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ELASTO-PLASTIC SEISMIC RESPONSE OF FRAMED STRUCTURES IN CONSIDERATION OF HINGED MECHANISM

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SUMMARY

The analytical method is presented to investigate the behaviours of non-linear response in framed structures subjected to earthquake ground motions in based on the plastic hinged mechanism in structural members of which framed structures are formed. From the results of numerical analysis, it is evident that energy absorption owing to hysteresis loop of plastic hinged mechanism have a great effect for dynamical response of framed structure considerably.

INTRODUCTION

Several method on seismic response in consideration of plastic hinged mechanism have been developed by Berge G.V. (Ref. 1) and other investigators. And, the part in this paper has been presented in AIJ on Oct.1973 (Ref. 2).
The elasto-plastic seismic response anlysis of framed stuctures in consideration of a crack and yield of stuctural members successively are developed in based on the idealized plastic anlysis. And also, shear deformation in beam, column and shear walls are sonsidered. Relationship between bending moments and plastic rotation in plastic hinged mechanism is extended to domain in bi-linear or tri-linear type by using the idealized elasto-plastic type which was proposed by Clough R.W. (Ref. 3). Based on the criterion of judgment on elastic and plastic conditions in structural members, non-linear response analysis of framed structures is accomplished successively and numerically by so-called incremental linear accerelation procedure. And, it is possible that the conditionin elastic and plastic zone is judged in regardless of a change of inflexion point in structural members by this method.

METHOD OF ANALYSIS

Assamption in this Analysis Multi-story buildings are idealized as multi-degree of freedom system, the structural members are replaced by straight line and analized. The deflection by bending moment, shearing force, axial thrust and the rigid zone in structural members are considered respectively.

Procedure in this analysis The plastic hinged mechanism in the end of rigid zone in structural members is illustrated in Fig. 1.
The relationship between incremental end force and incremental end deformation can be written:

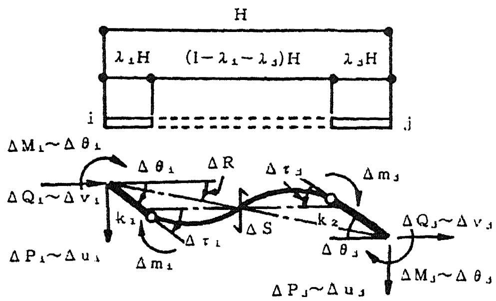

Fig. 1 Incremental End Force and Deformation in Structural Members

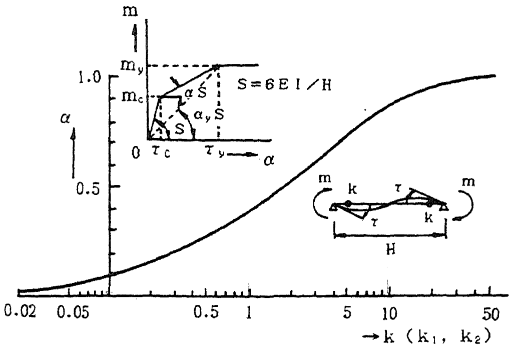

Fig. 2 Semi-Rigid Hinged Coefficient and Decreasing Factor in Rigidity

$$
\begin{Bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta M_i \\ \Delta P_j \\ \Delta Q_j \\ \Delta M_j \end{Bmatrix}
=
\begin{bmatrix}
\frac{A_{22}}{2k} & 0 & \frac{A_{12}}{k} & -\frac{A_{22}}{2k} & 0 & -\frac{B_{12}}{k} \\
 & \frac{AE}{He} & 0 & 0 & -\frac{AE}{He} & 0 \\
 & & \frac{A_{11}}{2k} & -\frac{HB_{22}+2B_{21}}{2k} & 0 & -\frac{2HB_{12}+B_{11}}{2k} \\
 & & & \frac{A_{22}}{2k} & 0 & \frac{B_{12}}{k} \\
 & \text{(SYM.)} & & & \frac{AE}{He} & 0 \\
 & & & & & \frac{B_{11}}{2k}
\end{bmatrix}
\begin{Bmatrix} \Delta u_i \\ \Delta v_i \\ \Delta \theta_i \\ \Delta u_j \\ \Delta v_j \\ \Delta \theta_j \end{Bmatrix}
\qquad (1)
$$

in which,

$$k=2(1-\lambda_i-\lambda_j)^4\left\{(k_1+1)(k_2+1)-\frac{1}{4}\right\}+\frac{24\kappa EI(1-\lambda_i-\lambda_j)^2}{\beta GAH^2}\left\{(k_1+1)(k_2+1)-\frac{1}{16}\right\}$$

$$A_{11}=\frac{4EI}{H}\left[4k_1k_2\{(1-\lambda_j)_3-\lambda_i^3\}+\frac{12k_1k_2\kappa EI}{\beta GAH^2}\{(1-\lambda_j)-\lambda_i\}+3k_1(1-\lambda_j)^2(1-\lambda_i-\lambda_j)+3k_2\lambda_i^2(1-\lambda_i-\lambda_j)\right]$$

$$A_{22}=\frac{12EI}{H^3}\left[4k_1k_2\{(1-\lambda_i)-\lambda_j\}+k_1(1-\lambda_i-\lambda_j)+k_2(1-\lambda_i-\lambda_j)\right]$$

$$A_{12}=\frac{6EI}{H^2}\left[2k_1k_2\{1-\lambda_j)^2-\lambda_i^2\}+k_1(1-\lambda_j)(1-\lambda_i-\lambda_j)+k_2\lambda_i(1-\lambda_i-\lambda_j)\right]$$

$$He=(1-\lambda_i-\lambda_j)H$$

in B_{11}, B_{12}, B_{21} and B_{22}, suffix of k and λ are exchenged 1, i for 2, j repectively. Where, EI ; bending rigidity, GA ; shear rigidity, EA ; axial thrust rigidity, κ; shape factor, β; decreasing factor in shear rigidity, λ; rigid zone ratio.

The semi-rigid hinged coefficient $k_1$and k_2 in Eq.(1) are given as follows: In the case that i and j joint are elastic condition, k_1,k_2 become ∞(about 10^5 in actural calculations), and in the case that i and j joint are plastic hinge condition, k_1,k_2 become 0, and also in the case that i and j joint are semi-rigidity condition as bring on crack, $k_1$and k_2 become $0<k_1,k_2<10^5$ (about 0.1~1.0 in actual calculations).

The relationship between semi-rigid hinged coefficient and decreasing factor of rigidity is shown in Fig. 2. Relationship between incremental bending moment m_i,m_j and incemental rotation angle τ_i,τ_j in end of rigid zone are given by substituting $\lambda_i = 0$, $\lambda_j = 0$, $u_i = u_j = v_i = v_j = 0$ into Eq.(1).

$$\begin{Bmatrix} \Delta m_i \\ \Delta m_j \end{Bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{Bmatrix} \Delta \tau_i \\ \Delta \tau_j \end{Bmatrix} \qquad (2)$$

in which,

$$a=\frac{1}{2k}\frac{2EI}{(1-\lambda_i-\lambda_j)H}\{8k_1k_2(1+\gamma)+6k_1\} \qquad b=\frac{1}{2k}\frac{2EI}{(1-\lambda_i-\lambda_j)H}\{4k_1k_2(1-2\gamma)\}$$

$$\gamma=\frac{3\kappa EI}{\beta GA(1-\lambda_i-\lambda_j)^2H^2} \qquad 2k=4(k_1+1)(k_2+1)(1+4\gamma)-(1+\gamma)$$

Incremental force in elastic and plastic zone are calculated by conjugating the semi-rigid hinged coefficient. Also, incremental end moment in joint are expressed by Eq.(3).

$$\begin{aligned} \Delta M_i &= \frac{1-\lambda_j}{1-\lambda_i-\lambda_j}\Delta m_i+\frac{\lambda_i}{1-\lambda_i-\lambda_j}\Delta m_j \\ \Delta M_j &= \frac{\lambda_j}{1-\lambda_i-\lambda_j}\Delta m_i+\frac{1-\lambda_i}{1-\lambda_i-\lambda_j}\Delta m_j \end{aligned} \tag{3}$$

And relationship between the incremental rotation angle in end of rigid zone and incremental rotation angle in joint is also expressed by Eq.(4).

$$\begin{aligned} \Delta\tau_i &= \frac{1-\lambda_j}{1-\lambda_i-\lambda_j}\Delta\theta_i+\frac{\lambda_j}{1-\lambda_i-\lambda_j}\Delta\theta_j-\frac{1}{1-\lambda_i-\lambda_j}\Delta R \\ \Delta\tau_j &= \frac{\lambda_i}{1-\lambda_i-\lambda_j}\Delta\theta_i+\frac{1-\lambda_i}{1-\lambda_i-\lambda_j}\Delta\theta_j-\frac{1}{1-\lambda_i-\lambda_j}\Delta R \end{aligned} \tag{4}$$

<u>Judgement on Elastic and Plastic Condition</u> When bending moment m_c increase and becomes yield moment m_y , the plstic hinge is constituted and bring on rotation angle in plastic zone τ_p . In after this condition, rotation angle τ_p in plastic zone are given geometically, and rotation angle τ_p are used to deciding the judgement in transition from elastic to plstic zone in general. The rotation angle τ in structural members are constituted by rotation angle τ_{ie} in elastic zone and τ_{ip} in plstic zone as be shown in Fig. 3-a, and relationship between bending moment m_i and rotation angle τ_{ip} has been assumed by Clough R.W. (Ref. 3) as be shown in Fig. 3-c for elasto-plastic.

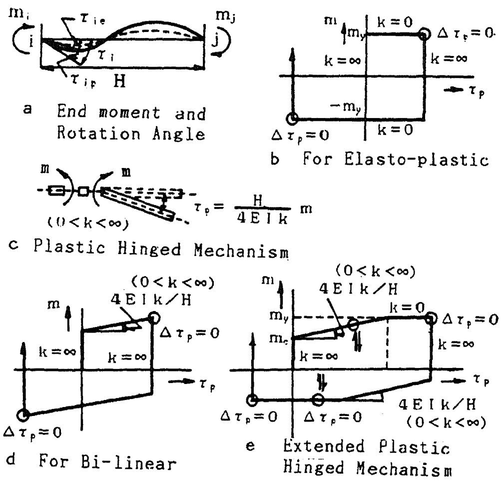

Fig. 3 Characteristics in Plastic Hinged Mechanism

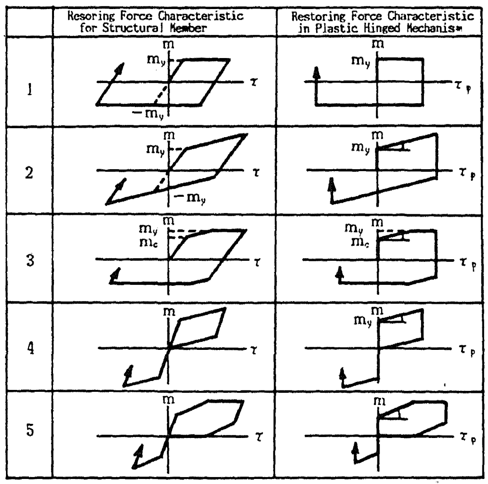

Fig. 4 Plastic Hinged Mechanism and Restoring Force

According to Fig. 3-c, rotation angle is $\Delta\tau_p=0$ and semi-rigid hinged coefficient become to $k=\infty$ in the elastic zone, and become to $\Delta\tau_p>0$ and $k=0$ in plasic zone. However, when bending moment m increase and reached to crack moment m_c, the rigidity decreases and become semi-rigid hinge, but moment increase. In this case, relationship between bending moment m and rotation angle τ_p in plastic zone are extended as be shown in Figs. 3-d and 3-e.

In the judgement for shearing deformation when the stress in structural members reached to yield level stress, joint translation angle R by shearing force becomes to a certain value (for example, $R=0.25\times10^{-3}$rad.). And in after this condition, the relationship between shearing force and joint translation angle may be assumed as bi-linear or tri-linear type. Well, relationship between plastic hinged mechanism and restoring force are shown in Fig. 4.

Incrementl stiffness matrix in structural frames are calculated by Eq.(1), and incremental displacement in joint are evaluated by means of incremental linear accerelation procedure. As be already mentioned, incremental rotation angle in the point becoming plastic condition are decided by Eq.(4), and incremental moment are given by Eq.(2). And also, incremental bending moment in end of structural members are evaluated by means of adding incremental bending moment successively. This method are also applied to seismic response anlysis including rocking and swaying vibration (Ref. 4).

EXAMPLE APPLICATIONS

The condition in occurance of crack and yield hinge, the maximum force and deflection in structural members are discussed in the following examples. And also, the results of response analysis in based on plastic hinged mechanism (call the flexural-shear system) are compared with the its as shear framed structure (call equivalent shear system). The fraction of viscous damping is assumed to be h=0.02 for each story in this anlysis. And following three kinds of earthquake ground motions are used: El-Centro, May 18, 1940, N-S, Taft-Calif., July 21, 1952, E-W, and Hachinohe, May 16, 1968, E-W. In which, one revises the acceleration of earthquake ground motions to make them max. 0.3g and 0.45g respectively, and leave the time axis..

Example(1) The reinforced precast concrete(RPC) structures of 11-story are modeled as be shown in Fig. 5, and are analized in considering rigid zone.
Case 1 in Example(1) The mass, crack moment Mc and yield moment My and rotation angle τ_c, τ_y etc. in structural members of which are used in response anlysis are estimated in based on the section of members, strength of concrete and reinforcement etc. in actual structures by the design standard for reinforced concrete structures (Ref. 5), but are not shown in this paper. The relationship between the moment and rotation angle are used tri-linear type as shown in Fig. 4-e.
Case 2 in Example(1) The mass, cracking moment and yield moment etc. are estimated by similar method with Case 1, The difference between Case 1 and 2 are as follows : (1) The base-shear coefficient increases from C=0.25 to C=0.35. (2) The value of reinforcement in structural members increase.
(3) The mass of structure only decrease. And (4) The standard strength of concrete increase from Fc=350 to Fc=400 kg/cm^2 for the under part from basement of column in 2nd story.

The maximum bending moments, the condition in occurence of cracks and yield hinges in structural members and maximum displacement in frames are illustlated in Figs. 7, 8, and 9. And also, the momentary forces of columns and beams, and deformation of frames for that the time is 3.0 sec. in during earthquake as the response in time history is shown in Fig. 6.
It is understood that the dynamic behaviour in during the earthquake has clarified from Figs. 6, 7, 8 and 9.

Example(2) The results of response anlysis in based on the plastic hinged mechanism (FSS) are discussed in compared with the results of analysis as equivalent shear system (ESS).
In the equivalent shear system, the initial and second rigidity are calculated

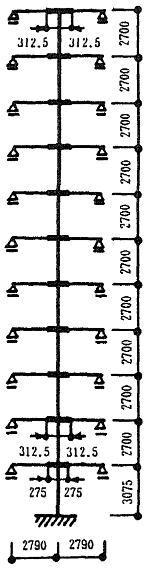

Fig. 5 Model

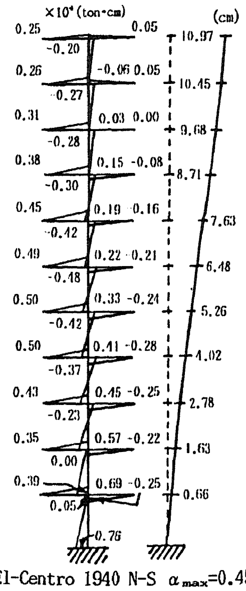

Fig. 6 Momentary Forces on T=3.0sec.in during Earthquake

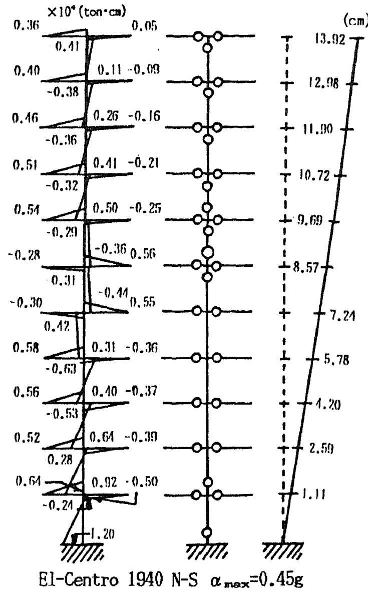

Fig. 7 Maximum Response, Crack(○) and Yield(●) Hinges

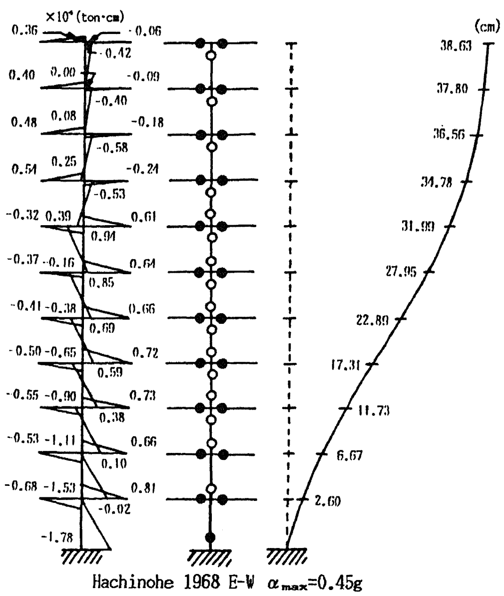

Fig. 8 Maximum Response, Crack(○) and Yield (●) Hinges

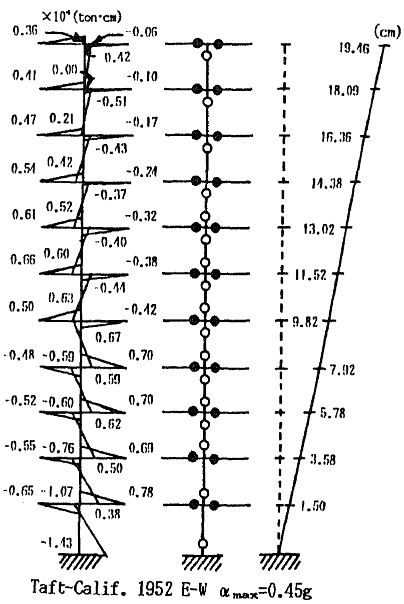

Fig. 9 Maximum Response, Crack(○) and Yield (●) Hinges

by the frame analysis (Ref. 5) in the case that horizontal forces of each story are assumed as contrary triangle distribution of seismic coefficient. And the crack and yield shearing force are gotten from crack and yield moments. The 1st natural period of this model becomes T_1=0.77033sec. and 2nd natural period is T_2=0.26618sec.. The difference between FSS and ESS are shown in Fig. 10.

The shearing force Q and shearing force coefficient C in the method by FSS are smaller than ones by ESS for three earthquake ground motions. And there is a differeces between the response in FSS and ESS for relative displacement δ. Then, it is expected that earthquake response are strictly analized as the flexural shear system.

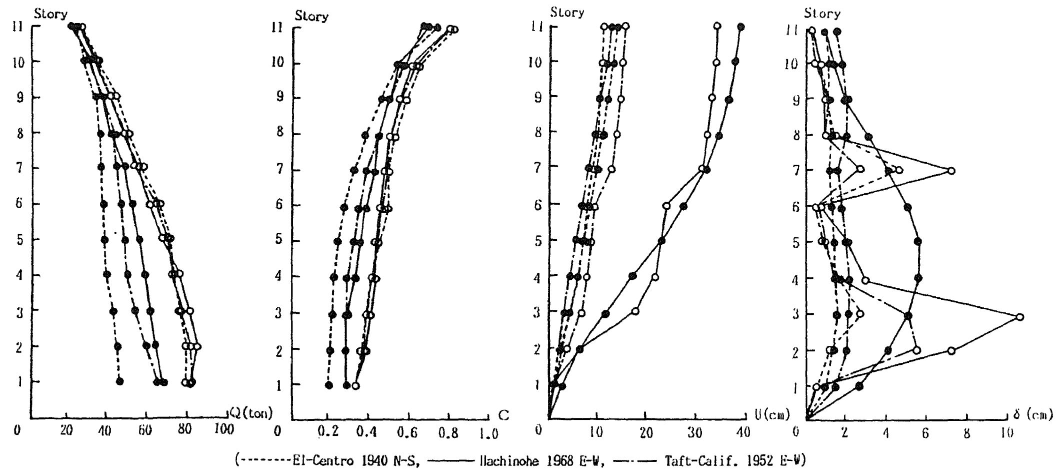

Fig. 10 Difference between the Response in FSS(●)and ESS(○)

CONCLUSION

The characteristics in this analytical method and the results in the analysis are as follows :

(1) The rigid zone in structural members are considered in response analysis.
(2) The semi-rigid hinged coefficient in plstic hinged mechanism are introduced, and are extended to the domain in bi-linear, tri-linear etc. by using the idealized elasto-plastic type which was proposed by Clough R. W..
(3) The behaviour of structures subjected to earthquake ground motions are explained in concerning with the elastic and plastic condition in structural members.
(4) The maximum force and deformation etc. are estimated, and these values on every time in during earthquake are gained by this method.
(5) It is generally analized as equivalent shear system in the seismic response, but it should be analized as flexural-shear system strictly.

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