THE STUDY ON THE COLLAPSE OF STEEL STRUCTURES
SUBJECTED TO THE EARTHQUAKE GROUND MOTION

Hisashi OKADA¹ and Teruo ODAKA²

¹Department of Architectural Engineering, Aichi Institute of Technology,
Toyota, Aichi, Japan
²Department of Architectural Engineering, Aichi Institute of Technology,
Toyota, Aichi, Japan

SUMMARY

It is necessary that the phenomena of collapse in structures are cleared
and the capacity of seismic resistant are estimated in based on the critical
condition of collapse in structures. In this paper, the phenomena of collapse
in structures is considered by using equations of motion for large deformation.
And the critical cumulative ductility factor in collapse is defined in based
on the results of analysis and the energy theory, and the fundamental character
is illustrated by the simulation on steel frames subjected to earthquake
ground motions.

INTRODUCTION

It is reported that many buildings were damaged or collapsed in Mexico
Earthquake, 1985 and Miyagi-Oki Earthquake, 1978 and so on. The study on
collapse phenomena has been developed (Refs. 1,2,3,4), but a little. In order
to prevent earthquake disaster like this, it is important to study the
collapsing process in the dynamic behaviours of structures which evolve from
a virgin condition to a shake down condition. And it is reasonable that the
capacity of seismic resistant in structures is estimated in based on collapse
phenomena. The plastic failure of frames during earthquake has been discussed
by Housner G.W.(Ref. 1). And the collapse criterion for multi-story steel
frames are developed by Kato B. and Akiyama H.(Ref. 2) under the assumption
that the frame arrives at collapsing condition when the restoring force becomes
zero. In this paper, the non-linear differential equations of motion are
introduced in order to investigate the process of collapse during earthquakes.
The effects of gravity and geometrical non-linearity for large deformation
in collapsing condition are considered in these equations. On the view point
of energy theory, collapse phenomena is examined in detail from the results
of numerical calculation on 1-story frames. And the critical cumulative ductility
factor is defined as the maximum allowable displacement in which structure
does not collapse. From results of simulation for steel frames of 2~5-story,
characteristics of critical cumulative ductility factor is discussed.

EQUATIONS OF MOTION

In order to study on the collapsing process, the gravity and geometrical
non-linearity should be considered in based on the equations of motion for
multi-story structures, and differential equations of motion are introduced
by Lagrange's equations. The multi-story frames are idealized as multi-degree of freedom system shown in Fig. 1. In which, \( m_i \), \( h_i \), and \( f_i \) denote mass, height of story, restoring force, and joint translation angle respectively.

Let us notice, the restoring force \( f_i \) applies to normal direction to columns shown in Fig. 1. The total kinetic energy \( T \) and potential energy \( V \) of this system are given by Eq.(1).

\[
\begin{align*}
T &= \frac{1}{2} \sum_{i=1}^{n} \left( \sum_{k=1}^{n} m_i \dot{h}_i^2 \right) \\
&+ \sum_{i=1}^{n} \left( \sum_{k=1}^{n} m_i h_i \ddot{h}_i \left( \sum_{k=1}^{n} \delta_k + \frac{1}{2} \left( \cos \theta_i - 1 \right) \right) \right) \\
V &= \sum_{i=1}^{n} f_i (h_i, \theta_i) d (h_i, \theta_i) - g \sum_{i=1}^{n} m_i h_i \left( 1 - \cos \theta_i \right)
\end{align*}
\]

Where, \( g \) denote the acceleration of gravity.

If the vertical component of earthquake ground motions is neglected, the virtual work by ground motions is given by Eq.(2).

\[
\delta W_i = -\ddot{y}_i \sum_{i=1}^{n} \left( m_i \delta \dot{h}_i \cdot \cos \theta_i \right)
\]

Where, \( \ddot{y}_i \) denote acceleration of ground motion.

![Diagram](image)

Fig. 1 Model Fig. 2 Characteristics of Restoring force in Steel Structures

And also, the virtual work by damping is given in Eq.(3).

\[
\delta W_d = \sum_{i=1}^{n} \left( C_i h_i^2 \ddot{h}_i \cdot \delta \theta_i \right)
\]

Where, \( C_i \) denote the coefficients of viscous damping.

Substituting Eq.(1), (2) and (3) into the Lagrange's equation, the equations of motion for multi-degree of freedom system are derived as follows:

\[
\begin{align*}
\left( \sum_{k=1}^{n} m_i \right) \ddot{h}_i &+ \sum_{k=1}^{n} \left( \frac{1}{2} \left( \cos \theta_i - 1 \right) \right) \ddot{h}_i \\
&+ C_i h_i \delta + f_i (h_i, \theta_i) - g \left( \sum_{k=1}^{n} m_i \right) \sin \theta_i \\
&= - \left( \sum_{k=1}^{n} m_i \right) \ddot{y}_i \cdot \cos \theta_i \quad (i=1,2,3,\ldots,n)
\end{align*}
\]

Eq.(4) are integrated numerically by Runge-Kutta's method in this paper. In the case that deflection is small, it is apparent that linear equations of motion is derived by substituting \( \sin(\theta_i - \theta_i) = 0 \) and \( \cos(\theta_i - \theta_i) = 1 \) into Eq.(4). The characteristics of restoring force for each story are illustrated in Fig. 2, in which \( C_{m1} \) denote the shearing force coefficient for \( i \)-th story.
PROCESS OF COLLAPSE

The dynamical behaviour of a structure which evolves from a virgin condition to a shake down condition by ground motions is discussed for a 3-story steel frame. In this frame, the natural period is 0.8 sec., and the characteristics of restoring force are assumed as $\alpha=0.0$, $\xi=0.1$, $\eta=16$ and $C_m=0.2$ (see Fig. 2). And also, El-Centro, May 1940, NS is used as an earthquake ground motion. The history of joint translation angle and phase-plane trajectory are shown in Figs. 3 and 4. From these figures, the process in which a structure evolves at a shake down condition is classified into three states. These states are considered by energy theory. If the joint translation angle in each story is assumed to be very small, the equilibrium of energy is given in Eq. (5).

$$E_u(t) + E_s(t) + E_o(t) = E_i(t)$$  \hspace{1cm} (5)

In which,

$$E_u(t) = \sum_{n=1}^{\infty} \frac{m_i \dot{x}_i^2}{2}$$

$$E_s(t) = \int (C_1 \ddot{x}_i + C_2 \dot{x}_i \dot{x}_i) \dot{x}_i \, dt$$

$$E_o(t) = \int (F_i - F_{i-1}) \ddot{x}_i \, dt$$

$$F_i = f_i - g(\sum_{k=1}^{n} m_k) x_i$$

$$x_{i-1} = \sum_{k=1}^{i} x_k$$

Moreover, $E_o(t)$ in Eq. (5) becomes following equation by using $x_i = x_{i,1} + x_{i,p}.$

$$E_o(t) = \int (F_i - F_{i-1}) \ddot{x}_i \, dt + \int (F_i - F_{i-1}) \ddot{x}_p \, dt = E_{u1}(t) + E_{pu}(t)$$

Where, $x_{i,1}$ and $x_{i,p}$ denote elastic and plastic component of $x_i$ respectively.

Let us now consider the derivative of $E_{pu}(t)$ with respect to time and try to correlate the derivative of $E_{pu}(t)$ with three states shown Fig. 3, namely,

$$E_{pu} \begin{cases} 0 & \text{STATE 1} \\ 0 & \text{STATE 2} \\ 0 & \text{STATE 3} \end{cases}$$  \hspace{1cm} (6)

And also, $E_{pu} = 0$ : the state in elastic response

$$E_{pu} \neq 0$$ : the state in elasto-plastic response \hspace{1cm} (7)

Fig. 3 History of Joint Translation Angle

Fig. 5 $E_{PU}(t)$, $E_{PU}(t)$ and Three States

Fig. 4 Phase-Plain Trajectory
Then, the history of $E_{pU}(t)$ and partial derivative $E_{pU}(t)$ in the previous example are shown in Fig. 5. At the point A shown Fig. 5, the frame does not collapse yet. But in the STATE 3, the value of $E_{pU}$ becomes to be greater than $E_0$, and the frame evolves at collapse condition with the increase of the energy for the vibration. Therefore, the cumulative ductility factor at the point A (see Fig. 5) is defined as the critical cumulative ductility factor. Then the critical cumulative ductility factor is defined as the cumulative ductility factor in the condition that the joint translation angle or the relative displacement in a story is satisfied Eq.(8).

$$E_{pU}(t)=[(T_i - (\frac{x_i}{h_i})^2)^2 + \frac{f_{v1}}{f_{v2}}] - \frac{f_{v1}}{f_{v2}}[(T_i - (\frac{x_i}{h_i})^2)^2 + \frac{f_{v1}}{f_{v2}}] \quad \text{satisfy}: h_i: f_{v2} = 0 \quad (8)$$

In which,

$$T_i = \frac{x_i}{h_i}, \quad f_{v1} = f_{v2}, \quad T^* = 2\pi \sqrt{\frac{x_i}{h_i} / k_i},$$

and $f_{v1}, x_{v1}$ denote the yield restoring force and displacement respectively.

**NUMERICAL CALCULATION AND CONSIDERATION**

The numerical calculation of Eq.(4) are carried out by means of Runge-Kutta's method. The critical cumulative ductility factor (CCDF) are estimated by Eq.(8), and the fundamental characteristics are considered. The following ground motions are used in this calculation: The impact wave and sin wave as be shown in Fig.6, Taft Calif., July 21, 1952, EW and El-Centro, May 18, 1940, NS. In which, one revises the accelerations of earthquake ground motions to make the same average velocity spectrum, and leave the time axis.

The damping ratio is assumed to be $\eta=0.02$ for each story. And some kinds of restoring force characteristics are used.

The case of 1-story structures When the structure is destroyed by one stroke of earthquake ground motion, critical cumulative ductility factor is introduced by analytical method. And relationship between critical cumulative ductility factor $\mu_0$ and virtual period $T^*$ is shown in Fig.7, in which $T^*=2\pi \sqrt{m_i / K_i}$ (natural period). It is illustrated in Fig. 7 that the critical cumulative ductility factor increase with increasing $\sigma_1$ in restoring force characteristics. And in the case of $\sigma_1<0$, critical ductility factor never become to be greater than a some value even if $T^*$ decreases.

![Fig. 6 Impact Wave and Sin Wave](image)

![Fig. 7 Influence of Restoring Force Characteristics](image)

V-390
Fig. 8 Effect of Repeating Load in Plastic Zone

From Fig. 8, it is shown that the CCDF $\mu_c$ in the case of sin wave is larger than the CCDF in the case of impact wave. It may be caused by the effect of repeating load in plastic zone. The CCDF for the frame subjected to Taft-Calif. and El-Centro are also shown in Fig. 8. The CCDF in the frame subjected to impact wave is quite agree with analytical solution, and is smaller than the other results.

The case of Multi-Story Frames For the case that the first story in 2~10-story frames are damaged, the CCDF $\mu_c$ are discussed. The CCDF of multi-story frames is not given by analytical solution. Then, the CCDF in 2 and 3-story frames are obtained by numerical calculation and are shown Figs. 9 and 11 repetitively. But, the CCDF by using impact wave are smaller than ones by using actual earthquake ground motions in the range of $T^*=0.8$ sec.

The influence of number of stories on the critical cumulative ductility factor is illustrated in Fig. 10. From this figure, the CCDF becomes small value as increasing the number of stories. In relation to the frames of $T^*=0.5$ sec., the areas denoted by hatch in Fig. 11, show the energy which absorbed by the hysteretic of restoring force until the CDF becomes the CCDF, in which $f_1/f_{\mu 1}$ and $X_1/X_{\mu 1}$. This means that absorbed energy becomes small as increasing the number of stories.

The relationship between critical cumulative ductility factor and mass ratio for 2-story frames are shown in Fig. 12. The virtual period of these frames is 1.0 sec. and 0.6 sec., $m_1/m_2$ is constant, and yield shearing force coefficient of each story are $C_{m1}=0.2$ and $C_{m2}=0.25$. According to Fig. 12, CCDF $\mu_c$ decreases with the increase of mass ratio $m_2/m_1$. 

(a) 2-story frames  (b) 3-story frames  (c) Number of Stories 

Fig. 9 CCDF $\mu_c$ for Multi-Story Steel Frames 
Fig. 10 Influence of the 
Fig. 11 Number of Stories
And also, the relationship between the critical cumulative ductility factor and ratio of period $T_2/T_1$ for 2-story frames is illustrated in Fig. 13. Where, the mass of these frames is $m_1 = m_2$, and virtual periods and yield shearing force coefficient are the same as that of frames in Fig. 12. It is shown that CCDF $\mu_\infty$ increases by increasing the ratio of period $T_2/T_1$.

CONCLUSION

The collapse process are discussed by energy theory and by simulation on dynamical behaviours which evolve to a shake down condition.

(1) The critical cumulative ductility factor is defined as cumulative ductility factor in the condition that the related displacement is satisfied Eq. (8).

Next, the fundamental characteristics of critical cumulative ductility factor are investigated by numerical analysis, and the following results are obtained.

(2) The critical cumulative ductility factor decreases increasing the virtual period of the story in which collapse occurs.

(3) The critical cumulative ductility factor becomes large value by the effect of repeating load in plastic zone during earthquake.

(4) When the number of upper stories from collapse story increases, the critical cumulative ductility factor decreases.

(5) The critical cumulative ductility factor is influenced by mass ratio and ratio of virtual period of each story. For 2-story steel frames, the critical cumulative ductility factor becomes large value when the mass ratio decreases and the ratio of virtual period increases.

REFERENCE


