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THE STUDY ON THE COLLAPSE OF STEEL STRUCTURES SUBJECTED TO THE EARTHQUAKE GROUND MOTION

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SUMMARY

It is necessary that the phenomena of collapse in structures are cleared and the capacity of seismic resistant are estimated in based on the critical condition of collapse in structures. In this paper, the phenomena of collapse in structures is considered by using equations of motion for large deformation. And the critical cumulative ductility factor in collapse is defined in based on the results of analysis and the energy theory, and the fundamental character is illustrated by the simulation on steel frames subjected to earthquake ground motions.

INTRODUCTION

It is reported that many buildings were damaged or collapsed in Mexico Earthquake, 1985 and Miyagi-Oki Earthquake, 1978 and so on. The study on collapse phenomena has been developed (Refs. 1,2,3,4), but a little. In order to prevent earthquake disaster like this, it is important to study the collapsing process in the dynamic behaviours of structures which evolve from a virgin condition to a shake down condition. And it is reasonable that the capacity of seismic resistant in structures is estimated in based on collapse phenomena. The plastic failure of frames during eathquake has been discussed by Housner G.W.(Ref. 1). And the collapse criterion for multi-story steel frames are developed by Kato B. and Akiyama H.(Ref. 2) under the assumption that the frame arrives at collapsing condition when the restoring force becomes zero. In this paper, the non-linear differential equations of motion are introduced in order to investigate the process of collapse during earthquakes. The effects of gravity and geometrical non-linearity for large deformation in collapsing condition are considered in these equations. On the view point of energy theory, collapse phenomena is examined in detail from the results of numerical calculation on 1-story frames. And the critical cumulative ductility factor is defined as the maximum allowable displacement in which structure does not collapse. From results of simulation for steel frames of 2~5-story, charactristics of critical cumulative ductility factor is discussed.

EQUATIONS OF MOTION

In order to study on the collapsing process, the gravity and geometrical non-linearity should be considered in based on the equations of motion for multi-story structures, and differential equations of motion are introduced

by Lagrange's equations. The multi-story frames are idealized as multi-degree of freedom system shown in Fig. 1. In which, m_i, h_i, and f_i denote mass, height of story, restoring force, and joint translation angle respectively.
Let us notice, the restoring force f_i applies to normal direction to columns shown in Fig. 1. The total kinetic energy T and potential energy V of this system are given by Eq.(1).

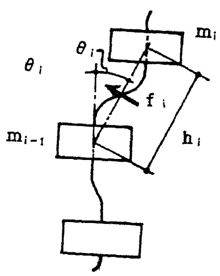
$$\begin{aligned} T &= \frac{1}{2} \Sigma_{i=1}^{n} \{ (\Sigma_{k=i}^{n} m_i) h_i^2 \dot{\theta}_i^2 \} \\ &+ \Sigma_{i=2}^{n} [(\Sigma_{l=i}^{n} m_l) h_i \dot{\theta}_i \{ \Sigma_{k=1}^{i-1} \dot{\theta}_k h_k \cdot \cos(\theta_k - \theta_i) \}] \\ V &= \Sigma_{i=1}^{n} \int f_i (h_i \theta_i)\, d(h_i \theta_i) - g \Sigma_{i=1}^{n} [m_i \Sigma_{l=1}^{i} h_l \{ (1 - \cos\theta_l) \}] \end{aligned} \quad (1)$$

Where, g denote the acceleration of gravity.

If the vertical component of earthquake ground motions is neglected, the virtual work by ground motions is given by Eq.(2).

$$\delta W_1 = -\ddot{y}_G \Sigma_{i=1}^{n} \{ m_i \Sigma_{l=1}^{i} (h_l \delta\theta_l \cdot \cos\theta_l) \} \quad (2)$$

Where, $\ddot{y}_G$ denote acceleration of ground motion.



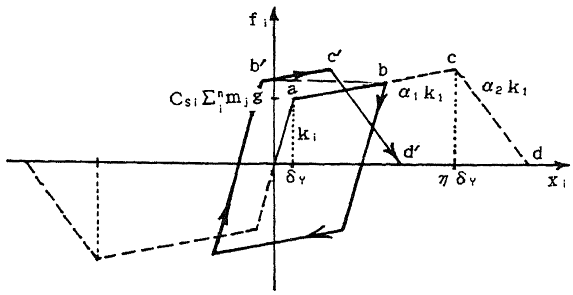


Fig. 1 Model Fig. 2 Characteristics of Restoring force in Steel Structures

And also, the virtual work by damping is given in Eq.(3).

$$\delta W_2 = \Sigma_{i=1}^{n} (C_i h_i^2 \dot{\theta}_i \cdot \delta\dot{\theta}_i) \quad (3)$$

Where, C_i denote the coefficients of viscous damping.

Substituting Eq.(1), (2) and (3) into the Lagrange's equation, the equations of motion for multi-degree of freedom system are derived as follows:

$$\begin{aligned} &(\Sigma_{k=i}^{n} m_k) \Sigma_{l=1}^{i} \{ h_l \ddot{\theta}_l \cdot \cos\cdot(\theta_l - \theta_i) - h_l \dot{\theta}_l^2 \cdot \sin(\theta_l - \theta_i) \} \\ &+ \Sigma_{l=i+1}^{n} [(\Sigma_{k=l}^{n} m_k) \{ h_l \ddot{\theta}_l \cdot \cos(\theta_i - \theta_l) - h_l \dot{\theta}_l^2 \cdot \sin(\theta_i - \theta_l) \}] \\ &+ C_i h_i \dot{\theta}_i + f_i (h_i \theta_i) - g (\Sigma_{k=i}^{n} m_k) \sin\theta_i \\ &= -(\Sigma_{k=i}^{n} m_k) \ddot{y}_G \cdot \cos\theta_i \qquad (i=1,2,3,\cdots\cdots,n) \end{aligned} \quad (4)$$

Eq.(4) are integrated numerically by Runge-Kutta's method in this paper. In the case that deflection is small, it is apparent that linear equations of motion is derived by substituting $\sin(\theta_i - \theta_l)=0$ and $\cos(\theta_i - \theta_l)=1$ into Eq.(4). The characteristics of restoring force for each story are illustrated in Fig. 2, in which C_{si} denote the shearing force coefficient for i-th story.

PROCESS OF COLLAPSE

The dynamical behaviour of a structure which evolves from a virgin condition to a shake down condition by ground motions is discussed for a 1-story steel frame. In this frame, the natural period is 0.8 sec., and the characteristics of restoring force are assumed as $\alpha_1=0.0$, $\alpha_2=-0.1$, $\eta=16$ and $C_{s1}=0.2$ (see Fig. 2). And also, El-Centro, May 1940, NS is used as a earthquake ground motion. The history of joint translation angle and phase-plain trajectory are shown in Figs. 3 and 4. From these figures, the process in which a structure evolves at a shake down condition is classfied into three states. These states are considered by energy theory. If the joint translation angle in each story is assumed to be very small, the equilibrium of energy is given in Eq.(5).

$$E_V(t)+E_h(t)+E_U(t)=E_I(t) \qquad (5)$$

In which,

$$E_V(t)=m_i\dot{x}_i^2/2\ , \qquad E_h(t)=\int_t(C_i\dot{x}_i-C_{i+1}\dot{x}_{i+1})\dot{x}_i dt$$

$$E_U(t)=\int_t(F_i-F_{i+1})\dot{x}_i dt, \qquad E_I(t)=-\int_t m_i(y_G+X_{i-1})\dot{x}_i dt$$

$$F=f_i-g(\Sigma_{k=i}^{n} m_k)x_i, \qquad X_{i-1}=\Sigma_{k=1}^{i} x_k$$

Moreover, $E_U(t)$ in Eq.(5) becomes following equation by using $x_i=x_{ei}+x_{pi}$.

$$E_U(t)=\int_t(F_i-F_{i+1})\dot{x}_{ei}dt+\int_t(F_i-F_{i+1})\dot{x}_{pi}dt=E_{eU}(t)+E_{PU}(t)$$

Where, x_{ei} and x_{pi} denote elastic and plastic compornent of x_i respectively.

Let us now consider the derivative of $E_{PU}(t)$ with respect to time and try to correlate the derivative of $E_{PU}(t)$ with three states shown Fig. 3, namely,

$$E_{PU}\begin{cases} =0 & : \text{STATE 1} \\ \geqq 0 & : \text{STATE 2} \\ \leqq 0 & : \text{STATE 3} \end{cases} \qquad (6)$$

And also,

$$E_{PU}\begin{cases} =0 & : \text{the state in elastic response} \\ \neq 0 & : \text{the state in elasto-plastic response} \end{cases} \qquad (7)$$

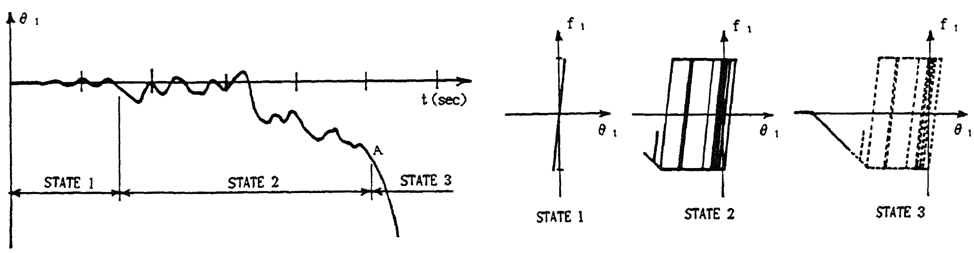
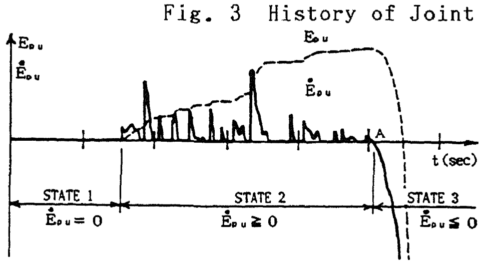


Fig. 3 History of Joint Translation Angle



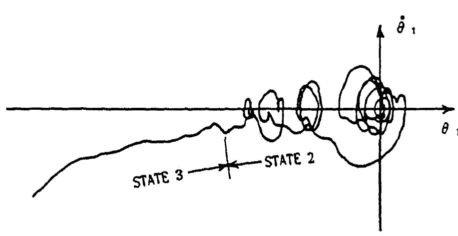


Fig. 5 $E_{PU}(t)$, $\dot{E}_{PU}(t)$ and Three States

Fig. 4 Phase-Plain Trajectry

Then, the history of $E_{PU}(t)$ and partial derivative $E_{PU}(t)$ in the previous example are shown in Fig. 5. At the point A shown Fig. 5, the frame does not collapse yet. But in the STATE 3, the value of E_v+E_{eP} becomes to be grater than E_I-E_h and the frame evolves at collapse condition with the increase of the energy for the vibration. Therefore, the cumulative ductility factor at the point A (see Fig. 5) is defined as the critical cumulative ductility factor. Then the critical cumulative ductility factor is defined as the cumulative ductility factor in the condition that the joint translation angle or the relative displacement in a story is satisfied Eq.(8).

$$E_{PU}(t)=\{[\tilde{f}_i-(\frac{g}{h_i})(\frac{T_i^*}{2\pi})^2\tilde{\theta}_i]-\frac{f_{Yi+1}}{f_{Yi}}[\tilde{f}_{i+1}-(\frac{g}{h_{i+1}})(\frac{T_{i+1}^*}{2\pi})^2\tilde{\theta}_{i+1}]\}\dot{\theta}_{Pi}h_i f_{Yi}=0 \qquad (8)$$

In which,

$$\tilde{x}_i=x_i/x_{Yi},\qquad \tilde{f}_i=f_i/f_{Yi}$$
$$\tilde{\theta}_i=\tilde{x}_i/h_i,\qquad T_i^*=2\pi\sqrt{(\sum_{k=i}^{n}m_k)/k_i}$$

and f_{yi},x_{yi} denote the yield restoring force and displacement respectively.

NUMERICAL CALCULATION AND CONSIDERATION

The numerical calculation of Eq.(4) are carried out by means of Runge-Kutta's method. The critical cumulative ductility factor(CCDF) are estimated by Eq.(8), and the fundamental characteristics are considered. The following ground motions are used in this calculation: The impact wave and sin wave as be shown in Fig.6, Taft Calif., July 21, 1952, EW and El-Centoro, May 18, 1940, NS. In which, one revises the accelerations of earthquake ground motions to make the same average velocity spectrum, and leave the time axis.. The damping ratio is assumed to be h=0.02 for each story. And some kinds of restoring force chracteristics are used.

<u>The case of 1-story structures</u> When the structure is destroyed by one stroke of eathquake ground motion, critical cumulative ductility factor is introduced by analytical method. And relationship between critical cumulative ductility factor μ_c and virtual period T^* is shown in Fig.7, in which $T^*=2\pi\sqrt{m_1/k_1}=T_0$ (natural period). It is illustlated in Fig. 7 that the critical cumulative ductility factor increase with increasing α_1 in restoring force characteristics . And in the case of $\alpha_2<0$, critical ductility fator never become to be greater than a some value even if T^* decreases.

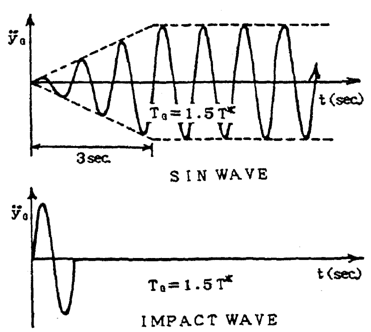


Fig. 6 Impact Wave and Sin Wave

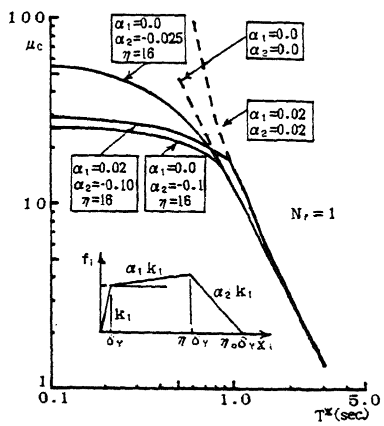


Fig. 7 Influence of Restoring Force Characteristics

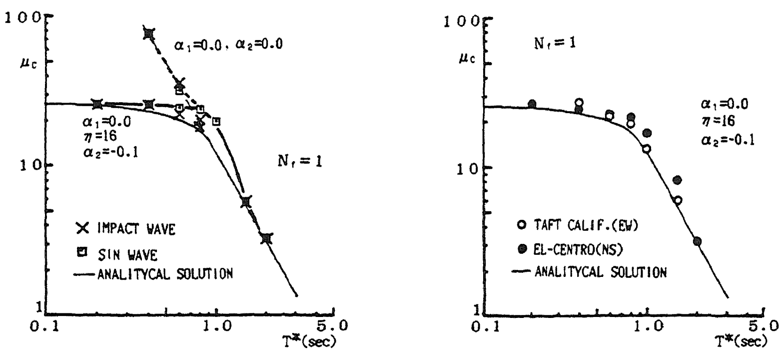
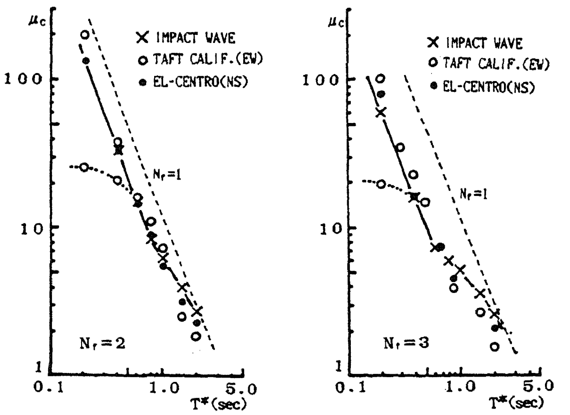


Fig.8 Effect of Repeating Load in Plastic Zone

From Fig. 8, it is shown that the CCDF μ_c in the case of sin wave is larger than the CCDF in the case of impact wave. It may be caused by the effect of repeating load in plstic zone. The CCDF for the frame subjected to Taft-Calif. and El-Centro are also shown in Fig. 8. The CCDF in the frame subjected to impact wave is quite agree with analytical solution, and is smaller than the other results.

The case of Multi-Story Frames For the case that the first story in 2~10-story frames are damaged, the CCDF μc are dicussed. The CCDF of multi-story frames is not given by analytical solusion .Then, the CCDF in 2 and 3-story frames are obtained by numerical calculation and are shown Figs. 9 and 11 repectively. But, the CCDF by using impact wave are smaller than ones by using actual earthquake ground motions in the range of $T^*<0.8$ sec..
The influence of number of stories on the critical cumulative ductility factor is illustrated in Fig. 10. From this figure, the CCDF becomes small value as increasing the number of stories. In relation to the frames of $T^*=0.5$ sec., the areas denoted by hatch in Fig. 11, show the energy which absorbed by the hysteresis of restoring force untill the CDF becomes the CCDF, in which $f_i=f_i/f_{yi}$ and $X_i=X_i/X_{yi}$. This means that absorbed energy becomes small as increasing the number of stories.
The relationship between critical cumulative ductility factor and mass ratio for 2-story frames are shown in Fig. 12. The virtual period of these frames is 1.0 sec. and 0.6 sec., m_1+m_2 is constant, and yield shearing force coefficient of each story are $C_{s1}=0.2$ and $C_{s2}=0.25$. According to Fig. 12, CCDF μ_c decreases with the increase of mass ratio m_2/m_1.



(a) 2-story frames (b) 3-story frames
Fig. 9 CCDF μ_c for Multi-Story Steel Frames

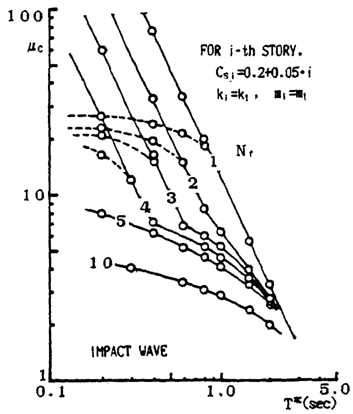


Fig. 10 Influence of the Number of Stories

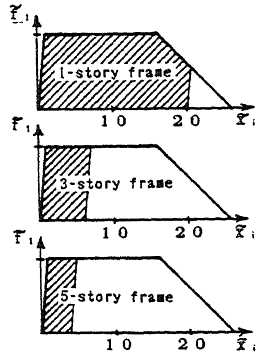


Fig. 11 Absorbed Energy

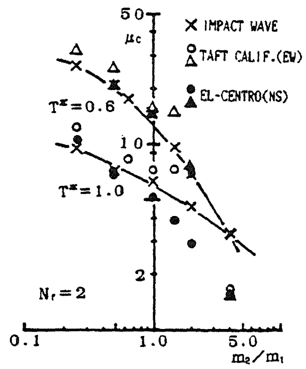


Fig. 12 μ_c and m_2/m_1

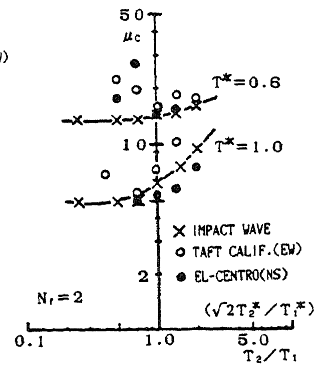


Fig. 13 μ_c and T_2/T_1

And also, The relationship between the critical cumulative ductility factor and ratio of period T_2/T_1 for 2-story frames is illustrated in Fig. 13. Where, the mass of these frames is $m_1=m_2$, and virtual periods and yield shearing force coefficient are the same as that of frames in Fig. 12. It is shown that CCDF μ_c increases by increasing the ratio of period T_2/T_1.

CONCLUSION

The collapse process are discussed by energy theory and by simulation on dynamical behaviours which evolve to a shake down condition.

(1) The critical cumulative ductility factor is defined as cumulative ductility factor in the condition that the related displacement is satisfied Eq.(8).

Next, the fundamental characteristics of critical cumulative ductility factor are investigated by numerical analysis, and the following results are obtained.

(2) The critical comulative ductility factor decreases increasing the virtual period of the story in where collapse occurs.

(3) The critical cumulative ductility factor becomes large value by the effect of repeating load in plstic zone during earthquake.

(4) When the number of upper stories from collapse story increases, the critical cumulative ductility factor decreases.

(5) The critical cumulative ductility factor is influenced by mass ratio and ratio of virtual period of each story. For 2-story steel frames, the critical cumulative ductility factor becomes large value when the mass raio decreases and the ratio of virtual period increases.

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