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SEISMIC RESPONSE OF FRAME ELASTIC BUILDINGS

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SUMMARY

The analogy between the vibrations of a shear frame elastic building and a shear beam shows that some special parameters characterise these constructions. This is particularly efficient when the motion is plane and the parameters reduce to one.

INTRODUCTION

Though many useful micro-computer automatic structural programs are today available for the engineers who make projects in the seismic areas of the world, the use of charts can be still useful to the comparison of results or to a first approach to the actual problems.

From this point of view the analogy existing between the motion of a shear plane elastic beam (it is well known that a shear beam is one which deforms under shear only) and a shear plane elastic frame under horizontal forces still now has same attraction among researchers (1).

R.E. Gibson, M.L. Moody, R.S. Ayre (2) extended this analogy to the tridimensional case including the torsion.

In both cases, the shear frame and the shear beam, we have the following properties:

- plane motions (of each floor or of each section);
- a shear centre 0 and a mass center G (3), (fixed if the mass and the rigidities are constant along the beam (building));
- two axes (principal axes) such that a force along one of them causes only a displacement in the sane direction (Fig. 1).

It can be shown that the three parameters of the motion of the shear beam, the two horizontal dispacements and the rotation of the plane section of the beam, are coupled together.

One of them uncouples only if G is on the correspondig axe. Recently (4) the author has shown how the three differential equations of the free vibrations of motion of the beam can be solved in closed form when the elastic parameters and the mass are constant along the whole beam (building).

In the present work the equations of motion of the beam are adimensionalised showing that some special parameters characterise the spatial frames: seismic forces are taken into account with the method of the spectral analysis.

When G is on 0 the displacements uncouple: one special parameter only rule frames. It is then possibile to picture plane charts giving the greatest sollecitations and displacements for each mode of vibration of the beam.

Equation of motion The three equations of motion of the shear elastic beam (4) can be written in an adimensional form in the following way:

1) $\delta_{ij}\, \partial^2 q_i/\partial \varepsilon^2 - m_{ij}\, \partial^2 q_j/\partial t^2 = - m_{ij}\, \partial^2 \eta_j/\partial t^2 \qquad i,j = 1,2,3$

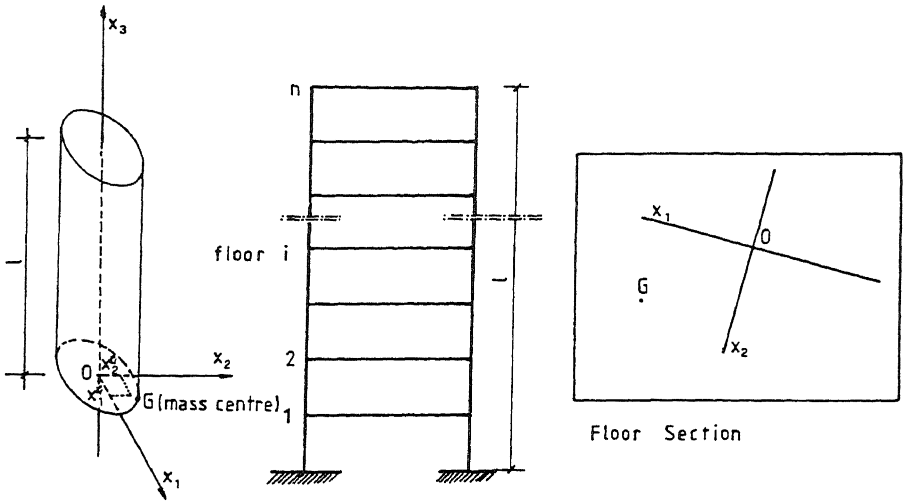

Fig. 1 - Shear beam, shear frame, principal axes; the position of G, the rigidities of the beam and of the columns are constant

where:

2) δ_{ij} is the Kronecker symbol

3) $m_{ij} = \begin{bmatrix} \mu l^2/GK_1 & 0 & -S_1 l/GK_1 \\ 0 & \mu l^2/GK_2 & S_2 l/GK_2 \\ -S_1 l^3/GJ_t & S_2 l^3/GJ_t & J_o l^2/GJ_t \end{bmatrix}$ 4) $q_i = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$ 5) $\eta_i = \begin{bmatrix} y_1/l \\ y_2/l \\ \vartheta_3/l \end{bmatrix}$

The meaning of the symbols here used are clearly explained in the following list.

- l = length of the beam;
- $\lambda_1 = u_1/l$, $\lambda_2 = u_2/l$ $\lambda_3 = \Psi_3/l$;
- u_1, u_2, Ψ_3 = movements along x_1, x_2, rotation with reference to x_3 of the beam section;
- μ = mass per unit length;
- S_1, S_2, J_o = static moments of μ with referece to x_1, x_2; polar moment of inertia with reference to 0;
- GK_1, GK_2, GJ_t = shear rigidities along x_1, x_2; GJ_t, torque rigidity;
- y_1, y_2, ϑ_3 = components of the ground motion: displacements along x_1, x_2; rotation with reference to x_3.

<u>Free vibrations</u> It can be shown that the following serie is the solution of the homogeneous system associated with 1), (4);

6) $$q_i = \sum_{1}^{3}{}_a \sum_{0}^{\infty}{}_n\, q_{in}^a (A_n^a \cos \Omega_n^a t + B_n \cos \Omega_n^a t)$$

with $q_{in}^a = \overline{q}_{in}^a$ sen $\alpha_n \varepsilon$ $\qquad \alpha_n = \pi/2 + n\pi \quad n = 0,1,\dots\infty, \; a = 1,2,3$

Ω_n^a is the pulsation;

$\bar{q}^a_{in}$ is the modal vector solution of

7) $$(\alpha_n^2 \delta_{ij} - (\Omega_n^a)^2 m_{ij})\ \bar{q}^a_{in} = 0$$

and $(\Omega_n^a)^2$ is the solution of the equation:

8) $$|\ \alpha_n^2\ \delta_{ij} - (\Omega_n^a)^2 m_{ij}| = 0$$

The three parameters of motion are clarly coupled in 7): motion uncouples if the shear centre and the mass centre coincide.

It is useful to notice that the three derivatives with respect to ε of 6) are proportional to shears and torque respectively.

Forced vibrations The solution of 1) has the form:

9) $$q_i = \sum_1^3{}_a \sum_0^\infty{}_n\ \bar{q}^a_{in}\ \mathrm{sen}\,\alpha_n \varepsilon\ .\ \Theta_n^a(t)$$

where Θ_n^a is a suitabele function of the time.

Inserting 9) in 1), integrating between 0 and 1, taking into account the orthogonality of the modes, we obtain that:

10) $$\ddot{\Theta}_n^a + (\Omega^a)^2\ \Theta_n^a = \bar{q}^a_{in}\ m_{ij}\quad \ddot{\eta}_j\ 4/\pi\ (1+2n)$$

We see that the functions Θ_n^a and Ω_n^a through 8) depend on the seven parameters in matrix 3).

Uncoupled motion Movements uncouple if G and 0 coicide. In this case supposing that the earthquake sollecitation η acts along one of the principal axes we can simplify 10):

11) $$\ddot{\Theta}_n + \Omega_n^2\,\Theta_n = \ddot{\eta}\ 4/\pi\ (1+2n)$$

and by making use of the spectral analysis we obtain the maximum value:

12) $$\Theta_{n\ max}\ .\ 1 = 4g\ C(\Omega_n)/\Omega_n^2\ \pi\ (1+2n)$$

while 8) simply becomes:

13) $$\Omega_n = \alpha_n \sqrt{GK/\mu\ 1^2}$$

Then with 12) we can picture the diagrams that give for each mode, in function of the adimensional quote displacements and shear for some values of the parameter $GK/\mu\ 1^2$, (Figs. 2, 3, 4, 5).

The parameter $GK/\mu\ 1^2$ characterises fully the plane frames. The figures are here calculated with the spectrum $C(\Omega_n)$ according to the Italian Code for a second category earthquake (5).

Numerical example Shear 6-floors concrete frame (Fig. 6). Given:
$h = 3.0$ m
$J = 160.000\ cm^4$ (J = moment of inertia of the column)
$\mu = 13.900$ kg/m
$1 = 18$ m
we obtain:
$GK/\mu\ 1^2 = 56.84\ sec^{-2}$, $\Omega_1 = 11.83\ sec^{-1}$, $T_1 = 0.53$ sec, (first mode); $\Omega_2 = 35.50\ sec^{-1}$, $T_2 = 0{,}17$ sec, (second mode). T_n is the period of the n-th mode.

From figure 4 we can see that the displacement of the top of the building is 0.63 cm and from figure 2 that the shear at the bottom of each column is about 35000N.

These results are compared with those obtained from the finite elements linear program SAP IV (6).

They are: T_1 = 0.55 sec; T_2 = 0.17 sec, the displacement is 0.67 cm, the shear 33000N, showing good agreement with the present shear beam theory.

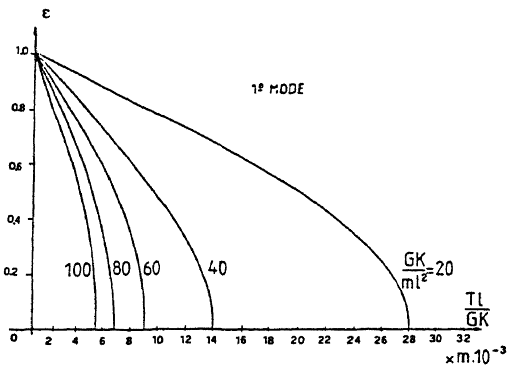

Fig. 2 - Shear per unit rigidity vs adimensional quote - First mode

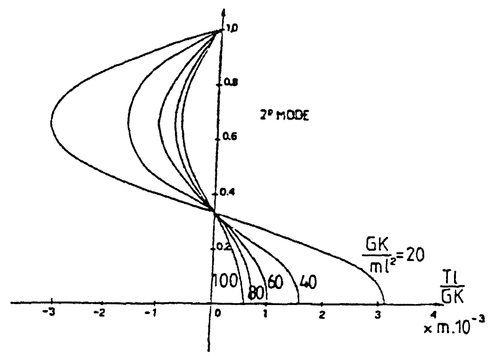

Fig. 3 - Shear per unit rigidity vs adimensional quote. Second mode

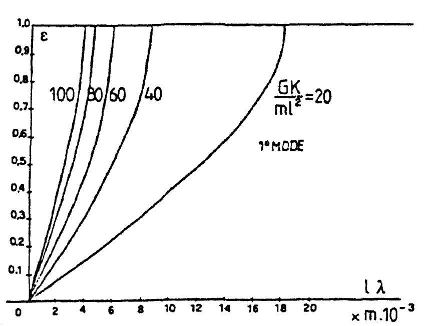

Fig. 4 - Displacement vs adimensional quote - First mode

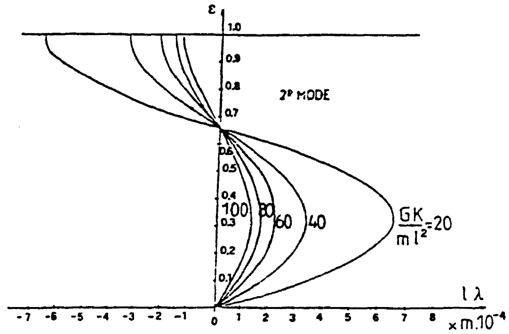

Fig. 5 - Displacement vs adimensional quote - Second mode

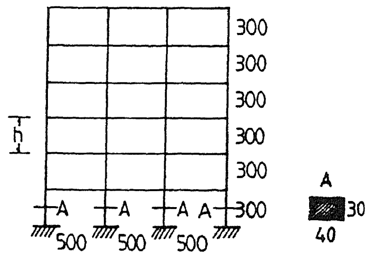

Fig. 6

Conclusions This research shows that seven parameters rule the earthquake vibrations of unsymmetrical elastic shear frame buildings: they reduce to one in the plane problems.

It is then possible to design bidimensional charts by making use of the spectral analysis which can be useful to a first approach to the actual problems.

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