\dddot{y} + 2\zeta\omega\,\ddot{y} + \omega^2\dot{y} = -\,\dddot{u}_g \tag{12}$$

so that summing both equations multiplied by the appropriate coefficients will yield the expression

$$(\ell\ddot{y} + L\dddot{y}) + 2\zeta\omega(\ell\dot{y} + L\ddot{y}) + \omega^2(\ell y + L\dot{y}) = -\,\ell\ddot{u}_g - L\dddot{u}_g \tag{13}$$

a result that can be interpreted as the identical satisfaction of the differential equation (9) by writing

$$Y(t) = \ell y(t) + L\dot{y}(t) \tag{14}$$

This function will then be the solution of the problem provided the initial conditions, which become somewhat complex under the two-component form of equation (14), are satisfied. However, this is of no concern if the structure is at rest at the begining of the earthquake, and if the ground acceleration builds up gradually, without a starting shock.

The two-component combination leads immediately to time history solutions in a modal superposition procedure. However, for spectral estimations of maximun response values, the application of these results needs further elaboration. The first and direct approach to the problem is to define a parametric family of spectra, giving directly the maximun value of the variable

$$z(t) = y(t) + T_p\dot{y}(t) \tag{15}$$

in terms of the parameter T_p, which has to be set as equal to the ratio between the two modal excitation factors of equation (10). However, both the definition and use of these parametric spectra is cumbersome, and when they are actually drawn for specific records, they strongly suggest that the parametric value can be directly computed from the displacement and velocity spectral values as

$$S_z^2(\omega,T_p) = S_d^2(\omega) + T_p^2 S_v^2(\omega) \tag{16}$$

In justification of this tendency, it can be argued that the variables $y(t)$ and $\dot{y}(t)$ are independent for weakly stationary Gaussian processes. However, the nonstationarity of earthquake records is an important factor in the response of multidegree of freedom systems with low damping (Ref. 9), so that a different explanation must be found. This can be done by resorting to the solution of the differential equation (11) in the form of a convolution integral, which neglecting damping, can be written as

$$y(t) = A(t)\sin\omega t - B(t)\cos\omega t \tag{17}$$

in which

$$A(t) = \frac{1}{\omega}\int_0^t \ddot{u}_g \sin\omega\tau\, d\tau \qquad B(t) = \frac{1}{\omega}\int_0^t \ddot{u}_g \cos\omega\tau\, d\tau \tag{18}$$

The functions A(t) and B(t) are simple integrals, so their derivatives are equal to their integrands. Hence, on differentiation of equation (17), the terms containing those derivatives conveniently cancel out to yield as a result

$$\dot{y}(t) = \omega A(t)\cos\omega t + \omega B(t)\sin\omega t \qquad (19)$$

Equations (17) and (19) can be restated as

$$y(t) = q(t)\sin(\omega t + \alpha) \qquad \dot{y}(t) = \omega q(t)\cos(\omega t + \alpha) \qquad (20)$$

where

$$q(t)^2 = A^2 + B^2 \qquad q(t)\cos\alpha = A \qquad q(t)\sin\alpha = B \qquad (21)$$

The modal coordinate Y(t) can then be expressed as

$$Y(t) = Q\ q(t)\sin(\omega t + \alpha + \beta) \qquad (22)$$

where

$$Q^2 = \ell^2 + L^2\omega^2 \qquad Q\cos\beta = \ell \qquad Q\sin\beta = L\omega \qquad (23)$$

These relationships can be easily used to show that equation (16) is indeed a good approximation, for moderate to large frequencies. A better approach is to directly recognize that the maximun value of Y(t), for that range of frequencies, is approximately equal to the the pseudo-amplitude Q q(t), due precisely to the high frequency associated with the accompanying quasi-sinewave motion, while for low frequencies, the phase angle β will tend to be small, so that there will be no great error in taking Y(t) to be equal to Q times the response y(t). In both cases, it will be appropriate to express the maximun of Y(t) as

$$|Y|_{max} = Q\ S_d(\omega) \qquad (24)$$

a formula which requires only the specifications of the displacement spectrum.

ADJUSTMENT TO CURRENT CODE ECCENTRICITIES

As was pointed out before, the ground velocity to be included in the definition of the rotational component should be adjusted to the eccentricities currently defined in most codes as five percent of the building dimension perpendicular to the direction of the force under consideration. Such a calibration of the parameter is necessary since those eccentricities can be regarded as leading to safe designs of regular buildings located in firm soils, for instance, for symmetrical buildings in a soil classified as Type 2 by the SEAOC recommendations (Ref. 10). The necessary adjustment can then be made by considering a symmetrical one-story building under the two-component excitation, which due to the uncoupling of the translational and rotational modes, leads to the results

$$V_{max} = k_x S_d(\omega_x) \qquad M_{max} = \frac{k_\theta \omega_\theta S_d(\omega_\theta)}{2c} \qquad (25)$$

from whence the implicit eccentricity can be calculated as

$$e = \frac{\rho^2\ \omega_\theta^3\ S_d(\omega_\theta)}{2c\ \omega_x^2\ S_d(\omega_x)} \qquad (26)$$

This result shows that in order to keep the eccentricity implied in the two-component scheme at the desired five percent level, the velocity c should be taken as

$$c = 10\ \frac{\rho^2\ \omega_\theta^3\ S_d(\omega_\theta)}{a\ \omega_x^2\ S_d(\omega_x)} \qquad (27)$$

in which a is the corresponding dimension of the building, and ρ is the inertial radius of gyration. This frequency dependent expression should be regarded as valid for deep cohesionless or stiff clay soils. The value of c has to be amplified by 1.5 for rock and stiff soils (Soil Type 1), or reduced by 1.5 for medium clays and sands (Soil Type 3), to account for the influence of soil (Ref. 10) properties in shear wave velocities, considering that in ideal conditions, the velocities are inversely proportional to the predominant soil periods.

CONCLUSIONS

An equivalent torsional excitation component for earthquake design of buildings that is proportional to the ground velocity and inversely proportional to a given wave propagation velocity characteristic of the local soil, has been proposed. A factor of two in the corresponding formula, with respect to known results, has been found. At present state of knowledge, the required wave velocity is to be evaluated as a frequency dependent parameter that introduces, for symmetrical buildings, implicit eccentricities equivalent to those specified by current codes. This equivalence is assumed to hold for intermmediate, firm soils, and modifications leading to amplification of torsional effects for soft soils and reduction for stiff soils, have been proposed. The spectral analysis under the combined action of the translational and the rotational components was discussed. The use of torsional spectra is not directly applicable, except for uncoupled symmetrical buildings, so a procedure which requires only of the displacement spectrum for computing the modal maxima, was presented.

ACKNOWLEDGMENTS

This research was funded by the Chilean Superior Council for Technological Development and the Research Division of the Catholic University of Chile.

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