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DYNAMIC BUCKLING OF SHALLOW SHELLS UNDER THE UP-AND-DOWN EARTHQUAKE EXCITATION

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SUMMARY

The dynamic behaviors to follow the dynamic buckling of reticulated single-layer dome are treated. Numerical analyses for two models are carried out to estimate the buckling load under the up-and-down earthquake excitation, and the results are shown as compared with the static buckling load.

INTRODUCTION

When a large span structures, such as shells and shell-like structures, are subject to compression membrane stresses, it is important to investigate the safety against the buckling load. There are many papers which presented the dynamic behaviors of continuous shells [1,2]. However, there are few papers which treat the same subject for reticulated single-layer shells. Above this situation, the present paper describes the dynamic buckling problem of reticulated single-layer domes by the step load modeled for the up-and-down earthquake excitation. The governing equations of the geometrically nonlinear vibration for pin-connected three-dimensional trusses are derived, and then a computer program is developed by the numerical integration basing on Newmark's β method. The numerical results for two reticulated single-layer models are obtained by this program, and compared with the static buckling loads.

GOVERNING EQUATIONS

When a cross section of truss member is constant, the equation of virtual work is described in the following equation.

$$\{\delta \mathbf{D}\}^{T}\cdot\{\mathbf{F}\} = EAl\,\delta\varepsilon^{T}\cdot\varepsilon \qquad (1)$$

in which $\mathbf{D}$ and $\mathbf{F}$ are displacement and external force vectors in the global coordinates. E,A and l are, respectively, Young's modulus, constant cross section and length of truss member, and ε shows strain, which is related with displacement components u,v and w by the expression:

$$\varepsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \qquad (2)$$

Introduction of Eq.(2) into Eq.(1) results in the following expression of the nonlinear stiffness matrix for pin-connected three-dimensional truss. Numerical integration based on Newmark's β method is used for numerical analyses where the following relations are used: $\Delta t=T/1000$, $\beta=1/4$, the number of calculation times=0∿5*T, in which T is the natural period of the first axisymmetric vibration.

NUMERICAL ANALYSES

Fig.1 shows a four bar model. Maximum displacement responses of this model for some step load levels applied at four supported points are plotted by circles in Fig.2 where the static load-displacement relation is depicted by the solid curve. In the case of the static load, concentrated vertical load is applied at the top. As a result, the step load reduces the buckling load to about half of the static buckling load. The following equations shows the relation among static buckling load P_{cr}, the equivalent acceleration $\ddot{y}_{0cr}$ and load parameter Λ.

$$P_{cr}=\frac{\rho A l\cdot n\cdot \alpha}{g}\ \ddot{y}_{0cr} \tag{3}$$

$$\Lambda=\frac{\text{step load}}{\ddot{y}_{0cr}} \tag{4}$$

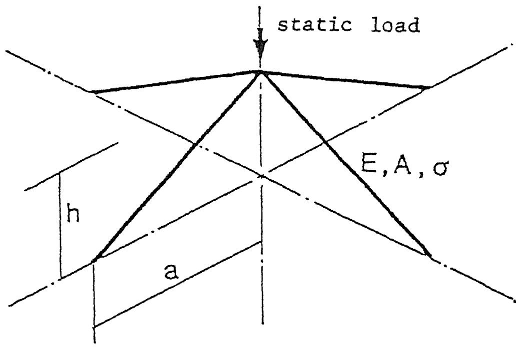
in which, ρAl is the weight of member, n, g and α are total number of member, the gravity acceleration and a mass parameter, respectively. In the case of this model, mass parameter α is fixed to 1/3. In other words, the equivalent acceleration to a static buckling load is estimated by the consistent mass of only free joint of top. But in the following model, mass parameter $\alpha=1$, that is, the whole weight of model is considered for the estimation of dynamic buckling load.

Fig.3 shows a dome model. This model, with the base diameter (2a) of 20m and the rise (h) of 3m, is composed of 156 members connected at 61 joints. As shown in Fig.4, there are 12 axisymmetric natural frequencies among 111 components of the whole natural vibration modes including asymmetric modes. The first 6 axisymmetric modes exist in the narrow and relatively lower zone of frequencies, that is 20 to 50 Hz, and the other 6 modes distribute in the higher zone of 300 to 500Hz. The first 6 axisymmetric mode shapes are drawn in Fig. 5. Fig.6 expresses the differences of nonlinear responses from linear responses of displacement and acceleration at the central joint with load parameter $\Lambda=0.2, 0.6$ and 1.0. Time histories of displacement responses of different four nodes with load parameter $\Lambda=0.6$ to 1.0 are shown in Fig.7. The dynamic buckling load by step load are estimated from this figure because the central joint(1) and the joint(20) in the vicinity of the central joint generate suddenly large deformations from around 10cm to 60cm at the load level of about $\Lambda=0.8$. Therefore the dynamic buckling load is estimated as about 80% of the static buckling load.

Fig.8 shows the comparison of Fourier spectra of acceleration responses at central joint(1) and eccentric joint(8). This figure means that the response component of frequencies exists in the narrow zone in the pre-buckling for $\Lambda=0.6$. But after buckling of $\Lambda=1$, the zone is enlarged, and then the prominent components of central joint and eccentric joint change from 42.8Hz to 53.5Hz and 80.3Hz, respectively. In Fig.9, the transition of dynamic deflection modes by the step load is compared with the static buckling modes due to the uniform vertical load. The dynamic buckling load will be also estimated from Figs.10 and 11 as the same method as treated for the simple four bar model. The dynamic buckling load of this dome model exists in about 80% of the static buckling load.

CONCLUSION

In this paper, the dynamic buckling loads due to step loads are numerically examined for reticulated single-layer shells. The results of two models ,that is a four bar model and a dome model,show the level of the dynamic buckling loads are in about 50% and 80% of the static buckling load,respectively.



Young's Modulus	E	$= 2.1 \times 10^6$	kgf/cm^2
Cross Section	A	$= 11.2$	cm^2
Density	σ	$= 7.85/10^3$	kgf/cm^2
Half Span	a	$= 500.0$	cm
Rise	h	$= 100.0$	cm

Fig.1 Four bar model

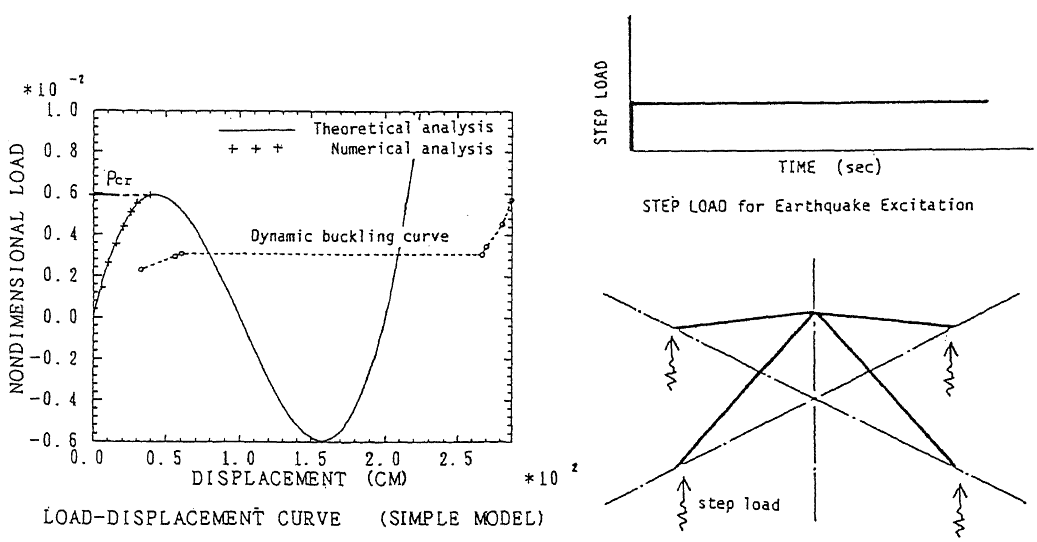


Fig.2 Comparison of buckling curve between static and dynamic analyses

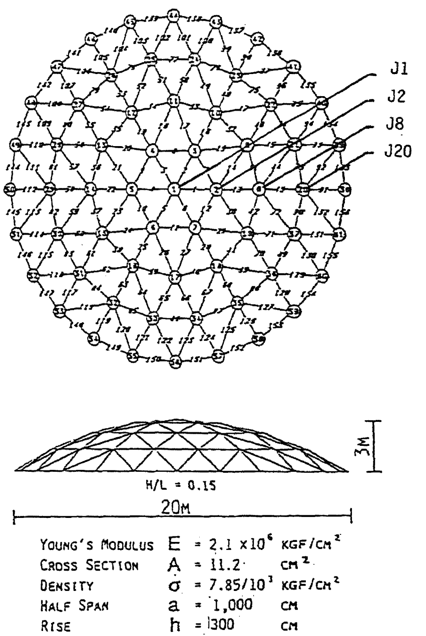


Fig.3 Dome model

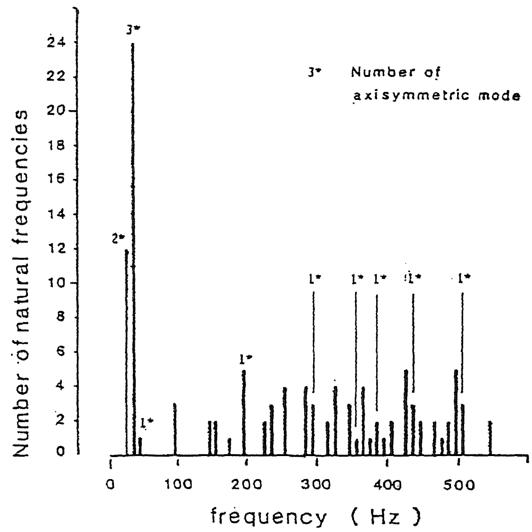


Fig.4 Histogram of distribution of natural frequencies

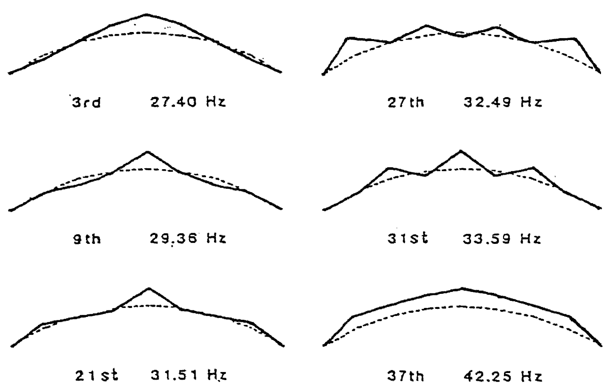
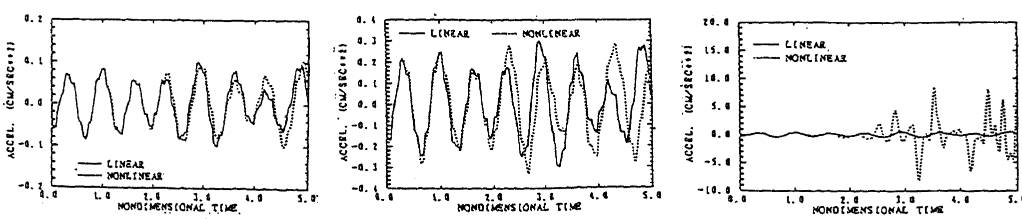


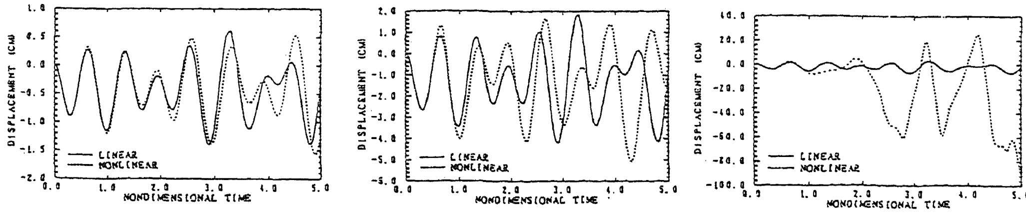
Fig.5 Axisymmetric mode shapes and the natural frequencies



Λ=0.2　Λ=0.6　Λ=1.0

Fig.6 Difference between linear and nonlinear response

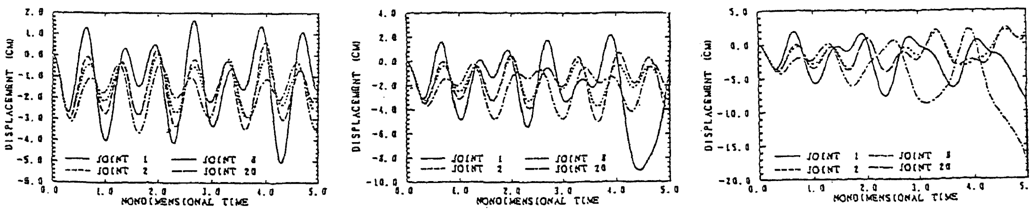
(a) Acceleration responses



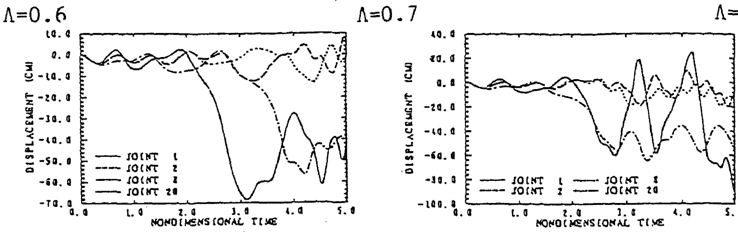
Λ=0.2 Λ=0.6 Λ=1.0

Fig.6 Difference between linear and nonlinear response

(b) Displacement responses

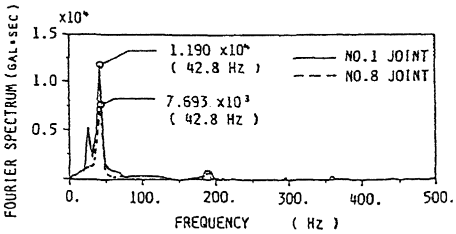


Λ=0.6 Λ=0.7 Λ=0.8

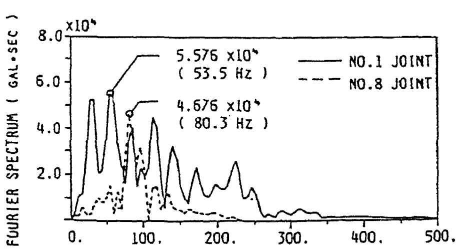


Λ=0.9 Λ=1.0

Fig.7 Time histories of displacement responses

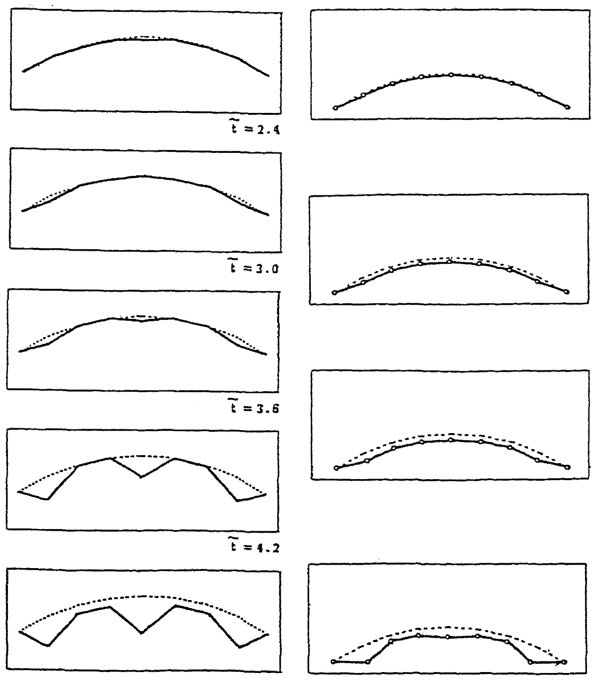


Λ=0.6



Λ=1.0

Fig.8 Comparison of Fourier spectra at joint(1) and joint(8)



(a) Step load (b) Static load

Fig.9 Transiton of deflection modes

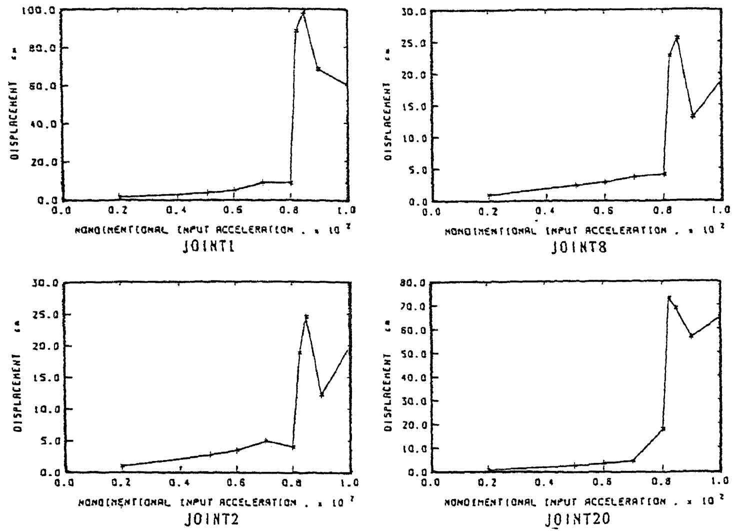


Fig.10 Variation of maximum response with step load

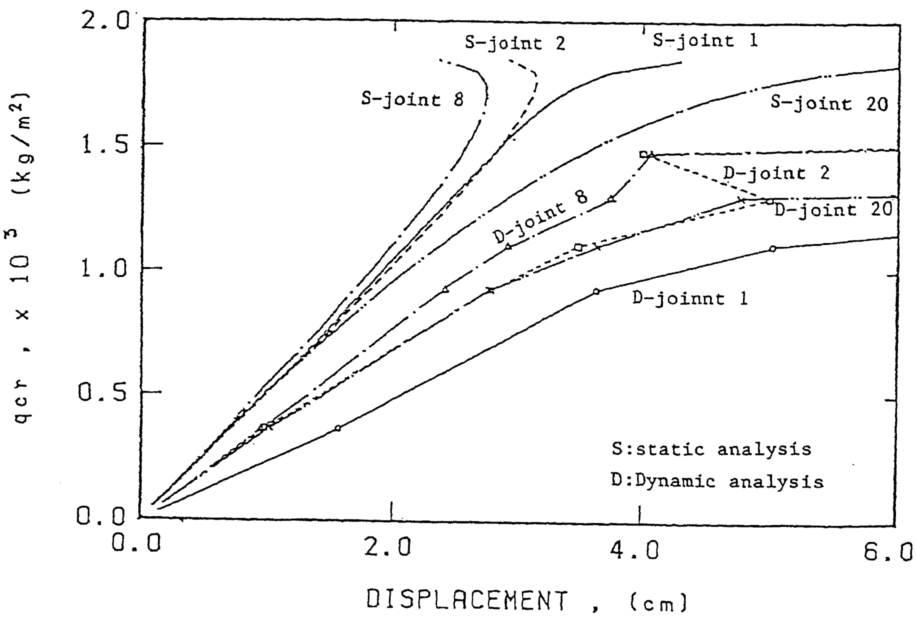


FIg.11 Comparison of buckling curves between static and dynamic analyses

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