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RESPONSE OF CONICAL SHELLS WITH EDGE BEAMS SUBJECTED TO SEISMIC FORCES IN THE VERTICAL AND HORIZONTAL DIRECTIONS

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SUMMARY

The object of the investigation is to clarify the influence of seismic forces in the vertical and horizontal directions regarding conical shells with/without edge beams. An edge beam is to be regarded as ring elements with triangular cross sections, and the shell as a set of conical frustum elements so that the finite element method can take into account the deformation of the edge beam in the cross section area. The authors attempted the study on free vibration analysis using the power method and dynamic response analysis using a step by step integration method.

INTRODUCTION

A large amount of research has been carried out on conical shells without edge beams, while very little has been done on the dynamic behavior of conical shells with edge beams in spite of the fact that conical shells in actual use usually have edge beams.

Moreover, in recent years a great deal of attention has been given to the dynamic response behavior of structures against seismic oscillation in a vertical direction. Shallow shells will be greatly affected by seismic oscillation in a vertical direction and high-rise shells by seismic oscillation in a horizontal direction. However, sufficient light has not been shed on the extent of influence of vertical and horizontal seismic oscillation in terms of the rise/span ratio.

To solve this problem by the use of the finite element, the authors considered an edge beam in an assemblage of ring elements with a triangular cross section, and a shell in an assemblage of conical frustum elements. In an attempt to find a solution, we proposed giving the ring element a degree of freedom of rotation. We also allowed a degree of freedom in each direction when measuring displacements in the perpendicular, circumferential and radial directions. We did this so that the ring element of the edge beam which had a triangular cross section, could be combined with the conical frustum element of the shell. We assumed the angle of rotation to be the mean rotation.

We attempted the dynamic analysis of conical shells with edge beams to consider the deformation of edge beams under certain boundary conditions and in doing this we changed the rise in the shells. As a result of our analysis we have been able to clarify some of the behaviors of conical shells with edge beams.

CONICAL SHELL

<u>Displacement Functions</u> Displacement functions of a conical shell are represented by polynomials in the meridional distance and the Fourier series in the circumferential angle.

$$\{f\}=\begin{Bmatrix} u \\ v \\ w \\ \chi \end{Bmatrix}=\begin{Bmatrix} \Sigma(\bar{\alpha}_1 + \bar{\alpha}_2 s)\cos n\theta \\ \Sigma(\bar{\alpha}_3 + \bar{\alpha}_4 s)\sin n\theta \\ \Sigma(\bar{\alpha}_5 + \bar{\alpha}_6 s + \bar{\alpha}_7 s^2 + \bar{\alpha}_8 s^3)\cos n\theta \\ \partial w/\partial s \end{Bmatrix} \quad (1)$$

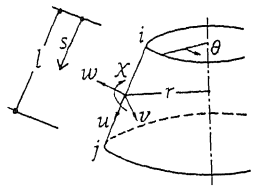

Fig.1 Displacements

<u>Strain-Displacement Relations</u> Strain-displacement relations of a conical shell are given as follows :

$$\{\varepsilon\}=\begin{Bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \gamma_{s\theta} \\ \kappa_s \\ \kappa_\theta \\ \kappa \end{Bmatrix}=\begin{Bmatrix} \dfrac{\partial u}{\partial s} \\ \dfrac{1}{r}\left(u\sin\phi + \dfrac{\partial v}{\partial \theta} + w\cos\phi\right) \\ \dfrac{1}{r}\left(\dfrac{\partial u}{\partial \theta} - v\sin\phi\right) + \dfrac{\partial v}{\partial s} \\ -\dfrac{\partial^2 w}{\partial s^2} \\ -\dfrac{\sin\phi}{r}\dfrac{\partial w}{\partial s} + \dfrac{1}{r^2}\left(\dfrac{\partial v}{\partial \theta}\cos\phi - \dfrac{\partial^2 w}{\partial \theta^2}\right) \\ \dfrac{1}{r}\left(\dfrac{\partial v}{\partial s}\cos\phi - 2\dfrac{\partial^2 w}{\partial s\partial\theta}\right) - \dfrac{2\sin\phi}{r^2}\left(v\cos\phi - \dfrac{\partial w}{\partial \theta}\right) \end{Bmatrix} \quad (2)$$

<u>Stress-Strain Relations</u> Stress-strain relations of a conical shell are :

$$\{\sigma\}=\left(N_s\ N_\theta\ N_{s\theta}\ M_s\ M_\theta\ M_{s\theta}\right)^T=[D]\{\varepsilon\} \quad (3)$$

where, $[D]$: elasticity marix of a shell

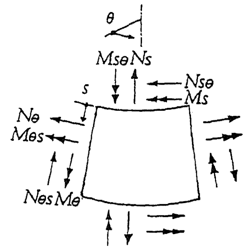

Fig.2 Stresses

EDGE BEAM

<u>Displacement Functions</u> We assume that the displacement functions of an edge beam are as follows.

$$\{f_b\}=\begin{Bmatrix} u^* \\ v^* \\ w^* \\ \chi^* \end{Bmatrix}=\begin{Bmatrix} \Sigma(\alpha_1 + \alpha_2 r + \alpha_3 z + \alpha_4 rz + \alpha_5 r^2)\cos n\theta \\ \Sigma(\alpha_6 + \alpha_7 r + \alpha_8 z)\sin n\theta \\ \Sigma(\alpha_9 + \alpha_{10} r + \alpha_{11} z + \alpha_{12} rz)\cos n\theta \\ \partial w^*/\partial z - \partial u^*/\partial r \end{Bmatrix} \quad (4)$$

<u>Strain-Displacement Relations</u> Strain-displacement relations of an edge beam are :

$$\{\varepsilon_b\}=\begin{Bmatrix}\varepsilon_z\\ \varepsilon_\theta\\ \varepsilon_r\\ \gamma_{rz}\\ \gamma_{\theta r}\\ \gamma_{z\theta}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u^*}{\partial z}\\ \dfrac{1}{r}\dfrac{\partial v^*}{\partial \theta}+\dfrac{w^*}{r}\\ \dfrac{\partial w^*}{\partial r}\\ \dfrac{\partial u^*}{\partial r}+\dfrac{\partial w^*}{\partial z}\\ \dfrac{\partial v^*}{\partial r}-\dfrac{v^*}{r}+\dfrac{1}{r}\dfrac{\partial w^*}{\partial \theta}\\ \dfrac{1}{r}\dfrac{\partial u^*}{\partial \theta}+\dfrac{\partial v^*}{\partial z}\end{Bmatrix} \qquad (5)$$

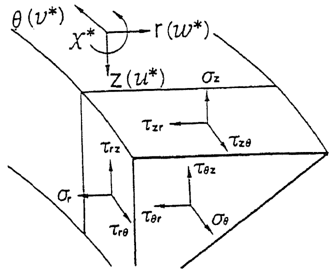

Fig.3 Displacements & Stresses

Stress-Strain Relations Stress-strain relations of an edge beam are given as follows.

$$\{\sigma_b\}=(\sigma_z\ \sigma_\theta\ \sigma_r\ \tau_{rz}\ \tau_{\theta r}\ \tau_{z\theta})^T=(D_b)\{\varepsilon_b\} \qquad (6)$$

where, (D_b) : elasticity matrix of an edge beam

EQUATIONS OF OSCILLATION

The equations of oscillation are represented in equation (7). We assume that the damping matrix is in proportion to the mass matrix.

$$[M^n]\{\ddot{\delta}\} + [C^n]\{\dot{\delta}\} + [K^n]\{\delta\} = \{F^n\} \qquad (7)$$

where, $[M^n]$, $[C^n]$, $[K^n]$: mass, damping and stiffness matrix

$\{\ddot{\delta}\}$, $\{\dot{\delta}\}$, $\{\delta\}$: acceleration, velocity and displacement vector

The equations were computed by a step by step integration method. Here, n=0 and n=1 represent seismic oscillations in the vertical and horizontal directions, respectively.

MODELS

Conical shell models and their boundary conditions are shown in Figure 4 and 5, respectively.

Shell
$E=1.38\times10^4 kg/cm^2$
$\nu=0$
$\rho=4.079531\times10^{-7} kg\cdot s^2/cm^4$
$t=1.0m$

Edge Beam
$E_b=2.1\times10^5 kg/cm^2$
$\nu_b=1/6$
$\rho_b=2.448979\times10^{-6} kg\cdot s^2/cm^4$

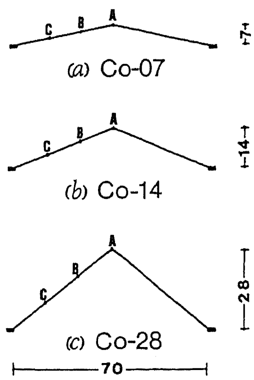

Fig.4 Models (m)

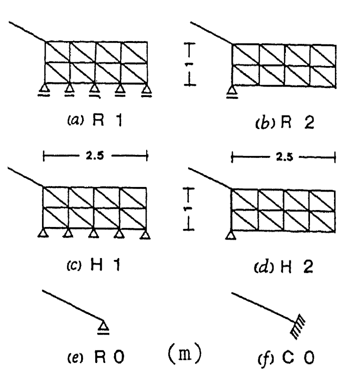

Fig.5 Boundary Conditions

RESULTS

Free Vibration   Vibration modes and natural frequencies are shown in Figure 6 and Table 1, respectively.

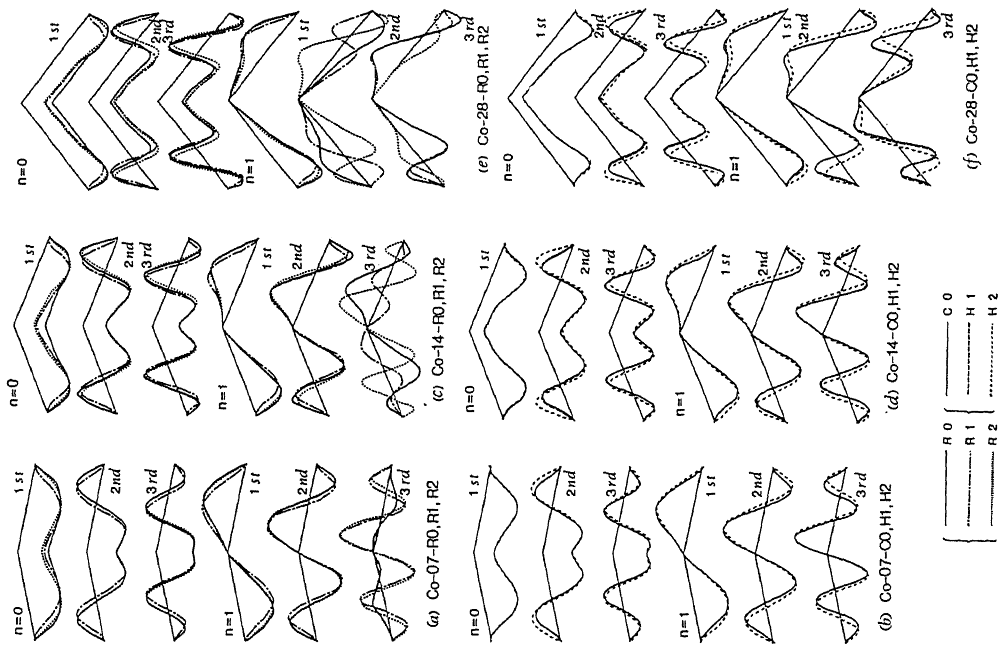

Fig.6 Vibration Modes

Dynamic Response   The maximum displacements of the shells subjected to seismic forces in vertical and horizontal directions are shown in Table 2. Figure 7 shows displacements and stresses when the vertical displacement u* at the top is at the maximum. Figure 8 shows displacements and stresses when the horizontal displacement w* at the top is at the maximum.

time interval
h= 0.004 s (UD)
h= 0.005 s (NS)

damping ratio $\zeta = 0.02$

input maximum acc.
200 gal (UD,NS)

Table 1 Natural Frequencies (Hz)

TYPE	HAM. NO.	ORDER	WITH EDGE BEAM R 1	WITH EDGE BEAM R 2	WITH EDGE BEAM H 1	WITH EDGE BEAM H 2	WITHOUT EDGE BEAM R 0	WITHOUT EDGE BEAM C 0
Co-07	n=0	1	2.37	2.19	2.88	2.88	1.03	2.90
		2	4.78	4.06	4.86	4.10	4.08	4.94
		3	7.83	7.01	7.93	7.43	6.98	8.07
		4	11.99	10.66	12.17	10.68	11.01	12.43
		5	14.27	14.02	17.70	15.63	15.50	18.06
	n=1	1	2.78	2.46	2.88	2.75	1.64	2.91
		2	5.30	4.60	5.32	4.67	4.61	5.41
		3	8.92	7.85	8.96	7.86	6.28	9.14
		4	9.86	9.31	13.91	11.74	8.02	14.20
		5	13.86	12.24	16.85	15.07	12.75	16.94
Co-14	n=0	1	3.87	3.69	4.75	4.74	1.60	4.78
		2	6.67	5.82	6.87	5.83	5.86	6.97
		3	9.67	8.86	9.89	9.44	8.83	10.02
		4	13.08	12.07	13.86	12.11	12.41	14.11
		5	15.04	14.28	18.87	16.96	16.01	19.20
	n=1	1	4.36	4.00	4.52	4.40	2.31	4.56
		2	7.00	6.26	7.02	6.29	6.30	7.12
		3	9.59	8.87	10.37	9.17	6.38	10.55
		4	10.40	9.59	14.72	12.58	9.56	14.95
		5	14.71	13.21	15.81	14.92	13.76	15.92
Co-28	n=0	1	5.87	5.73	7.05	7.05	2.41	7.11
		2	8.57	7.69	8.98	7.70	7.77	9.07
		3	11.20	10.44	11.47	11.11	10.48	11.61
		4	13.36	12.81	14.65	13.02	13.06	14.84
		5	15.70	14.56	18.42	17.09	16.43	18.67
	n=1	1	6.39	6.04	6.49	6.31	2.95	6.54
		2	8.49	8.11	8.80	8.12	6.64	8.90
		3	9.07	8.24	11.37	9.86	8.24	11.48
		4	11.80	10.95	12.75	12.66	11.09	12.83
		5	14.92	13.84	15.35	13.90	14.28	15.55

Table 2 Displacement Responses (cm)

EARTHQUAKE	TYPE		WITH EDGE BEAM								WITHOUT EDGE BEAM			
			R 1		R 2		H 1		H 2		R 0		C 0	
			VER.	HOR.	VER.	HOR.	VER.	HOR.	VER.	HOR.	VER.	HOR.	VER.	HOR.
MIYAGI PREFEC. UD (JAPAN) JUN.1978	C_o- 07	A	2.779	0	2.236	0	2.585	0	2.536	0	12.673	0	2.725	0
		B	3.360	0.175	2.826	0.148	3.621	0.295	3.659	0.296	13.667	0.230	3.851	0.318
		C	2.789	0.199	2.820	0.252	3.126	0.411	3.029	0.390	12.819	0.192	3.339	0.439
	C_o- 14	A	1.082	0	1.065	0	0.403	0	0.424	0	3.867	0	0.414	0
		B	1.267	0.097	1.218	0.083	0.523	0.063	0.556	0.067	3.949	0.049	0.548	0.068
		C	1.475	0.275	1.596	0.321	0.796	0.233	0.790	0.221	4.230	0.214	0.817	0.238
	C_o- 28	A	0.369	0	0.277	0	0.139	0	0.193	0	2.678	0	0.139	0
		B	0.380	0.028	0.283	0.025	0.149	0.017	0.206	0.025	2.868	0.028	0.146	0.017
		C	0.492	0.176	0.392	0.141	0.262	0.413	0.266	0.415	2.901	0.256	0.250	0.136
IMPERIAL VALLEY NS (USA) MAY.1940	C_o- 07	A	0	0.229	0	0.182	0	0.054	0	0.071	0	1.138	0	0.052
		B	0.163	0.221	0.310	0.174	0.419	0.077	0.812	0.143	0.875	1.026	0.409	0.075
		C	0.171	0.200	0.301	0.183	0.431	0.086	0.699	0.152	1.376	0.973	0.411	0.082
	C_o- 14	A	0	0.157	0	0.172	0	0.058	0	0.090	0	1.169	0	0.060
		B	0.103	0.155	0.219	0.221	0.182	0.067	0.349	0.099	0.799	0.977	0.198	0.066
		C	0.147	0.141	0.263	0.154	0.353	0.130	0.429	0.184	1.728	0.817	0.367	0.125
	C_o- 28	A	0	0.241	0	0.253	0	0.117	0	0.110	0	1.505	0	0.116
		B	0.144	0.217	0.070	0.247	0.165	0.141	0.166	0.146	0.608	1.063	0.164	0.138
		C	0.282	0.238	0.219	0.198	0.400	0.299	0.400	0.301	1.397	0.686	0.367	0.296

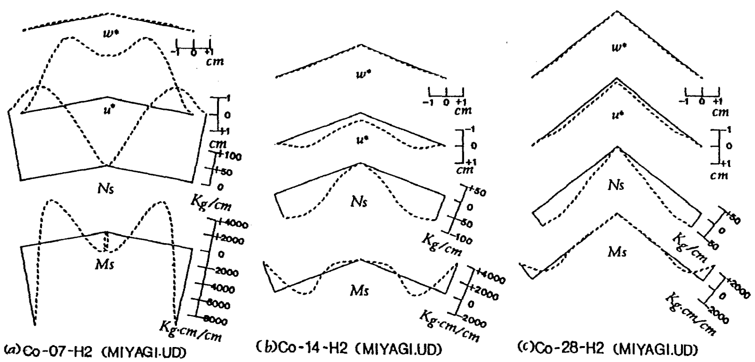

Fig.7 Displacements & Stresses (Miyagi Pre. UD)

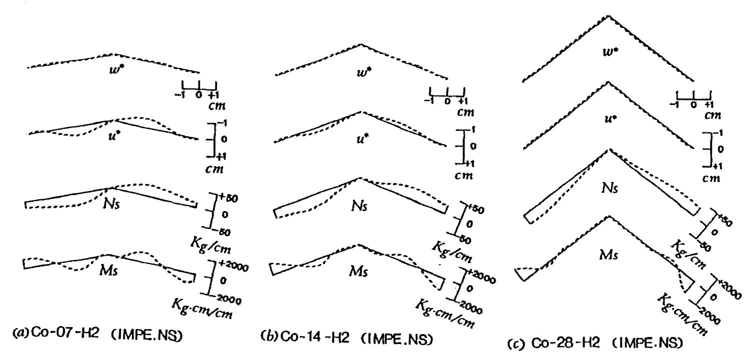

Fig.8 Displacements & Stresses (Imperial Val. NS)

CONCLUSIONS

Free Vibration We can conclude the following from the results of the free vibration analysis of conical shells :

(1) In the case of a roller support, the natural frequency of the first order is remarkably low and the shape of the mode is very different, compared to other types of support.
(2) As the rise increases, the shell approaches rigidity, and its natural frequency becomes higher.
(3) Addition of an edge beam will raise the rigidity of the shell, which will, therefore, run closer to the value in the case of a clamped support.

Seismic Response We can conclude the following from the results of the seismic response analysis of conical shells :

(1) In the case of a roller support without an edge beam, the action of seismic oscillation in either the vertical or horizontal direction will cause major displacements and stresses to conical shells. The addition of an edge beam can significantly reduce the displacements and stresses.
(2) Subject to vertical seismic oscillation, conical shells with and without edge beams will show a smaller fall in displacement and stress as the rise increases.
(3) When horizontal seismic oscillation takes place, each rise in the shell has a very slight displacement in both vertical and horizontal directions. Stresses are also small, compared to the vertical displacement of a shallow conical shell subjected to vertical seismic oscillation.

When we design a shallow conical shell with a rise/span ratio of less than approximately 0.1, full consideration needs to be given to the deformations and stresses against vertical seismic oscillation.

REFERENCES

1. Tajimi, H. : "Introduction to Structural Dynamics", Corona Publishing Co., Ltd., Tokyo, 1986 (in Japanese)
2. Kawai, T. : "Matrix Method Vibrations and Responses", Baifukan, Tokyo, February, 1971 (in Japanese)
3. Kawashima, S. : "Dynamic Response Analysis", Baifukan, Tokyo, July, 1972 (in Japanese)
4. Zienkiewicz, O.C. and Cheung, Y.K. : "The Finite Element Method in Structural and Continuum Mechanics", McGraw-Hill, London, 1967
5. Nishimura, T. and Shingu, K. : "Study on Dynamic Response Analysis of Rotational Shell with Edge Ring", World Congress on Shell and Spatial Structures, Madrid, pp. 2.107-2.121, September,1979
6. Nishimura, T. and Shingu, K. : "Study on Dynamic Response of Rotational Shells with Edge Beams Subjected to Forces in the Vertical and Horizontal Directions", Trans. of the Architectural Institute of Japan, No.326, pp.47-59, April, 1983 (in Japanese)
7. Nishimura, T. and Shingu, K. : "Theoretical and Experimental Studies on Natural Vibration of a Submerged Conical Shell", Ocean Space Utilization '85, Proceedings of the International Symposium, Springer-Verlag, pp.393-400, June, 1985
8. Shingu, K. and Nishimura, T. : "A Study on Dynamic Response of Submerged Conical Shells", Shells, Membranes and Space Frames, Proceeding IASS Symposium, Elsevier, pp.81-88, September, 1986