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## SEISMIC RESPONSE OF FRAME STRUCTURES WITH MOVABLE LOADS

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### SUMMARY

In recent years, some investigations of aseismic property of large automated warehouses, especially, large scale shaking table tests have been carried out as reported in Ref.1-3, and the results have shown the occurrence of slip motion of heavy loads over a certain maximum excitation level which is about 100 Gal, and consequently the remarkable reduction of frame response, in other words, a large equivalent damping effect. In this study, such dynamic effects of slip motion have been investigated by means of analytical simulations and experimental verifications.

### INTRODUCTION

A slipping of heavy loads on the floor during strong earthquakes could affect on the dynamic behaviour of supporting main frame such as a large scale automated warehouse. Vibration characteristics of this type structure is the variation of inertia mass with time and effect of friction due to slipping. Some basic studies have been conducted in this research field (such as Ref.4,5) in the past years.

The author have applied the basic idea already known on slip motion and friction effect to the analysis of multi-story frame structures and verified by small scale experiment. This method was also applied to the real scale structure tested by a large shaking table (Ref.1), and found to be useful for a large model. In a large model analysis, a pseudo-force technique was used for calculation. In this study, unlimited slip motions of load masses were supposed, and then the collision of a mass and frame was neglected.

### ANALYSIS METHOD AND EXPERIMENT

Basic Equation of A Slip Model A former described type of structure is simply represented by a vibration model (called a slip model here) as Fig.1. Mass matrix  $[M]$  (supposed diagonal) is usually considered as fixed values in vibration analysis of multi-degree of freedom system. But, in a slip model, a part of  $[M]$  can go into slip motion over a certain response level, and effective mass matrix of frame is given by

$$[M_e] = [M] - [\xi][m] \quad (1)$$

where  $[m]$  represents movable parts of total mass  $[M]$ ,  $\xi_{i1} = \xi_{i2} = 0$  (no slipping at

node i),  $\xi_i=1$  (slipping at i). Considering this and effect of friction, basic equations of a slip model of multi-story structure are given as follows:

(frame)

$$[M]\{\ddot{x}_i\}+[C]\{\dot{x}_i\}+[K]\{x_i\} = -\ddot{u}[M]\{1\}+[\xi][m]\{\ddot{x}_i+\ddot{u}\}+\{F_i\} \quad (2)$$

(movable mass in slipping)

$$m_i(\dot{y}_i+\ddot{x}_i+\ddot{u})=-F_i \quad (3)$$

(dynamic friction force)

$$F_i=\overline{F}_i\xi_i, \quad \overline{F}_i=-\text{sgn}(\ddot{x}_i+\ddot{u})\cdot m_i g \mu_{di} \quad (4)$$

(beginning of slip)

$$\xi_i=0 \quad \text{and} \quad |\ddot{x}_i+\ddot{u}|>g\cdot\mu_{si}\rightarrow\xi_i=1 \quad (5)$$

(stop of slip)

$$\xi_i=1 \quad \text{and} \quad \dot{y}_i=0\rightarrow\xi_i=0 \quad (6)$$

where dynamic friction  $\mu_{di}$  is supposed as velocity independant, and  $\text{sgn}(\ast)$  represents the sign of  $\ast$  at the beginning of slip motion. The response can be estimated by methods such as a direct integration of above equations.

Experiment of 3 Storied Model Frame In order to assess the above analytical method, a small shaking table test was performed using a model as Fig.2. A heavy load mass on the each floor of this frame can move along the guide rail in free condition, and also can be fixed to the floor if necessary. Measurements of acceleration and slip displacement are shown in Fig.2. The natural frequencies were measured as the followings, and the damping values were about 0.2-0.4%.

f1=5.1Hz, f2=14.5Hz, f3=21.7Hz (no load)

f1=3.2Hz, f2= 9.1Hz, f3=13.5Hz (with fixed load)

The system constants estimated from these values are shown in Table 1. The friction constants  $\mu_{s1}$ ,  $\mu_{d1}$  on the each floor were estimated from acceleration records of load masses at the beginning and in the midst of slipping. Fig.3 shows a typical resonance curve measured by sinusoidal excitation of this slip model. This figure also shows the calculated resonance curve for mass-fixed model using measured damping values  $h1=h2=0.4\%$ ,  $h3=0.28\%$ . A slip effect can be seen near the resonances.

Fig.4 shows a comparison of numerical simulation and experiment by a modified earthquake input. The response of non-slip (mass fixed) model is also shown in this figure. This figure shows the fairly good agreement of simulation and experiment for slip model, and further, incorrect prediction by a non-slip model.

#### PSEUDO-FORCE METHOD FOR A LARGE SCALE MODEL

Pseudo-Force Method A direct integration method formerly applied will require too much time of computation for a large model. Besides, it is not suitable for measured modal constants. Then, a type of pseudo-force method (Ref.5) have been

applied here. Using this method, the basic equation (2) becomes the modal expression as follows:

$$\begin{aligned} [M_k]\{\ddot{q}\} + [C_k]\{\dot{q}\} + [K_k]\{q\} \\ = -\ddot{u}[M_k]\{\beta\} + [\phi]^T[\xi][m]\{\ddot{x}_i + \ddot{u}\} + [\phi]^T\{F_i\} \end{aligned} \quad (7)$$

where  $\{q\}$ : normal function,  $[\phi]$ : modal matrix,  $[\beta]$ : participation factor,  $k$ : mode,  $i$ : freedom.

The second and third term of right side of Eq.(7) represents a pseudo-force. This pseudo-force also depends upon the system response, and then, calculation of convergence for equality of both side of Eq.(7). The response of movable masses are directly calculated using the former equation (3). Fig.5 shows a comparison of pseudo-force method and direct integration method applied to the former 3-storied model. A pseudo-force method was much effective for saving calculation time of this type model over 20 degree of freedom.

A Large Model Simulation The above method was applied to estimate a large model response of Fig.6, which was tested by a large shaking table, using some input waves of Fig.7, constant modal damping and constant friction  $\mu_s=0.28$ ,  $\mu_a=0.24$ . From this analytical simulation, the maximum shear force versus input excitation level was estimated as Fig.9. The analytical prediction by slip model is close to the experimental value (Ref.1), and the figure shows the ceiling effect by slip motion as shown in Fig.10. Fig.11 shows the same manner estimations in different damping conditions. From these figures, slip motion effects would be expected at 100-200 Gal excitation level or over.

A Simple Estimation of Minimum Effective Input As described before, a slip model shows constant value of response over a certain level of input. This minimum effective input ( $A_e$ ) depends on structural parameters including friction. One of simple estimations can be given by the following:

$$A_e = \frac{\mu_s g}{\bar{a}} A \quad \bar{a} = \frac{1}{N} \beta_1 S_1(h_1, \omega_1) \sum_{k=1}^N |\phi_{k1}| \quad (8)$$

In above,  $A_e$  is determined as a input level for response  $\mu_s g$  when response  $\bar{a}$  to input  $A$ , and representative response  $\bar{a}$  is an averaged response acceleration of frame considering only the first mode. Table 2 shows a comparison of this estimation ( $\mu_s=0.28$ ) from response spectra of Fig.12 ( $A=341\text{Gal}$ ) and numerical simulation (Fig.11). It is seen above simplified method can give fairly reasonable values in this case.

## CONCLUSION

The results of the study are summarized as follows:

- 1) The analysis and experiment of three stories slip model showed the availability of a analytical model in which static and dynamic friction constants are used.
- 2) The method of pseudo-force modal superposition have been found to be applicable for a large model analysis.
- 3) The numerical simulation of a real scale model have explained well the results of large scale shaking table test.
- 4) A simplified estimation method of slip motion effect have been presented in a form of minimum effective input, and it's adequacy for rough estimation have been shown.

## ACKNOWLEDGMENTS

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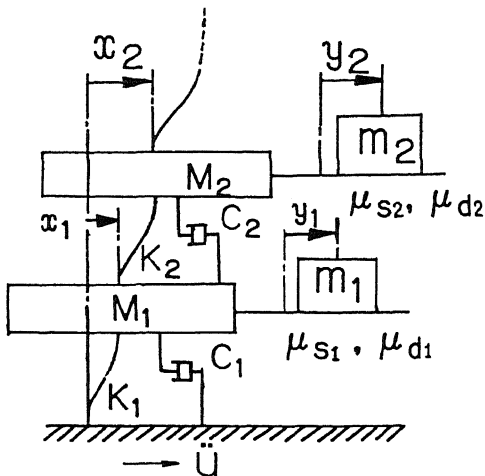


Fig.1 Vibration model (multi-story)

Tab.1 Measured properties of test model

i	$M_i$	$m_i$	$k_i$	$C_i$	$\mu_{s i}$	$\mu_{d i}$
	kgf·s <sup>2</sup> /cm	kgf·s <sup>2</sup> /cm	kgf/cm	kgf·s/cm		
1	0.019	0.023	68	0.01	0.272	0.231
2	0.019	0.023	96	0.01	0.387	0.355
3	0.015	0.023	92	0.01	0.278	0.231

$M_i$ : fixed mass of i-story  
 $m_i$ : movable mass of i-story  
 $k_i$ : spring const. of i-1-i story  
 $C_i$ : damping force coefficient of i-1-i story  
 $\mu_{s i}$ : static friction coefficient of i-story  
 $\mu_{d i}$ : dynamic friction coefficient of i-story

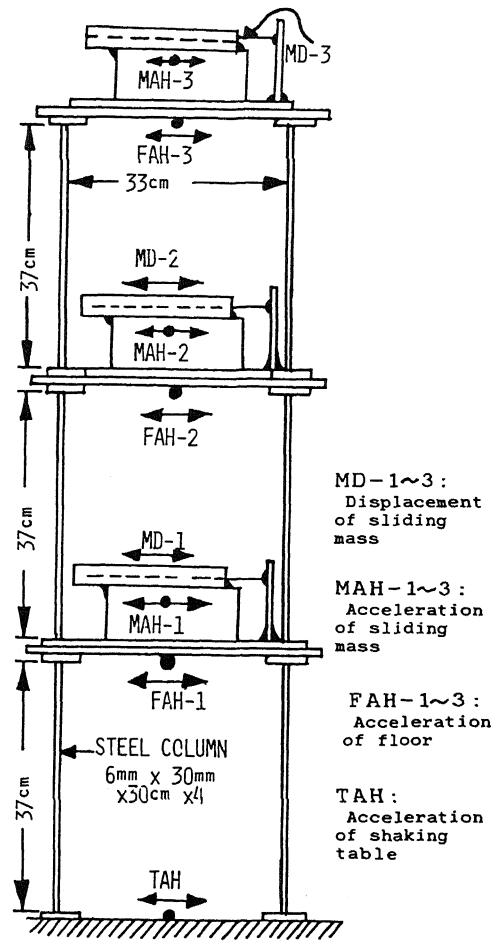


Fig.2 Multi-story test model

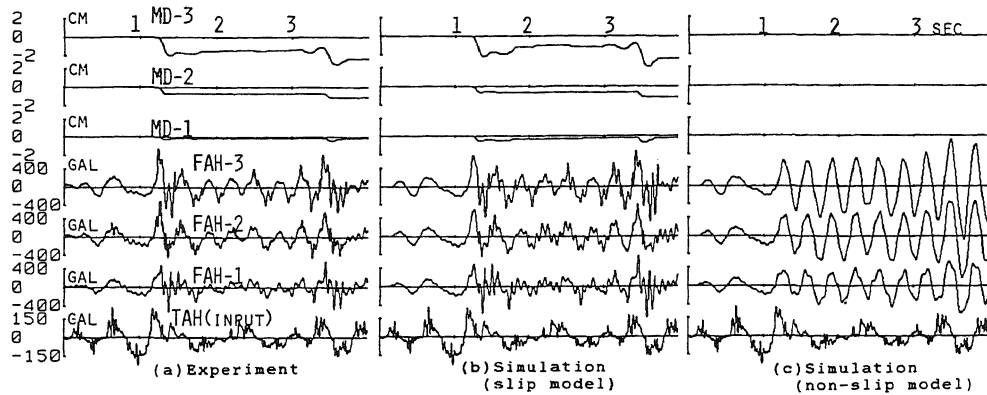


Fig. 4 Comparison of response waves of 3-story test model by experiment and simulation (input : table response wave by modified EL CENTRO NS)

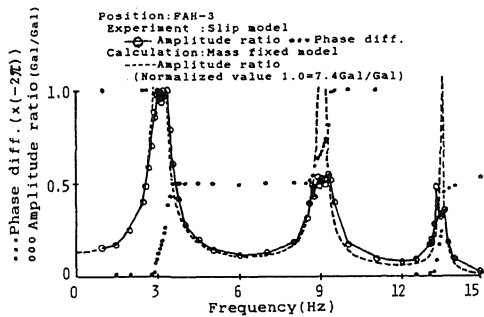


Fig. 3 Typical resonance curve of test model (fundamental amplitude of excitation : 50 Gal)

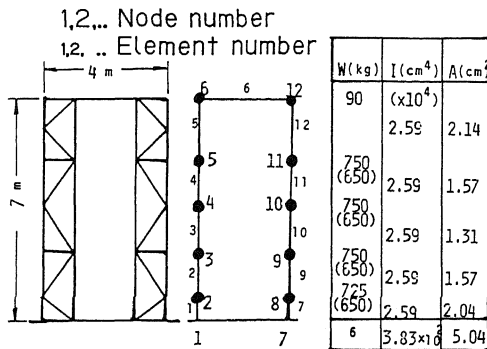


Fig. 6 Large scale model (by Ref. 1)  
 W : lumped mass, ( ) = movable part  
 A : effective area of shear  
 I : moment of sectional area

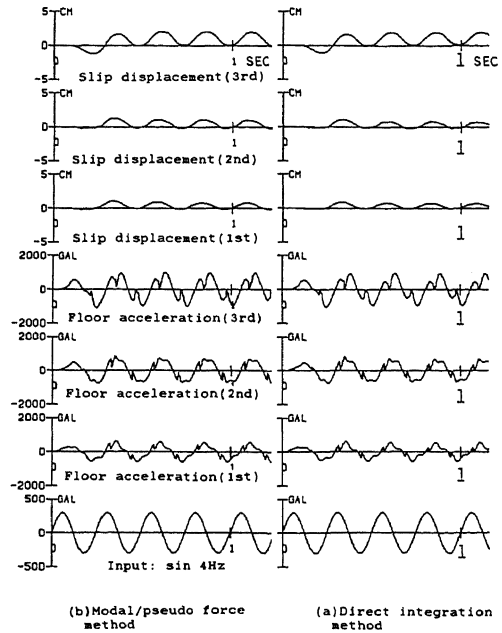


Fig. 5 Comparison of two simulation methods (3-story test model)

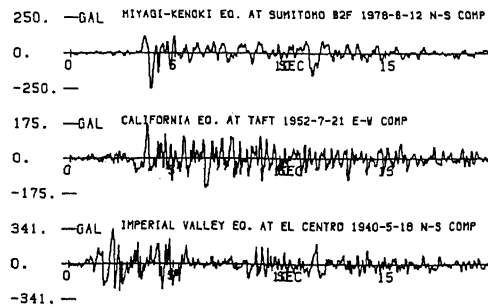


Fig. 7 Input waves for simulation

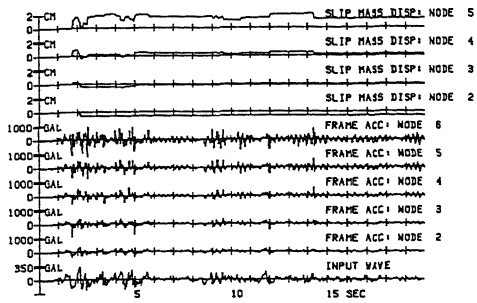


Fig.8 Typical response waves by earthquake excitation (EL CENTRO 350 Gal)

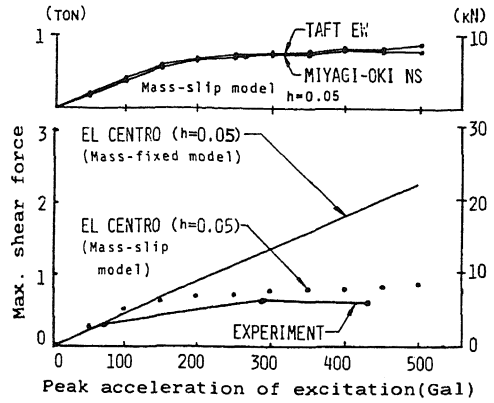


Fig.9 Comparison of response shear forces by large scale experiment and simulation of slip model

Experiment : average measured value of elements 2, 9 (Ref. 1)

Simulation : response value at element 2

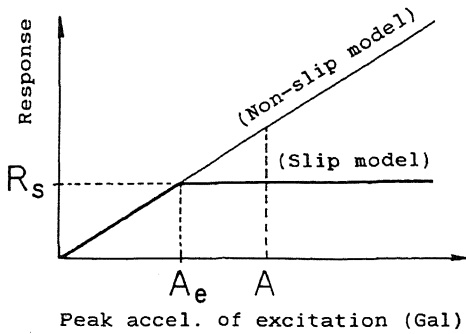


Fig.10 Schematic illustration of slip motion effect (by Ref. 1)

$A_e$  : minimum effective value of input  
 $A$  : design level of input

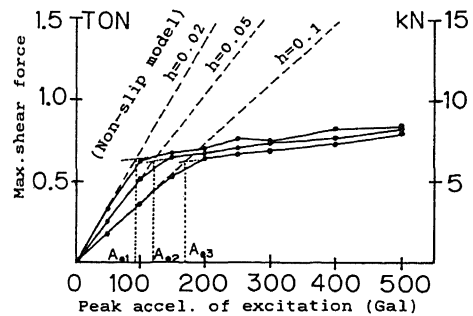


Fig.11 Effect of damping value (at element 2,  $\mu_s=0.28$ ,  $\mu_d=0.24$ , input : ELCENTRO)

Tab.2 Calculation of minimum effective input by simplified estimation method and comparison with the result of simulation.

Node	$\phi_{K1}$	$\beta_1$	$S_A(h_1, \omega_1)$ Gal		
			$h=0.02$	$h=0.05$	$h=0.1$
2	0.026	2.011	1060	850	650
3	0.169	"	"	"	"
4	0.41	"	"	"	"
5	0.607	"	"	"	"
6	0.854	"	"	"	"
$A_e$ by Eq. (8)			106Gal	132	173
$A_e$ by FIG.-11			93Gal	123	168

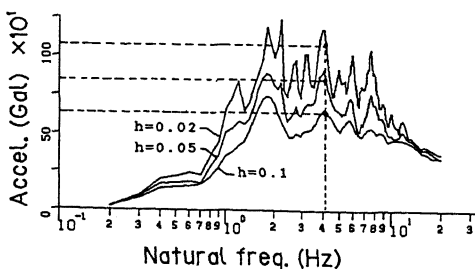


Fig.12 Response spectrum of input acceleration (ELCENTRO NS, max=341 Gal, 20 sec)  
 1st natural frequency of the model=4.1 Hz