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SEISMIC RESPONSE OF MULTIPLY COUPLED SECONDARY SYSTEMS USING GENERALIZED MODAL SPECTRA

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SUMMARY

A method is explored of modeling a coupled system of main structure and substructures by way of modal analysis. The generalized coupled model is constructed from a sequence of static displacements fields generated from uniform acceleration modes in the expected direction of excitation. By reducing the vector space to pairs of primary and secondary structure modes an approximation is obtained from which a generalized response spectrum method and a corresponding numerical scheme can be derived. The present generalized modal response spectrum method shows a number of advantages over conventional methods, such as fewer modal degrees of freedom, no truncation error, simpler modal superposition and direct use of earthquake response spectra.

INTRODUCTION

The design of nuclear power plant interior structures requires the consideration of a large number of loading, operating and emergency conditions. The analysis of a subsystem attached to the main structure at several locations, and subjected to the motion of the main structure from earthquake excitation is traditionally carried out in a decoupled fashion using floor response spectra. For earthquake loadings acceleration time histories have first to be constructed from the given earthquake acceleration spectra.

The analysis of the main structure proceeds in three steps. First, a free vibration analysis is carried out up to a frequency selected by the analyst based on design specifications. Second, a modal time history analysis is performed to determine the acceleration time histories at all the attachment points of subsystem supports. Third, the acceleration time histories are transformed into acceleration spectra according to standard nuclear power plant design practice.

With this prerequisite main structure analysis work completed the subsystem analysis can proceed using the response spectrum method including the effect of multiple suport excitations (Refs.1,2,3).

This cascaded computational procedure to predict the maximum response of a substructure attached to a main structure gives rise to the following considerations for improving the computational procedure:

- direct construction of the floor response spectra when the excitation of the main structure is defined by spectra;

- evaluation of phase relationships in the motion of the support points;
- coupling effects of the subsystem with the main structure;
- superposition rules of maxima resulting from different load components and different response modes;
- correction of modal truncation errors;
- construction of simplified analysis procedures.

The present paper addresses the reduction of effort for seismic analysis of subsystems by modifying the free vibration analysis of the main structure (Ref.4), by avoiding the construction of floor response spectra and by shifting to a higher degree of computational automation. The other aspect is the improvement of accuracy by attending to the list of desirable features mentioned above through the construction of a dynamic modal main structure - subsystem combination.

ANALYSIS OF THE MAIN STRUCTURE

The main structure is characterized by three system matrices, the mass matrix $\underline{M}_m$, the damping matrix $\underline{C}_m$ and the stiffness matrix $\underline{K}_m$. The loading consists of a base acceleration time history $\underline{\eta}\,\underline{\ddot{U}}_g(t)$. The matrix $\underline{\eta}^m$ is the spatial distributions of the excitations throughout the main structure. For each separate time history there is one vector in the matrix $\underline{\eta}$.

The complete equation of motion of the main structure is given by

$$\underline{M}_m\underline{\ddot{U}} + \underline{C}_m\underline{\dot{U}} + \underline{K}_m\underline{U} = -\underline{M}_m\underline{\eta}\,\underline{\ddot{U}}_g(t) \tag{1}$$

This equation is then transformed by the introduction of the free vibration modes $\underline{\Psi}$ resulting from the eigenproblem

$$\left[\underline{K}_m - \underline{\omega}^2\underline{M}_m\right]\underline{\Psi} = \underline{0} \quad , \tag{2}$$

so that

$$\underline{U} = \underline{\Psi}\,\underline{X} \quad . \tag{3}$$

Seismic excitation can now be dealt with through either modal time history analysis or a modal response spectrum analysis (Ref.5) using the modal participation factors

$$\underline{L} = \underline{\Psi}^T\underline{M}_m\underline{\eta} \tag{4}$$

The use of free vibration modes decouples the equations through the diagonal matrices

$$\underline{M}_{xx} = \underline{\Psi}^T\underline{M}_m\underline{\Psi} = \underline{I} \tag{5a}$$

$$\underline{C}_{xx} = \underline{\Psi}^T\underline{C}_m\underline{\Psi} = \text{diag}\left[2\xi_i\omega_i\right] \tag{5b}$$

$$\underline{K}_{xx} = \underline{\Psi}^T\underline{K}_m\underline{\Psi} = \text{diag}\left[\omega_i^2\right] \tag{5c}$$

The computation of the free vibration modes $\underline{\Psi}_i$ and frequencies ω_i is very time consuming. More modes than the number of modes actually needed for a particular load distribution $\underline{\eta}$ are computed. At a later stage in the computation, for a large number of modes, the modal participation factors in eq.(4) are negligibly small.

Conventional dynamic substructuring methods are usually biased towards component free vibration modes (Refs.6,7). Early applications of dynamic substructuring techniques in large complex space flight structural systems (Refs.8,9) have used three types of modes

- component free vibration modes
- static displacement modes
- uniform acceleration modes

In particular the use of uniform acceleration modes has shown a significant advantage over free vibration modes and arbitrary static displacement modes (Ref. 10). Uniform acceleration modes are generated in a way similar to the Vianello-Stodola method for successive computation of approximations to free vibration modes (Refs.11,12).

The first uniform acceleration mode is computed from

$$\underline{K}_m \tilde{\underline{\Psi}}_1^* = \underline{M}_m \underline{\eta} \tag{6}$$

where $\underline{\eta}$ is the particular load spatial distribution. This vector is normalized by requiring

$$\tilde{\underline{\Psi}}_1^T \underline{M}_m \tilde{\underline{\Psi}}_1 = 1 \tag{7}$$

The higher modes are computed from the recurrence relation

$$\underline{K}_m \tilde{\underline{\Psi}}_i^* = \underline{M}_m \tilde{\underline{\Psi}}_{i-1} \tag{8}$$

The mode $\tilde{\underline{\Psi}}_i^*$ is then orthogonalized with respect to $\underline{M}_m$ to all previous modes

$$\tilde{\underline{\Psi}}_1^{**} = \tilde{\underline{\Psi}}_i^* - \sum_{j=1}^{i-1} c_j \tilde{\underline{\Psi}}_j \quad , \qquad c_j = \tilde{\underline{\Psi}}_j^T \underline{M}_m \tilde{\underline{\Psi}}_i^* \quad , \tag{9}$$

which is then normalized according to eq.(7)(Ref.10).

When a sequence of uniform acceleration modes are used for the modal transformation of the equations of motion, the transformed damping and stiffness matrices are not diagonal. By solving the eigenvalue problem

$$\left[\tilde{\underline{K}}_m - \underline{\omega}^2 \underline{I}\right] \underline{\alpha} = 0 \quad , \tag{10}$$

where

$$\tilde{\underline{K}}_m = \tilde{\underline{\Psi}}^T \underline{K}_m \tilde{\underline{\Psi}} \quad , \tag{11}$$

the equation of motion can again be diagonalized. This produces the modes which can now be used for transforming the original equation of motion. They are

$$\underline{\Psi} = \tilde{\underline{\Psi}} \, \underline{\alpha} \quad . \tag{12}$$

Since these uniform acceleration modes are directly related to the load pattern $\underline{\eta}$ and the mass distribution fewer of them are needed compared to the use of free vibration modes (Refs.8,9,10). The above procedure is called the WYD Ritz vector procedure (Ref.14). Improved versions of this procedure, called LWYD, have been investigated in (Ref.14) showing more stable orthogonality of the Ritz vectors generated. The computational procedure is much simpler and mor effective than the solution of large scale eigenvalue problems. Eqs.(6) through (12) have to be computationally performed, of course, for each load pattern $\underline{\eta}$. Earthquake loadings require one load pattern for each of the three translatory motions. Base rotations require additional vectors $\underline{\eta}$.

Modal time history and modal response spectrum analysis can be carried out as usual with the additional advantage, however, that no correction for modal truncation errors is required. Even if there is a large number of load patterns the approach described here is still more effective than the use of free vibration modes

ANALYSIS OF THE SUBSTRUCTURE

The substructure is described by its system matrices $\underline{M}_s$, $\underline{C}_s$ and $\underline{K}_s$ for mass, damping and stiffness, respectively. The excitation of the substructure is entirely due to support acceleration. The equation of motion of the substructure is given by

$$\underline{M}_s \underline{V} + \underline{C}_s \underline{V} + \underline{K}_s \underline{V} = \underline{\eta}_z \underline{F}_z(t) \tag{13}$$

The vector $\underline{\eta}$ indicates each support degree of freedom, one at a time. The functions $\underline{F}_z(t)$ are the corresponding interface forces acting on the supports.

The substructure displacements $\underline{V}$ are transformed by two typed of modes. The substructure free vibration modes assume all support points to be fixed. They are computed from

$$\left[\underline{K}_s - \omega^2 \underline{M}_s\right] \underline{\Phi} = \underline{0} \quad . \tag{14}$$

In addition, static displacement modes $\underline{\eta}$ are computed by assuming a unit displacement at each support degree of freedom, one at a time,

$$\underline{K}_s \underline{\eta} = \underline{K}_z \underline{\eta}_z \quad . \tag{15}$$

The matrix contains the support springs. The substructure displacement vector $\underline{V}$ is thus approximated by

$$\underline{V} = \underline{\Phi}\ \underline{Y} + \underline{\eta}\ \underline{Z} \quad . \tag{16}$$

The modes $\underline{\Phi}$ can be computed in the same way as suggested for the main structure, from eqs.(6) through (12), using the substructure matrices $\underline{K}_s$ and $\underline{M}_s$ and the vectors $\underline{\eta}$ obtained from the static solution of eq.(15). For each support degree of freedom a set of modes $\underline{\Phi}$ derived from uniform acceleration modes is required. Most interior subsystems are attached to the main structure of a power plant at just a few points so that the number of vectors $\underline{\eta}$ remains low. Possibly, for a large number of attachment points, however, the regular free vibration modes may be easier to handle. The diagonal matrices of the transformed problem are

$$\underline{M}_{yy} = \underline{\Phi}^T \underline{M}_s \underline{\Phi} = \underline{I} \quad , \tag{17a}$$

$$\underline{C}_{yy} = \underline{\Phi}^T \underline{C}_s \underline{\Phi} = \text{diag}\ 2\zeta_j \omega_j \quad , \tag{17b}$$

$$\underline{K}_{yy} = \underline{\Phi}^T \underline{K}_s \underline{\Phi} = \text{diag}\ \omega_j^2 \quad . \tag{17c}$$

The other matrices involve products with the matrix $\underline{\eta}$:

$$\underline{M}_{zz} = \underline{\eta}^T \underline{M}_s \underline{\eta} \ , \quad \underline{M}_{yz} = \underline{M}_{zy}^T = \underline{\Phi}^T \underline{M}_s \underline{\eta} \ , \tag{18}$$

and similarly for $\underline{C}_{zz}$, $\underline{C}_{yz}$, $\underline{K}_{zz}$ and $\underline{K}_{yz}$.

MODAL SUBSTRUCTURE MODEL

The system to be analyzed consists of a main structure (power plant building) and a major substructure (piping system, mechanical somponent or equipment). The two modally transformed matrix models are coupled via the interface forces $\underline{F}_z(t)$ (Refs.15,16). For the main structure the modal equation of motion is

$$\ddot{\underline{X}} + \underline{C}_{xx} \dot{\underline{X}} + \underline{K}_{xx} \underline{X} = - \underline{L}\ \ddot{\underline{U}}_g(t) - \underline{\Psi}_z^T \underline{F}_z \tag{19}$$

in which $\underline{\Psi}_z$ is the modal matrix evaluated at the interface locations of the substructure. For the substructure the modes $\underline{\Phi}$ are fixed at the support points and the support motion is described by the vectors $\underline{\eta}$. The subsystem modal equation of motion is

$$\begin{bmatrix} \underline{I} & \underline{M}_{yz} \\ \underline{M}_{zy} & \underline{M}_{zz} \end{bmatrix} \begin{bmatrix} \ddot{\underline{Y}} \\ \ddot{\underline{Z}} \end{bmatrix} + \begin{bmatrix} \underline{C}_{yy} & \underline{C}_{yz} \\ \underline{C}_{zy} & \underline{C}_{zz} \end{bmatrix} \begin{bmatrix} \dot{\underline{Y}} \\ \dot{\underline{Z}} \end{bmatrix} + \begin{bmatrix} \underline{K}_{yy} & \underline{K}_{yz} \\ \underline{K}_{zy} & \underline{K}_{zz} \end{bmatrix} \begin{bmatrix} \underline{Y} \\ \underline{Z} \end{bmatrix} = \begin{bmatrix} \underline{\eta}_z \underline{F}_z \\ 0 \end{bmatrix} \tag{20}$$

Both modal models are combined by eliminating the interface forces from eqs.(19) and (20) and observing that the interface motions are determined by the main structure modes $\underline{Z}=\underline{\Psi}_z\underline{X}$. The combined set of modal equations contains l generalized main structure coordinates $\underline{X}$ and m generalized substructure coordinates $\underline{Y}$. The n interface coordinates $\underline{Z}$ have been eliminated from the set of equations. The next step in constructing a modal substructure model concerns the diagonality of the generalized mass, damping and stiffness matrices. These matrices are diagonalized by neglecting the off-diagonal terms except the ones which couple main and substructure modes. This decouples the modal equation of motion to pairs of generalized coordinates, X_i and Y_i,

$$\begin{bmatrix} 1+m_{ii} & m_{ij} \\ m_{ji} & 1 \end{bmatrix}\begin{bmatrix} \ddot{X}_i \\ \ddot{Y}_j \end{bmatrix} \begin{bmatrix} 2\xi_i\omega_i+c_{ii} & c_{ij} \\ c_{ji} & 2\xi_j\omega_j \end{bmatrix}\begin{bmatrix} \dot{X}_i \\ \dot{Y}_j \end{bmatrix} \begin{bmatrix} \omega_i^2+k_{ii} & k_{ij} \\ k_{ji} & \omega_j^2 \end{bmatrix}\begin{bmatrix} X_i \\ Y_j \end{bmatrix} = \begin{bmatrix} -L_i\ddot{U}_g(t) \\ 0 \end{bmatrix} \tag{21}$$

The coefficients m_{ii}, m_{ij}, c_{ii}, k_{ij} are computed from

$$m_{ii} = \underline{\Psi}_{iz}^T \underline{M}_{zz} \underline{\Psi}_{iz} \qquad m_{ij} = \underline{\Psi}_{iz}^T \underline{\eta}^T \underline{M}_s \underline{\Phi}_j \quad , \tag{22}$$

and similarly for the damping and stiffness terms. There are l by m nested pairs of equations as indicated in eq.(21). The solution to the l by m pairs of coupled two-degree-of-freedom equations may proceed along the path of time history and response spectrum analysis. If a time history solution is desired each pair of modal equations of eq.(21) is integrated over time and the solutions superimposed onto each other.

RESPONSE SPECTRUM METHOD WITH THE COUPLED MODEL

The solution of eq.(21) by the response spectrum method requires three computational steps before the modal maxima can be superimposed following one of the various superposition rules.

First, the main structure response maxima are determined from a modal response spectrum solution of eq.(19) in which the substructure properties are contained only in an approximate manner in the same way as all the other substructures attached to the main structure are represented. From this response spectrum solution the modal displacement maxima are obtained as

$$\hat{\ddot{X}}_i^s = L_i S_a(\omega_i, \xi_i) \tag{23}$$

The interface forces in eq.(19) are momentarily disregarded. It is assumed that the choice of modes derived from the uniform acceleration modes will make the modal truncation error negligible. Seismic acceleration spectra are given as input to the analysis directly with no conversion to time histories necessary.

Second, a time history solution of the coupled two-degree-of-freedom equations is obtained in which a modal unit impulse load is applied on the main structure generalized coordinate X_i in eq.(21). The unit impulse response can also be obtained by subjecting the system to an initial modal velocity $\dot{X}_i(\tau)$ with the Y--components all at rest up to $t=\tau$. The response to this excitation is obtained either analytically as in (Ref.15) or can be computed numerically for all combinations of main structure modes i and substructure modes j. From these time history results the maxima $\hat{X}_i^o$ and $\hat{Y}_j^o$ are recorded.

Third, the maxima of modal response of the substructure are obtained by assuming that the ratios of the actual maxima $\hat{X}_i^s$ and $\hat{Y}_j^s$ are identical to the ratios of the maxima from the unit impulse function, $\hat{X}_i^o$ and $\hat{Y}_j^o$. With $\hat{X}_i^s$ known from eq.(23) and $\hat{X}_i^o$ and $\hat{Y}_j^o$, the desired modal response maxima are for the pair (i,j)

$$\hat{Y}_j^s(i,j) = (\hat{X}_i^s/\hat{X}_i^o)\hat{Y}_j^o \quad . \tag{24}$$

The contributions of all primary structure modes are found by superposition over all i modes. This formulation has been called "generalized response spectrum method" in (Ref.15). The main structure maxima $\hat{X}_i^s$ need to be computed only once for each main structure excitation. The number of required modes depends on the choice of modal transformation functions in the definition of the generalized degrees of freedom. The further superpositions follow the rules of conventional response spectrum methods. The main difference is the approach for finding the modal maxima of the substructure. The method can be easily incorporated in a general purpose finite element computer program.

CONCLUSIONS

The method to analyse substructure attached to main structures has been redesigned to improve accuracy and to reduce computational effort. There are several features of the present computational procedures which compare favorably with the conventional approach

- generation of uniform acceleration modes from load pattern by recurrence relation instead of large eigenvalue problem
- transformation to modal degrees of freedom with fewer generalized degrees of freedom
- construction of response maxima of the main structure from seismic input
- computation of coupled substructure response maxima for idealized load,i.e. unit impulse function, with nested 2-dof systems ("generalized response spectrum method")
- scaling of the idealized response from unit impulse loading of the 2-dof systems to match the actual response of the main structure
- superposition of all modal substructure maxima for each mode.

Numerical computations with uniforn acceleration modes and the partially diagonalized coupled 2-dof modal substructure show promising results.

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