A NETWORK MODEL WITH SUBSTRUCTURES
FOR SEISMIC ANALYSIS OF R/C FRAMES

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SUMMARY

A network model and a related computer program for nonlinear seismic analysis of plane reinforced concrete frames, presented in a previous paper, are here improved by a substructuring technique, so that computing time is drastically reduced and preparation of input becomes simpler and more systematic. Improved model and program are applied in a simple, economical and, at same time, accurate way, to seismic analysis of buildings with a soft first storey.

INTRODUCTION

Recently, a network model and a related computer program, were presented (Ref. 1), for nonlinear seismic analysis of plane reinforced concrete frames. This model and program were applied to seismic analysis of an one-bay, two-storey frame, for which experimental data are available (Ref. 2). And agreement between computational and experimental results was satisfactory. However, above model and program consumed a lot of computing time and had a complicated way of preparation of input data. In order to cure these deficiencies, an improved model and a related computer program were developed.

In new model, regions of frame joints are assumed rigid, as shown in Fig. 1a. So, network model of frame splits into substructures, one for every beam. As will be shown in following, this substructuring has as consequences, first, a significant saving of computing time and, second, a simpler and more systematic preparation of input data.

Improved model and program can be, in a simple, economical and, at same time, accurate way, applied to seismic analysis of buildings with a soft first storey. Indeed, as shown in Fig. 2, upper part of building can be roughly assumed as a big rigid joint of the frame, as the details of its dynamic behavior are not of interest. On the contrary, columns of soft first storey, which constitute the most sensible, to earthquake, part of the building, are accurately simulated by networks.

In present paper, improved, by substructures, model and program are briefly documented. And some applications of the above are given, with emphasis to buildings with a soft first storey.
DESCRIPTION OF NETWORK MODEL

In Fig. 1, it is shown how a plane reinforced concrete frame can be simulated by proposed here network model with substructures. Regions at frame joints are assumed rigid; to every rigid joint, mass m and rotational inertia \( m_\phi \) of neighboring part of frame are assigned (Fig. 1a). The beams (columns and girders) are first divided to rectangles, which are, then, simulated by networks (Fig. 1b), with bars obeying nonlinear uniaxial stress-strain laws of concrete and steel (Fig. 1c, d). In Ref. 1, formulae are given for determination of sections of bars of model. Rigid frame joints, as well as nodes of network substructures, can bear static loads, like P in Fig. 1a. Seismic excitation is introduced as known history of ground displacements, like \( u \) in Fig. 1a, with respect to time \( t \) (Fig. 1e).

INPUT AND OUTPUT OF SEISMIC ANALYSIS

Input data for seismic analysis of proposed network model are the following:
- For every frame joint: support conditions (free joint or ground support), initial coordinates \( X, Y \) of its center, mass \( m \) and rotational inertia \( m_\phi \), static loads \( P_X, P_Y \).
- For every network substructure, first, input data of every node are given: support conditions (free node or connected to a frame joint), initial coordinates \( x, y \) and static loads \( P_X, P_Y \); then, for every bar, are given: the two nodes it connects, material (concrete or steel), section \( A \) and underformed length \( l_0 \).
- Histories of seismic ground displacements \( u_X, u_Y \) with respect to time \( t \).

Output data of seismic analysis are the histories of coordinates \( X, Y \) and inclinations \( \phi \) of frame joints, and of coordinates \( x, y \) of nodes of network substructures, as well as the histories of any other variables dependent on the above ones.
EQUATIONS OF THE PROBLEM

For every free node of a network substructure, two equations of static equilibrium along $x, y$ are considered. Whereas, for every free rigid frame joint, three equations of dynamic equilibrium are considered, corresponding to translations along $x, y$ and rotation; damping is omitted.

Geometric, constitutive (stress-strain), static and dynamic equations of problem, with input data, form an initial value problem. For its numerical integration, we shall use a step-by-step algorithms. So, time $t$ is divided to intervals $\Delta t$, which are small enough, so that, within each of them, we can assume that all equations of problem are linear. For every bar, we write, within $\Delta t$, the simple linear incremental stress-strain relation $\Delta \sigma = E \Delta \varepsilon$, where $E$ is called tangential elasticity modulus of the bar and is constant within $\Delta t$.

We form the state vector $\mathbf{y} = (x, v)$ where $x$ includes coordinates and inclination $x, y, \phi$ for every frame joint and $v = \mathbf{f}$.

Now, dynamic equations of problem can be written:

$$\ddot{y} = q(y) \quad \text{or} \quad \begin{bmatrix} x' \\ v' \end{bmatrix} = \begin{bmatrix} v \\ y(x) \end{bmatrix}$$

(1)

$$\dot{y} = M^{-1} \mathbf{f}$$

(2)

where $y(x) = M^{-1} f(x)$.

$\gamma$ contains accelerations of free joints, $M$ is the diagonal mass matrix and $f$ contains forces and moment $F_x, F_y, M$ for every frame joint. Function of eq.(2) is known and linear within $\Delta t$.

In order to integrate numerically the above ordinary differential equation (1), we can use the step-by-step algorithm of trapezoidal rule, combined with predictor-corrector technique, with two corrections per step, as recommended in Ref. 1.

DESCRIPTION OF THE PROGRAM

Based on the above chosen algorithm, a simple and short computer program (435 Fortran instructions) has been developed. This program, consisting of main program and six subroutines, is briefly described below.

First, the main program is described, following flow-chart of Fig. 3. Beginning of algorithm, input data are read as mentioned previously. Then, subroutine NONL is called to determine, on nonlinear accurate basis, variables of the problem corresponding to time $t = 0$. Also, subroutine STIF is called to form tangential stiffness matrices of substructures, for $t = 0$.

For every degree of freedom of every joint, a small displacement $\Delta x, \Delta y$ or $\Delta \phi$ is performed. Each time, subroutine LIN is called to find corresponding acceleration increments $\Delta y$ of joints. So, we form the Jacobian matrix $\Delta y/\Delta x = M^{-1} \Delta f/\Delta x = M^{-1} K$. Then, as recommended in Ref. 1, we find an upper bound for eigenfrequencies of structure $\omega^2 = || M^{-1} K ||$ and determine the constant time steplength of algorithm $\Delta t = 2.0 \text{ rad}/\omega_0$.

Any step of algorithm. First, increments of seismic ground displacements, corresponding to present $\Delta t$, are performed. Then a step of algorithm of trapezoidal rule, combined with predictor-corrector technique, with two corrections, is applied, as shown in flow-chart of Fig. 3. We observe that, in this P-C process, subroutine LIN is called three times to estimate function $q(y)$ which reduces to $y(x)$ (eq. 1, 2).

Subroutine LIN is called, for a fourth time, in order to find the displacements $\Delta x, \Delta y$ of nodes of network substructures.
After updating of coordinates \( X, Y, \phi \) of frame joints and \( x, y \) of nodes of network substructures, subroutine NONL is called to calculate the variables of problem, corresponding to present time instant; any of these variables can be printed out.

Then, subroutine STIF is called to form new tangential stiffness matrices of network substructures. And we proceed to next step of the algorithm.

**End of algorithm.** When present time \( t \) exceeds total time interval \( t_n \) in which we wish to study dynamic behavior of structure, algorithm is interrupted.

Present program has the following six subroutines.

Subroutine NONL applies nonlinear accurate equations of problem. From coordinates \( x, y \) of nodes, strains \( \epsilon \) of bars are calculated. For every bar, subroutine CONCR or STEEL is called to update its stress \( \sigma \) and tangential elasticity modulus \( E_t \). Then, axial force of bar is found, \( N = \sigma \Delta \), and projected along \( x, y \). These projections are assembled to forces and moments \( F_x, F_y, M \) of frame joints; from these, finally, accelerations \( \ddot{x}, \ddot{y}, \ddot{\gamma} \) of frame joints are found.

Subroutine STIF forms, for every network substructure, its tangential stiffness matrix.

Subroutine LIN performs linear transformations. From displacements \( \Delta x, \Delta y, \Delta \phi \) of frame joints, first, displacements \( \Delta x, \Delta y \) are found for these nodes of network substructures which are connected with frame joints. Every network substructure is statically analysed, for the above support displacements, by calling of subroutine GAUS, and displacements \( \Delta x, \Delta y \) of its free nodes are obtained. From these, reactions \( \Delta F_x, \Delta F_y \) of support nodes are found, which are assembled to forces and moments \( \Delta F_x, \Delta F_y, \Delta M \) of frame joints; these, finally, give acceleration increments \( \Delta x, \Delta y, \Delta \gamma \) of frame joints.

Subroutine GAUS solves a linear algebraic system.

Subroutine CONCR describes the nonlinear unaxial stress-strain law of a concrete bar.

Subroutine STEEL describes the nonlinear unaxial stress-strain law of a steel bar.

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**Fig. 3:** Flow-chart of main program
Fig. 4: Second application. Seismic analysis of a building with a soft first storey.

APPLICATIONS

1. The same one-bay, two-storey frame of Ref. 1, for which experimental data are available (Ref. 2), was analysed by the improved model and program and gave again satisfactory agreement between computational and experimental results. As in Ref. 1, in improved model, too, attention must be paid to reduction of widths of beams.

Improved program run in an IBM-4381 computer, with a speed of 20 sec of dynamic analysis per 1 hr of CPU time, that is, it proved three times faster than program of Ref. 1. This saving of computing time is justified by the fact that, the big global algebraic system of static equilibrium, solved four times within each step of the algorithm (each time that subroutine LIN is called), splits into small systems, one for every substructure.

Another improvement of new program is that preparation of input data becomes, thanks to substructuring, simpler and more systematic, mainly because we are relieved now from the care of optimum numbering of nodes (needed in Ref. 1, in order to minimize the bandwidth of global stiffness matrix of structure).

2. In Fig. 4a–d, input data are given for seismic analysis of a plane frame of a building with a soft first storey, where, as mentioned previously, upper building is assumed rigid, whereas columns of first storey are accurately simulated by networks. This application run in an IBM-4381 computer with a step length $\Delta t = 16.7$ msec. This large $\Delta t$ is justified by the big mass of upper building.
which means low eigenfrequencies of structure, which, in turn, give a large $\Delta t$ and therefore, a significant saving of computing time. Present application spent only 5 min of CPU time.

In Fig. 4e-h, some diagrams are shown, based on output of present application. Fig. 4e describes the history of horizontal, relative to ground, displacement $\Delta u$ of upper building, whereas Fig. 4f the history of sum of shear forces EQ of columns.

Fig. 4g, resulting from combination of two previous figures, presents the diagram of variation of EQ versus $\Delta u$, by its virgin curve and envelope of hysteretic loops. Finally, Fig. 4h presents, for time $t = 5.1$ sec, the deformed configuration of the structure and its dynamic equilibrium under the action of self-weight and inertia forces of upper building, as well as ground reactions of columns.

3. In previous application were only changed: height of upper building to 24 m and mass to 800 t. This higher and heavier building was found resisting only until time = 5.0 sec, when strong seismic action begins, as shown in Fig. 4d, and, then, it collapsed.

CONCLUSIONS

1. A network model and a related computer program, for nonlinear seismic analysis of plane reinforced concrete frames, have been improved by a substructuring technique. Improved program is simple and short, and a brief documentation of it has been presented here.

2. Thanks to substructures, the big global algebraic system of static equilibrium, solved four times within each step of dynamic algorithms used, splits into small systems, one for every substructure; so, computing time is drastically reduced. Also, thanks to substructures, preparation of input data becomes simpler and more systematic, mainly because we are relieved from the care of optimum numbering of nodes.

3. Improved model and program can be, in a simple, economical and, at same time, accurate way, applied to seismic analysis of buildings with a soft first storey. Columns of first storey are accurately simulated by networks, whereas upper building is assumed rigid; and its big mass means low eigenfrequencies of structure, which, in turn, give a large time steplength $\Delta t$ of algorithm and, therefore, a significant saving of computing time.

REFERENCES