IDENTIFICATION OF VIBRATION SYSTEMS AND NONLINEAR
DYNAMIC CHARACTERISTICS OF STRUCTURES
UNDER EARTHQUAKE EXCITATIONS

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SUMMARY

Vibration systems of the earthquake responses of structures and their non-
linear characteristics are examined in terms of two approaches, one is based on
the coherence functions and transfer functions between two components of the
responses, and the other is on the parameter estimation by a moving window
procedure using linear one-mode model. Furthermore, the nonlinear responses are
simulated by the amplitude-dependency of the estimated parameters, and examined
by the coherence function. It is concluded that the system nonlinearities around
the first natural mode can almost be explained by the amplitude-dependent
variations of the vibration parameters.

INTRODUCTION

Under the excitations of various sources, the vibration systems of struc-
tures are generally varying, fluctuating, and are off from the linear system.
This fact is due to various effects, such as nonlinear behavior caused by
materials' and elements' natures, influences of a coupling of other modes and
its nonstationalities, a mixing of non-correlated other signals, and so on. It
is important to identify the structure of the system having these wide-sense
nonlinearities for the system parameters estimation and the response evaluation.

In the present paper, the vibration systems of the earthquake responses of
buildings, especially of the first mode, and their nonlinear properties are
examined by two different approaches and their comparisons. One is based on the
coherence function characteristics between two components of the responses. The
shapes of the coherence functions in frequency domain are effective information
on the influences of such nonlinearities of the systems mentioned above. The
authors have examined the ambient vibration systems of buildings by coherence
functions, and have revealed the system properties subjected to the wind and
ground inputs (Ref.1). The other approach is the parameter estimation by a
moving window identification, which employs a linear one-mode model. This
procedure gives the equivalent linear parameters in each short time segment.

COHERENCE FUNCTION CHARACTERISTICS OF THE FIRST MODE RESPONSE OF STRUCTURES

Statistical Evaluation of Coherence Functions To show the properties of
vibration system in frequency domain, a transfer function $H(f)$ and a coherence
function coh2(f) between an input(x) and a output(y) are employed as follows:

\[ H(f) = \frac{S_{XY}(f)}{S_{XX}(f)} \quad (1) \quad coh^2(f) = \frac{|S_{XY}(f)|^2}{S_{XX}(f)S_{YY}(f)} \quad (2) \]

where Sxx and Syy are power spectra of two signals x and y, and Sxy is their cross spectrum. These spectra are calculated statistically by use of ensemble averages of FFT spectra of many records as:

\[ \begin{align*}
S_{XX}(f) &= \frac{1}{N} \sum_{i=1}^{N} |X_i(f)X_i^*[f]|I_i \\
S_{XY}(f) &= \frac{1}{N} \sum_{i=1}^{N} |X_i(f)Y_i^*[f]|I_i \\
S_{YY}(f) &= \frac{1}{N} \sum_{i=1}^{N} |Y_i(f)Y_i^*[f]|I_i \\
I_i &= \frac{\int_{0.2}^{10} |X_i(f)|^2 df}{I_1} 
\end{align*} \quad (3) \]

where \( \ast \): complex conjugate, N: total number of data, Ii: the weighting factor to evaluate the data of different energies. The transfer function shows the averaged linear properties for many data by the relation of x and y. Besides, the coherence function indicates the degree of linearity of the system, in other words, the system constantness for the many earthquakes at every frequency point.

Outline of the Buildings and the Earthquake Records The earthquake response records used in this study were observed at two test buildings with the same superstructures. One is an ordinary constructed 3-story RC structure and the other is a base-isolated building with laminated rubbers and viscous oil dampers. Fig.1 depicts the outline of these buildings and accelerograph setting points. The accelerograms were obtained for 30 earthquakes, which occurred in about a year, from 24 June 1986 (No.1) till 16 July 1987 (No.30). Their maximum acceleration values are in the range of 1-40 gal for the ground surface, the largest maximum response acceleration is 270 gal for the ordinary constructed building and 40 gal for the base-isolated one.

Coherence Function Characteristics of Observed Responses Transfer functions and coherence functions between the 1st floor and the roof of the ordinary-constructed building using 30 earthquake records are shown in Fig.2. The peak at 4 Hz is the first mode of N-S direction. The remarkable characteristic of the coherence function is the degrading around the natural frequency of the first mode, which indicates the existence of system nonlinearities. These functions between basement floor and roof of the base-isolated structure are shown in Fig.3 where (a) is the result calculated for 30 earthquakes, (b) and (c) are the ones for the biggest 5 records and for the smallest 6 records. As the second mode exists at about 6 Hz, we pay attention to the coherence characteristics in the frequency range under 3 Hz to examine the first mode properties. The coherence function of (a) also drops in this range, but the coherence functions of (b) and (c), calculated from the records of nearly the same response revel are greater (closer to 1.0) than that of (a). These results suggest that system nonlinearities for the base-isolated construction are closely related to the response level. The shapes of the transfer functions are also different according to the response level.

![Fig.1 Outline of the buildings and accelerograph setting points](image)

(Only the components of accelerograph used in this study are indicated)
Fig. 2
Transfer function and coherence function between observed inputs and outputs.
(The ordinary construction, 30 earthquakes)

Fig. 10
Transfer function and coherence function between observed inputs and simulated outputs.
(The ordinary construction, 30 earthquakes)

(a) for 30 earthquakes

(b) for biggest 5 earthquakes

(c) for smallest 6 earthquakes

Fig. 3
Transfer functions and coherence functions between observed inputs and outputs.
(The base-isolated construction)

Fig. 11
Transfer functions and coherence functions between observed inputs and simulated outputs.
(The base-isolated construction)
VARIATION OF VIBRATION PARAMETERS ESTIMATED THROUGH THE LINEAR MODEL

Procedure Parameter identification is performed through optimizing the linear one-mode model, by use of the observed responses of the 1st floor (the basement floor for the base-isolated structure) and the roof as the input and the output, respectively. To detect the time- and amplitude-varying characteristics of the vibration systems, the identification is performed in short time segments which are taken from all the duration time of every earthquake record (Ref.2). Every segment is 5sec. in length, and the step of their mid-interval time is 2sec.. Estimated parameters through such moving window identification procedure imply the equivalent linear properties in a short interval of time.

Fig.4 and Fig.5 show the time-variation of the estimated parameters of the two buildings during the No.18 earthquake (which is the one of the biggest records). These parameters are redepicted in Fig.6 and Fig.7 as a function of the r.m.s. value of the input wave of each segment, for the Nos.16, 17 and 18 earthquakes. Fig.8 and Fig.9 indicate the estimated parameters and normalized estimation errors of all segments of 30 earthquakes. The parameters estimated from a segment in which normalized error was not smaller than 0.05(0.1 for the base-isolated construction) are rejected. The results of ambient vibration tests are also depicted in these figures as the function of input intensity. Since damping factors tend to be coupled with participation factors(Ref.2) and the estimated participation factors of the base-isolated construction are fairly different from 1.0, the damping factors divided by the participation factors are also shown for the base-isolated construction. This parameter corresponds to the value 1/2A, where A is the peak value of the transfer function.

Properties of Estimated Parameters Figs.4-9 suggest the following properties of the variations of the parameters of the two buildings during earthquakes. For the ordinary constructed building, during each earthquake, the natural frequency of the first mode is lowered as the input intensity increases, and then gradually raises as the earthquake ended. This variation is nearly the linear function of logarithmic value of r.m.s. of the input acceleration. Being affected by the series of earthquakes, this amplitude-dependent properties shifted downward. This change of the natural frequency is also demonstrated by the ambient vibration measurements. The variation of the damping factors is fairly great, but does not show the distinct properties. As for the base-isolated building, the parameters show the distinctive features of the amplitude-dependent variation, which are probably caused by the nonlinearities of the base-isolation system.

![Ground Acceleration](image1)

![Natural Frequency](image2)

![Damping Factor](image3)

Fig.4 Estimated parameters of the ordinary construction during No.18 earthquake.

![Natural Frequency](image4)

![Damping Factor](image5)

![Participation Factor](image6)

Fig.5 Estimated parameters of the base-isolated construction during No.18 earthquake.
Fig. 6 Variation of estimated parameters of the ordinary construction during each earthquake.

Fig. 7 Variation of estimated parameters of the base-isolated construction during each earthquake.

- $o$ and $*$: parameters from segments before and after the maximum input, respectively.
- ---: amplitude-dependency of parameters used in the simulation of nonlinear responses.

Fig. 8 Amplitude-dependency of estimated parameters of the ordinary construction.

Fig. 9 Amplitude-dependency of estimated parameters of the base-isolated construction.

Each dot is the parameter estimated from 5 sec segment of 30 earthquakes. The parameters from the segment in which the normalized error is greater than the lines in (a) are rejected.

- $o$ represents the parameter estimated by ambient tests.
  - No. A1: after No. 6 earthquake
  - No. A2: after No. 26 earthquake
SIMULATION OF NONLINEAR RESPONSES AND ITS COHERENCE CHARACTERISTICS

Procedure  Following the amplitude-dependency of parameters mentioned above, nonlinear responses of 1-d.o.f. system subjected to the observed waves are simulated. For this purpose, a step-by-step numerical integration procedure using a state transition matrix is employed. The natural frequency and the damping factor in this matrix are determined as a function of an input amplitude level around the time of every step of integration, and this amplitude-dependency of the parameters are given as follows. For the ordinary-construction, the natural frequencies are lowered as a linear function of logarithmic value of input revel while the damping is in a constant value, and the functions are determined for each earthquake according to the properties of the estimated parameters. For example, these values are drawn in Fig.6 for the Nos.16-18 earthquakes. For the base-isolated construction, the natural frequency and the damping coefficient are given by the lines depicted in Fig.7 for all the earthquakes.

Coherence Function Characteristics of Simulated Response  Transfer functions and coherence functions between observed inputs and calculated outputs are shown in Fig.10 and Fig.11(placed in the 3rd page). As compared with that in Fig.2, the simulated coherence function of the ordinary construction is generally in accord to the observed coherence function, but is somewhat greater than the observed one, particularly in the range of higher frequency than the first natural frequency. This fact indicates that the real response is affected by other behavior than the simulated amplitude-dependent characteristics. For the base-isolated construction, the agreement of the characteristics between observed and simulated responses are fairly good in the frequency range under 3 Hz, except for the transfer function amplitude from the small earthquakes. This is due to the fact that the amplitude-dependent nonlinear characteristics are fairly large and definite, especially for large response level as shown in Fig.9., and other factors are relatively small.

CONCLUSION

1) By use of the coherence functions, the nonlinear characteristics around the frequency range of the first mode were shown. This nonlinear characteristics represented by the coherence functions can almost be explained in terms of the amplitude-dependent characteristics of vibration parameters. The coherence function shape in frequency domain is effective to examine the system nonlinearities.

2) The characteristics of the parameters variations of the two buildings, the base-isolated construction and the ordinary one, were shown in detail. They have the distinct amplitude-dependent properties.

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REFERENCES
