ELASTO-PLASTIC SEISMIC RESPONSE OF FRAMED STRUCTURES IN CONSIDERATION OF HINGED MECHANISM

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SUMMARY

The analytical method is presented to investigate the behaviours of non-linear response in framed structures subjected to earthquake ground motions in based on the plastic hinged mechanism in structural members of which framed structures are formed. From the results of numerical analysis, it is evident that energy absorption owing to hysteresis loop of plastic hinged mechanism have a great effect for dynamical response of framed structure considerably.

INTRODUCTION

Several method on seismic response in consideration of plastic hinged mechanism have been developed by Berge G.V. (Ref. 1) and other investigators. And, the part in this paper has been presented in AIJ on Oct.1973 (Ref. 2). The elasto-plastic seismic response analysis of framed structures in consideration of a crack and yield of structural members successively are developed in based on the idealized plastic anlysis. And also, shear deformation in beam, column and shear walls are considered. Relationship between bending moments and plastic rotation in plastic hinged mechanism is extended to domain in bi-linear or tri-linear type by using the idealized elasto-plastic type which was proposed by Clough R.W. (Ref. 3). Based on the criterion of judgment on elastic and plastic conditions in structural members, non-linear response analysis of framed structures is accomplished successively and numerically by so-called incremental linear accreration procedure. And, it is possible that the conditionin elastic and plastic zone is judged in regardless of a change of inflexion point in structural members by this method.

METHOD OF ANALYSIS

Assumption in this Analysis. Multi-story buildings are idealized as multi-degree of freedom system, the structural members are replaced by straight line and analized. The deflection by bending moment, shearing force, axial thrust and the rigid zone in structural members are considered respectively.

Procedure in this analysis. The plastic hinging mechanism in the end of rigid zone in structural members is illustrated in Fig. 1. The relationship between incremental end force and incremental end deformation can be written:
Fig. 1 Incremental End Force and Deformation in Structural Members

\[
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i \\
\Delta M_i \\
\Delta P_j \\
\Delta Q_j \\
\Delta M_j
\end{bmatrix} = \begin{bmatrix}
\frac{A_S}{2k} & 0 & -\frac{A_M}{2k} & 0 & -\frac{B_i}{k} \\
\frac{A_E}{He} & 0 & 0 & -\frac{A_E}{He} & 0 \\
\frac{A_S}{2k} & -\frac{HB_i + 2B_i}{2k} & 0 & -\frac{2HB_i + 8B_i}{2k} & 0 \\
\frac{A_S}{2k} & 0 & -\frac{B_j}{k} & 0 & \frac{B_j}{2k} \\
\frac{A_E}{He} & 0 & 0 & -\frac{A_E}{He} & 0 \\
\frac{B_i}{2k} & \frac{B_j}{2k} & 0 & \frac{B_j}{2k} & 0
\end{bmatrix}\begin{bmatrix}
\Delta u_i \\
\Delta v_i \\
\Delta \theta_i \\
\Delta u_j \\
\Delta v_j \\
\Delta \theta_j
\end{bmatrix}
\]

in which,

\[
k = 2(1 - \lambda_1 - \lambda_2)^3 \left\{ (k_i + 1)(k_j + 1) \frac{1}{4} + \frac{24k_S EH (1 - \lambda_1 - \lambda_2)^2}{\beta GAH^2} \left( (k_i + 1)(k_j + 1) \frac{1}{16} \right) \right\}
\]

\[
A_{ii} = \frac{4EI}{H} \left[ 4k_S k_i (1 - \lambda_1 - \lambda_2) + \frac{12k_S k_i E_i}{\beta GAH} \left( (1 - \lambda_i - \lambda_1) + 3k_i (1 - \lambda_1 - \lambda_2) (1 - \lambda_1 - \lambda_2) + 3k_i \lambda_i (1 - \lambda_1 - \lambda_2) \right) \right]
\]

\[
A_{jj} = \frac{12EI}{H} \left[ 4k_S k_i (1 - \lambda_1 - \lambda_2) + k_j (1 - \lambda_i - \lambda_1) + k_j (1 - \lambda_i - \lambda_2) \right]
\]

\[
A_{ij} = \frac{6EI}{H} \left[ 2k_S k_i (1 - \lambda_1 - \lambda_2) + k_j (1 - \lambda_i - \lambda_1) + k_j (1 - \lambda_i - \lambda_2) \right]
\]

\[
He = (1 - \lambda_1 - \lambda_2) H
\]

in \( B_{ii}, B_{jj}, B_{ij} \) and \( B_{ji} \), suffix of \( k \) and \( \lambda \) are exchanged \( i \) and \( j \) respectively. Where, \( E_i \); bending rigidity, \( GA \); shear rigidity, \( E_A \); axial thrust rigidity, \( \beta \); shape factor, \( \beta \); decreasing factor in shear rigidity, \( \lambda \); rigid zone ratio.

The semi-rigid hinged coefficient \( k_i \) and \( k_j \) in Eq.(1) are given as follows: In the case that \( i \) and \( j \) joint are elastic condition, \( k_i, k_j \) become \( \infty \) (about \( 10^5 \) in actual calculations), and in the case that \( i \) and \( j \) joint are plastic hinge condition, \( k_i, k_j \) become \( 0 \), and also in the case that \( i \) and \( j \) joint are semi-rigidity condition as bring on crack, \( k_i \) and \( k_j \) become \( 0 < k_i, k_j < 10^5 \) (about \( 0.1 - 1.0 \) in actual calculations).

The relationship between semi-rigid hinged coefficient and decreasing factor of rigidity is shown in Fig. 2. Relationship between incremental bending moment \( \Delta m_i, \Delta m_j \) and incremental rotation angle \( \Delta \tau_i, \Delta \tau_j \) in end of rigid zone are given by substituting \( \lambda_i = 0 \), \( \lambda_j = 0 \), \( u_i = u_j = v_i = v_j = 0 \) into Eq.(1).

\[
\begin{bmatrix}
\Delta m_i \\
\Delta m_j
\end{bmatrix} = \begin{bmatrix}
a & b \\
b & a
\end{bmatrix}\begin{bmatrix}
\Delta \tau_i \\
\Delta \tau_j
\end{bmatrix}
\]

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in which,

\[ a = \frac{1}{2k(1-\lambda_1-\lambda_2)} \left( 8k, k_1(1+\gamma) + 6k_1 \right) \]

\[ b = \frac{1}{2k(1-\lambda_1-\lambda_2)} \left( 4k, k_2(1-2\gamma) \right) \]

\[ \gamma = \frac{3\pi E}{Gk(1-\lambda_1-\lambda_2)} \]

\[ 2k = 4(k_1+1)(k_2+1)(1+4\gamma) - (1+\gamma) \]

Incremental force in elastic and plastic zone are calculated by conjugating the semi-rigid hinged coefficient. Also, incremental end moment in joint are expressed by Eq. (3).

\[ \Delta M_i = \frac{1-\lambda_i}{1-\lambda_i-\lambda_j} \Delta m_i + \frac{\lambda_j}{1-\lambda_i-\lambda_j} \Delta m_j \]

\[ \Delta M_j = \frac{\lambda_j}{1-\lambda_i-\lambda_j} \Delta m_i + \frac{1-\lambda_i}{1-\lambda_i-\lambda_j} \Delta m_j \]

And relationship between the incremental rotation angle in end of rigid zone and incremental rotation angle in joint is also expressed by Eq. (4).

\[ \Delta \theta_i = \frac{1-\lambda_i}{1-\lambda_i-\lambda_j} \Delta \theta_i + \frac{\lambda_j}{1-\lambda_i-\lambda_j} \Delta \theta_j - \frac{1}{1-\lambda_i-\lambda_j} \Delta \theta \]

Judgement on Elastic and Plastic Condition When bending moment \( m \) increase and becomes yield moment \( m_y \), the plastic hinge is constituted and bring on rotation angle in plastic zone \( \tau_p \). In after this condition, rotation angle \( \tau \) in plastic zone are given geometically, and rotation angle \( \tau_p \) are used to deciding the judgement in transition from elastic to plastic zone in general.

The rotation angle \( \tau \) in plastic zone are constituted by rotation angle \( \tau_p \) in elastic zone and \( \tau_p \) in plastic zone as be shown in Fig. 3-a, and relationship between bending moment \( m \) and rotation angle \( \tau_p \) has been assumed by Clough R.W. (Ref. 3) as be shown in Fig. 3-c for elasto-plastic.

According to Fig. 3-c, rotation angle is \( \Delta \tau_p = 0 \) and semi-rigid hinged coefficient become to \( k = \infty \) in the elastic zone, and become \( \Delta \tau_p > 0 \) and \( k = 0 \) in plastic zone. However, when bending moment \( m \) increase and reached to crack moment \( m_c \), the rigidity decreases and become semi-rigid hinge, but moment increase. In this case, relationship between bending moment \( m \) and rotation angle \( \tau_p \) in plastic zone are extended as be shown in Figs. 3-d and 3-e.

Fig. 3 Characteristics in Plastic Hinged Mechanism

Fig. 4 Plastic Hinged Mechanism and Restoring Force
In the judgement for shearing deformation when the stress in structural members reached to yield level stress, joint translation angle $R$ by shearing force becomes to a certain value (for example, $R=0.25 \times 10^{-3}$ rad.). And in after this condition, the relationship between shearing force and joint translation angle may be assumed as bi-linear or tri-linear type. Well, relationship between plastic hinged mechanism and restoring force are shown in Fig. 4.

Incremental stiffness matrix in structural frames are calculated by Eq.(1), and incremental displacement in joint are evaluated by means of incremental linear acceleration procedure. As be already mentioned, incremental rotation angle in the point becoming plastic condition are decided by Eq.(4), and incremental moment are given by Eq.(2). And also, incremental bending moment in end of structural members are evaluated by means of adding incremental bending moment successively. This method are also applied to seismic response anlysis including rocking and swaying vibration (Ref. 4).

EXAMPLE APPLICATIONS

The condition in occurance of crack and yield hinge, the maximum force and deflection in structural members are discussed in the following examples. And also, the results of response analysis in based on plastic hinged mechanism (call the flexural-shear system) are compared with the its as shear framed structure (call equivalent shear system). The fraction of viscous damping is assumed to be $h=0.02$ for each story in this anlysis. And following three kinds of earthquake ground motions are used: El-Centro, May 18, 1940, N-S, Taft-Calif., July 21, 1952, E-W, and Hachinohe, May 16, 1968, E-W. In which, one revises the acceleration of earthquake ground motions to make them max. 0.3g and 0.45g respectively, and leave the time axis.

Example(1) The reinforced precast concrete (RPC) structures of 11-story are modeled as be shown in Fig. 5, and are analized in considering rigid zone.

Case 1 in Example(1) The mass, crack moment $M_c$ and yield moment $M_y$ and rotation angle $\theta_c$, $\theta_y$ etc. in structural members of which are used in response analysis are estimated in based on the section of members, strength of concrete and reinforcement etc. in actual structures by the design standard for reinforced concrete structures (Ref. 5), but are not shown in this paper. The relationship between the moment and rotation angle are used tri-linear type as shown in Fig. 4-e.

Case 2 in Example(1) The mass, cracking moment and yield moment etc. are estimated by similar method with Case 1. The difference between Case 1 and 2 are as follows: (1) The base-shear coefficient increases from $C=0.25$ to $C=0.35$. (2) The value of reinforcement in structural members increase. (3) The mass of structure only decrease. And (4) The standard strength of concrete increase from $f_c=350$ to $f_c=400$ kg/cm$^2$ for the under part from basement of column in 2nd story.

The maximum bending moments, the condition in occurence of cracks and yield hinges in structural members and maximum displacement in frames are illustrated in Figs. 7, 8, and 9. And also, the momentary forces of columns and beams, and deformation of frames for that the time is 3.0 sec. in during earthquake as the response in time history is shown in Fig. 6.

It is understood that the dynamic behaviour in during the earthquake has clarified from Figs. 6, 7, 8 and 9.

Example(2) The results of response anlysis in based on the plastic hinged mechanism (FSS) are discussed in compared with the results of analysis as equivalent shear system (ESS).

In the equivalent shear system, the initial and second rigidity are calculated
Fig. 5 Model

El-Centro 1940 N-S $\alpha_{max}=0.45g$

Fig. 6 Momentary Forces on T=3.0sec.in during Earthquake

Fig. 7 Maximum Response, Crack(○) and Yield(●) Hinges

El-Centro 1940 N-S $\alpha_{max}=0.45g$

Fig. 8 Maximum Response, Crack(○) and Yield(●) Hinges

Hachinohe 1968 E-W $\alpha_{max}=0.45g$

Fig. 9 Maximum Response, Crack(○) and Yield(●) Hinges

Taft-Calif. 1952 E-W $\alpha_{max}=0.45g$

by the frame analysis (Ref. 5) in the case that horizontal forces of each story are assumed as contrary triangle distribution of seismic coefficient.

And the crack and yield shearing force are gotten from crack and yield moments.

The 1st natural period of this model becomes $T_1=0.77033$sec. and 2nd natural period is $T_2=0.26618$sec.. The difference between FSS and ESS are shown in Fig. 10.
The shearing force $Q$ and shearing force coefficient $C$ in the method by FSS are smaller than ones by ESS for three earthquake ground motions. And there is a difference between the response in FSS and ESS for relative displacement $\delta$. Then, it is expected that earthquake response are strictly analyzed as the flexural shear system.

Fig. 10 Difference between the Response in FSS(●) and ESS(○)

CONCLUSION

The characteristics in this analytical method and the results in the analysis are as follows:

1. The rigid zone in structural members are considered in response analysis.
2. The semi-rigid hinged coefficient in plastic hinged mechanism are introduced, and are extended to the domain in bi-linear, tri-linear etc. by using the idealized elasto-plastic type which was proposed by Clough R. W.
3. The behavior of structures subjected to earthquake ground motions are explained in concering with the elastic and plastic condition in structural members.
4. The maximum force and deformation etc. are estimated, and these values on every time in during earthquake are gained by this method.
5. It is generally analyzed as equivalent shear system in the seismic response, but it should be analyzed as flexural-shear system strictly.

REFERENCES