Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-5-15

STRUCTURAL IDENTIFICATION OF A NONLINEAR MDOF SYSTEM BY EXTENDED KARMAN FILTER

R.Y. TAN and C.C. CHEN

Department of Civil Engineering, National Taiwan University, Taipei, ROC.

SUMMARY

This paper applies the extended Kalman filter to estimate the system parameter values of a nonlinear structure subjected to the ground motion so that the dynamic characteristics of the structure can be known once the appropriate observed data are available. The system is a three-stroy shear building with restoring force exhibiting bilinear hysteresis of kinematic type. The measured data are generated artificially and the measurement error is assumed to be band-limited white noise. Numerical results show that the filter is very powerful in identification problem when the proposed two-stage iteration procedure is employed.

INTRODUCTION

Engineering structures are often subjected to dynamic forces. Generally, the main effort in structural dynamics is concerned with defining environmental loads, establishing analytical structural models, and developing suitable numerical schemes for calculating the corresponding response. The usefulness of such analytical solutions is, however, limited by the degree of realistic representation of the formulated mathematical models. Obviously, a logical prelude to the prediction of the dynamic response of system is the determination of its dynamic properties. On the other hand, to evaluate the safety or reliability of structures following a natural hazard, e.g. a strong earthquake, engineers also need to know the current state of structural characteristics. Such is the system identification problem.

In fact, with the recent interest in the aseismic design of structures, more structures are instrumented with strong motion accelerographs so that the structural properties can be determined from records obtained from major earthquakes. Many structures have been instrumented with two accelerographs, one is placed in the basement of the structure while the other is placed at some floor level. It is clear that a reliable identification technique is needed to take advantage of such recorded information.

The study intends to examine the feasibility of an identification scheme by which system parameter values can be estimated. Since the structural behavior usually becomes nonlinear under the threat of severe damage and the measurement error is random in nature, the present study applies the prediction-filtering theory to perform the parameter identification. Indeed, by regarding each of the parameters involved in the system as an augmented state variable, the extend Kalman filter may sequentially estimate the parameter values [1,2]. The system is a nonlinear three-story shear building. Eighteen state variables are involved to represent three sets of floor displacement, velocity, damping, stiffness, post-yielding stiffness, and yielding strength. The system parameter is obtained by two-stage identification approach and is compared with its true value to explore the degree of accuracy offered by such approach.

FILTERING THEORY

A nonlinear continuous system with measurements on discrete time instants is described by

$$x = f\ (x,t) \tag{1}$$

$$z(k) = Hx(k) + r(k) \tag{2}$$

where x is an n-dimensional state vector for the system, H is the measurement matrix, z is an m-dimensional measurement vector and r is the measurement error. Both x(k) and z(k) are assumed to be Gauss-Markov sequences. In addition, the measurement error process is an m-dimensional Gaussian white sequence with zero mean and covariance matrix R(k). This sequence is independent of the initial state x(0) since physically we expect the mechanism from which the measurement errors arise to be independent of the one leading to the initial state.

Through proper linearization of Eq.1 along the reference state x^*, we have the computational cycle which proceeds as follows:

(a) Compute the predicted state and its error covariance matrix

$P(k+1 \mid k)$

$$\bar{x}(k+1|k) = \bar{x}\ (k|k) + \int_{t_k}^{t_{k+1}} f(\bar{x}(t|t_k), t)dt \tag{3}$$

(b)Compute the Kalman gain matrix K(k+1) which depends on $P(k+1|k)$, H(k+1) and R(k+1).

(c)Compute the filtered state and its error covariance matrix $P(k+1|k+1)$ with the aid of measurement z(k+1)

$$\bar{x}(k+1|k+1) = \bar{x}(k+1|k) + K(k+1)\ [z(k+1) - H(k+1)\bar{x}(k+1|k)] \tag{4}$$

(d)Increase k to k+1 and return to step (a).

STRUCTURAL MODEL

The system equation of a three-story shear building with the restoring force exhibiting bilinear hysteresis of kinematic type is [3]

$$\dot{x} = f(x,t)$$

$$= \begin{cases} x_4 \\ x_5 \\ x_6 \\ -\ddot{x}_g + \frac{1}{m_1}(-x_4x_7 - x_4x_8 + x_5x_8 - x_{10}h_1(x_1) + x_{11}h_2(x_2-x_1)) \\ -\ddot{x}_g + \frac{1}{m_2}(x_4x_8 - x_5x_8 - x_5x_9 + x_6x_9 - x_{11}h_2(x_2-x_1) + x_{12}h_3(x_3-x_2)) \end{cases}$$

$$\left. \begin{array}{l} -\ddot{x}_g + \frac{1}{m_3} (x_5 x_9 - x_6 x_9 - x_{12} h_3 (x_3 - x_2)) \\ 0 \\ \vdots \\ 0 \end{array} \right. \quad (5)$$

with state vector

$$x^T = [x_1, x_2 \ldots, x_{18}] = [V_i, \dot{V}_i, c_i, k_i, b_i, a_i] \qquad i=1,2,3 \quad (6)$$

where V_i=displacement at the i-th floor, c_i=damping coefficient, k_i=unyielding stiffness, a_i=stiffness ratio, b_i=ratio of yielding strength to k_i, m_i=mass, $h_i(\cdot)$=restoring force, and $\ddot{x}_g$=ground acceleration.

The measurement is a six-dimensional vector consisting of displacement and velocity time histories. It is generated by adding a Gaussian white nosie to the structural response which is computed on the basis of the assumed parameter value and the simulated ground acceleration $\ddot{x}_g$. The intensity of noise is taken to be 6% of the response in terms of RMS values. The upper frequency is 25 Hz. By pretending that the parameter are unknown and performing the previous filtering theory based on such artificial measurement as well as simulated input, we may compare the identified parameter values with the true values which are actually the above assumed ones. This makes it possible to justify the convergence and accuracy offered by this identification technique.

NUMERICAL EXAMPLES

If excitation is small, based on the corresponding linear system equation of Eq. 5 and the global iteration technique [2], we obtain the system parameters which are shown in Table 1. To compare the accuracy, the error index is defined by averaging the normalized error for each measurement component, e.

$$e = [\sum_{k=1}^{N} (z(k) - x(k|k))^2]^{\frac{1}{2}} / [\sum_{k=1}^{N} z^2(k)]^{\frac{1}{2}} \quad (7)$$

where N is number of measurement sequence. The result is good and the error index being 4.2% reflects the effect of noise.

Table 1. Identified paremters of linear system based on global iteration

Parameters / Iteration No.	c_1	c_2	c_3	k_1	k_2	k_3	error index
0	5.00	5.00	5.00	50.0	50.0	50.0	
1	2.36	0.70	1.44	110.9	105.4	91.0	0.054
2	1.74	0.97	1.55	120.7	96.8	85.9	0.043
3	1.48	1.18	1.45	121.5	96.9	83.4	0.042
4	1.49	1.23	1.29	120.3	99.2	80.8	0.042
5	1.50	1.23	1.29	120.2	99.3	80.7	0.042
6	1.50	1.23	1.29	120.2	99.3	80.7	0.042
True velue	1.50	1.20	1.20	120.0	100.0	80.0	

The system behaves nonlinearly when intensity of input excitation increases. If the previous identified values are employed to perform the identification procedure for linear system, we expect the divergence of parameter values. Figs. 1, 2, and 3 show time histories of k_i, c_i, and V_1, respectively. In Figs. 1 and 2 parameter values change abruptly around t=0.4 sec. The interpretation that the system becomes nonlinear at this time allows us to estimate the yielding displacement being close to 1 cm. This information is important since the identified parameter values are particularly sensitive to the initial estimate of yielding strength.

To identify the nonlinear system, we propose a two-stage iteration procedure. First, the filtering technique is carried out on an equivalent linear system as mentioned above in order that certain parameters can be estimated with confidence. Followed is the performance of filtering technique on original hysteretic system based on those estimated parameter values. The outcome is summarized in Table 2. Except the stiffness ratio, all initial parameter values are same in eight trials.

Table 2. Identified parameters of nonlinear system

initial value of a_i	c_1	c_2	c_3	k_1	k_2	k_3	b_1	b_2	b_3	a_1	a_2	a_3	error index
0.1	1.75	1.30	1.35	117.5	98.9	81.8	.56	.69	.66	.32	.20	.18	.094
0.2	1.67	1.20	1.31	118.1	99.2	81.1	.59	.66	.63	.31	.24	.23	.072
0.3	1.62	1.31	1.29	119.3	98.9	80.7	.60	.63	.61	.30	.28	.27	.064
0.4	1.56	1.36	1.33	119.8	98.7	80.5	.60	.61	.58	.29	.29	.29	.062
0.5	1.56	1.38	1.34	120.2	98.6	80.5	.61	.60	.54	.28	.31	.33	.064
0.6	1.50	1.34	1.39	120.0	98.9	80.3	.60	.58	.50	.29	.32	.37	.066
0.7	1.44	1.32	1.39	120.1	99.0	79.8	.58	.55	.43	.30	.33	.42	.075
0.8	1.42	1.36	1.35	120.0	98.4	81.3	.63	.52	.43	.27	.36	.38	.088
true value	1.50	1.20	1.20	120.0	100	80.0	.60	.60	.60	.30	.30	.30	

The parameter values from the iteration with a minimum error index are regarded as the final identified values. It is interesting to note that the index is close to the noise intensity. This implies that the sequential identification scheme is able to filter the measurement noise and yield a more accurate parameter value.

CONCLUSIONS

Numerical results show that the extended Kalman filter is very powerful in identification problems of a linear and nonlinear MDOF system when the proposed two-stage iteration procedure is employed. Further investigation in identification of a tall building with recorded data from installed seismographs is encouraged.

ACKNOWLEDGEMENTS

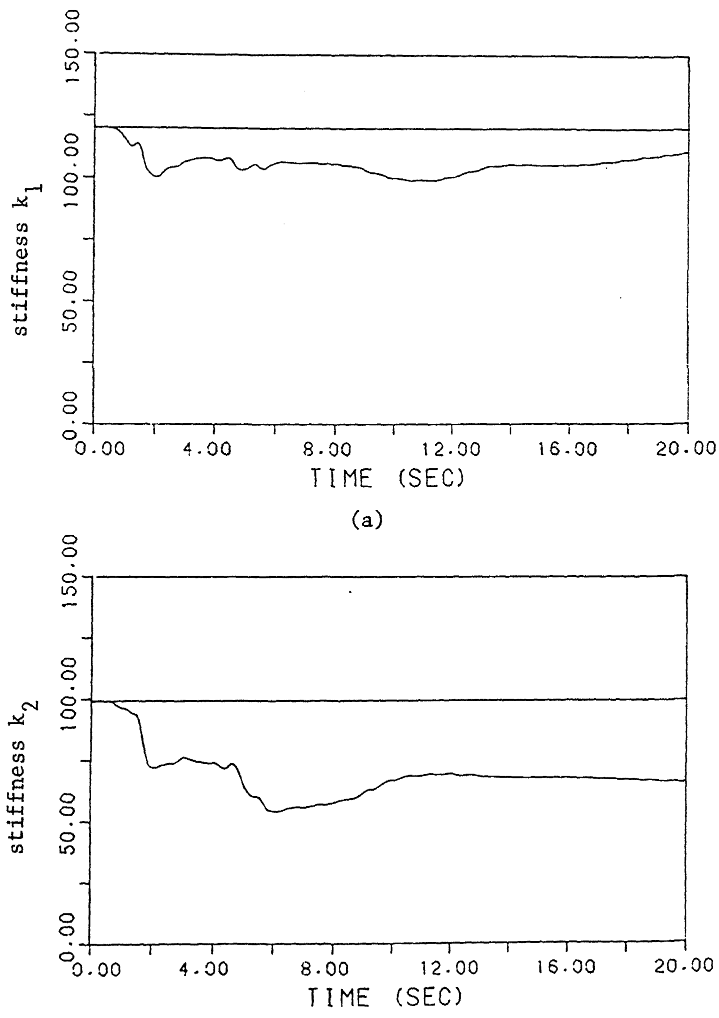
This paper is the partial result of research project supported by the National Science Council, ROC, through Grant NSC-75-0410-E002-27. The support is sincerely appreciated.

REFERENCES

1. Yun, C-B, and Shinozuka, M., "Identification of Nonlinear Structural Dynamics Systems", J. Struct. Mech 8(2), pp. 187-203, 1980.

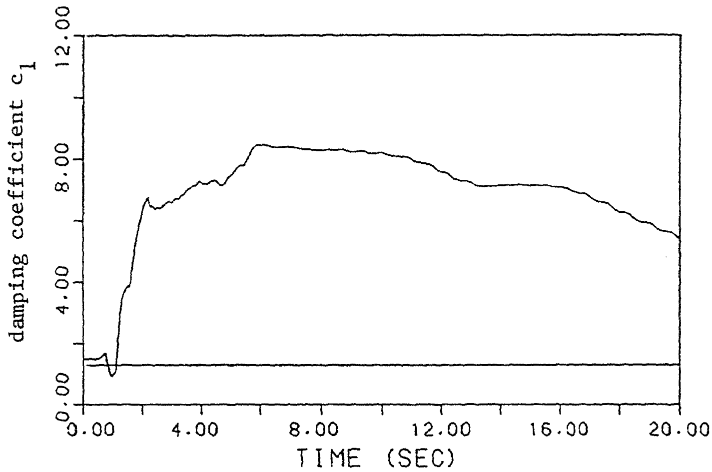
2. Hoshiya, M., and Saito, E., "Structural Identification by Extended Kalman Filter", J. of Engineering Mechanics, ASCE, Vol. 110, No. 12, pp. 1757-1770, December 1984.

3. Tan, R.Y., and Chen, C.C., "Feasiability of the Application of Kalman Filter in identification of MDOF System", Technical Report 75-09, Center for Earthquake Engineering Research, Taiwan University, August 1986.

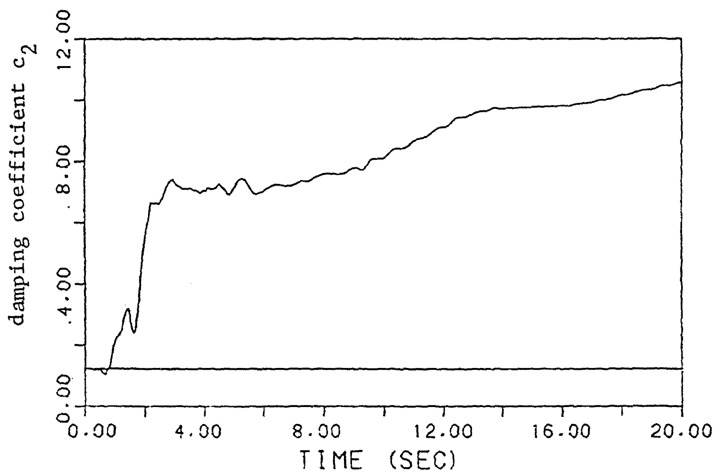


(b)

Fig.1 Identified stiffness based on linear state equation



(a)



(b)

Fig.2 Identified damping coefficient based on linear state equation

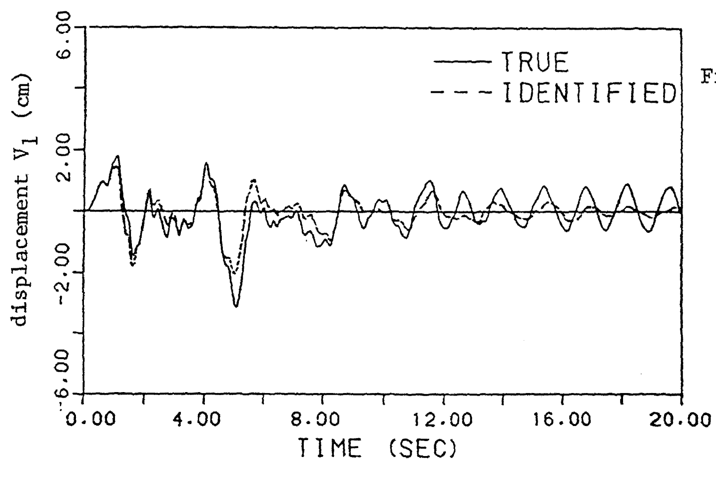


Fig.3 Comparison of identified displacement with observed one based on linear state equation