Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-5-10

FREQUENCY RESPONSE ANALYSIS OF PROBABILISTIC STRUCTURES USING FINITE ELEMENT METHOD

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SUMMARY

This paper illustrates the application of an interpolation function method to the stochastic FEM of structures which have uncertain properties when they are subjected to earthquake loading. In the method developed here the nonlinearity of the response is approximated by an interpolation function similar to a frequency transfer function of a SDOF system. The traditional perturbation method poorly approximates nonlinearity in the response caused by variance in material properties. The interpolation function allows us to better model the nonlinearity of response. Monte Carlo simulations are performed in order to verify the accuracy of the proposed method. Consequently it is found that the technique proposed here is well suited for actual dynamic response problems.

INTRODUCTION

The stochastic finite element method (SFEM) which is based on the first order perturbation method has been developed in order to solve many probabilistic structural problems , and has been widely used as an efficient and accurate method in safety and reliability analyses 1)2)3). As for dynamic problems , however , the SFEM is not always accurate when an analyzed structure has uncertain elasticities and/or densities 4). The reason is mainly that variations of mass and/or stiffness matrices change eigenvalues of the structure and then partial derivatives of the response with respect to probabilistic variables cannot be approximated to be constant.

In this paper an interpolation function method for the frequency response analysis using the SFEM is presented. The form of the interpolation function is similar to a frequency transfer function of a SDOF system.

This idea is based on an interpolation method with respect to a frequency axis, which is generally used in the frequency response analysis, and which is expanded with respect to an uncertain parameter axis instead of a frequency axis in the proposed method.

ANALYSIS PROCEDURE

The outline is as follows;

(1) in order to get the frequency transfer function $H(\omega; \boldsymbol{a}=\bar{\boldsymbol{a}}=0)$ and its partial derivative with respect to uncertain parameters, $\partial H(\omega;\boldsymbol{a})/\partial a_i|_{\boldsymbol{a}=\mathbf{0}}$, the well-known SFEM based on the first order perturbation method is performed,

(2) the interpolation functions to each uncertain parameter are determined by the values of $H(\omega;\boldsymbol{a}=\mathbf{0})$ and $\partial H(\omega;\boldsymbol{a})/\partial a_i|_{\boldsymbol{a}=\mathbf{0}}$,

(3) statistical values of the dynamic response, the mean and the variance, are computed by Hermite-Gauss quadrature schemes (HGQ),

where ω is the circular frequency; $\boldsymbol{a}$ and a_i are a vector of uncertain parameters and its i-th element, respectively; and the superposed bar denotes the mean.

<u>The first order expansion</u> The frequency transfer function is expanded about the probabilistic parameter a_i via Taylor series using the perturbation method as follows;

$$H(\omega;\boldsymbol{a}) = H(\omega;\boldsymbol{a}=\mathbf{0}) + \sum_{i=1}^{n} \left.\frac{\partial H(\omega;\boldsymbol{a})}{\partial a_i}\right|_{\boldsymbol{a}=\mathbf{0}} a_i \tag{1}$$

where n is the number of uncertain parameters.

<u>Approximation of the transfer function</u> In the frequency response analysis, the frequency transfer function of a SDOF system, which is

$$H(\omega) = -\frac{m}{k - m\omega^2 + 2hki} \tag{2}$$

where m, k, h and i are a mass, a spring coefficient, a damping coefficient and the imaginary unit, respectively, is widely used as an interpolation function to the frequency axis. Generally speaking, the first order perturbation method seems to lead to noticeable error in the neighborhood of $\partial H(\omega;\boldsymbol{a})/\partial a_i|_{\boldsymbol{a}=\mathbf{0}}=0$. In the new method proposed here, the frequency transfer function is approximated by an interpolation function to each uncertain parameter instead of equation (1). Regarding the SDOF system transfer function as the fundamental form, the interpolation function is supposed of the following form

$$H(\omega; a_i) = \frac{c_1^i + c_2^i a_i}{1 + c_3^i a_i} \quad , \tag{3}$$

where $c_1{}^i, c_2{}^i, c_3{}^i$ are complex constants. Although the constants is determined by $H(\omega; \mathbf{a})$ and $\partial H(\omega; \mathbf{a}) / \partial a_i|_{\mathbf{a}=0}$ in equation (1), these are not always $c_j{}^i \neq 0$ because of characteristic of each parameter.

<u>Statistical values of the dynamic response</u> The dynamic response and its statistical values to every uncertain parameter are computed as follows;

$$g_i(a_i; t) = \int_{-\infty}^{\infty} F(\omega) H(\omega; a_i) e^{i\omega t} dt \tag{4}$$

$$m_i(a_i; t) = \int_{-\infty}^{\infty} g_i(a_i; t) f_A(a_i) da_i \tag{5}$$

$$\sigma_i^2(a_i; t) = \int_{-\infty}^{\infty} \{g_i(a_i; t) - m_i(t)\}^2 f_{Ai}(a_i) da_i \tag{6}$$

where $g_i(a_i; t)$, $m_i(a_i; t)$, and $\sigma^2{}_i(a_i; t)$ represent the response, its mean and its standard division, respectively; $F(\omega)$ is the input wave spectrum; and $f_{Ai}(a_i)$ is the density function of the uncertain parameter. And for the calculations of the above integrations, HGQ 5) is employed.

As for the statistical values to all uncertain parameters and the whole system, convenient procedures, which should be verified by MCS, are assumed as follows;

$$m(t) = \frac{1}{n} \sum_{i=1}^{n} m_i(a_i; t) \tag{7}$$

$$\sigma^2(t) = (n-l) \sum_{i}^{n} \sum_{j}^{n} \sigma_i(a_i; t) \sigma_j(a_j; t) \delta_{ij} \tag{8}$$

where $m(t)$ and $\sigma^2(t)$ are the overall mean and the overall variance, respectively; δ_{ij} is the Kronecker delta; and l is the number of densities and elasticities which are varied simultaneously. In order to apply the present method to various, practical problems widely, the effect of correlation is approximated in a convenient way by the following equation;

$$\left.\frac{\partial H(\omega;\mathbf{a})}{\partial a_i}\right|_{\mathbf{a}=\mathbf{0}} = \left.\frac{\partial H(\omega;\mathbf{a})}{\partial a_i}\right|_{\mathbf{a}=\mathbf{0}} + \sum_{i=1(i\neq j)}^{n} \left.\frac{\partial H(\omega;\mathbf{a})}{\partial a_j}\right|_{\mathbf{a}=\mathbf{0}} E[a_i a_j] \tag{9}$$

NUMERICAL EXAMPLES

Some numerical examples are calculated by the proposed method , MCS and the second MCS in order to verify this method and evaluate its applicability. The second MCS in this paper is MCS relaxed restrictions on the generation of probabilistic parameters such as the parameters are generated only on each parameter's axis. Although the ensemble input should be employed in a statistical sense , the experienced motion , that is the El-Centro accelerogram , is done because of ease in understanding the phenomena in a practical sense.

A single degree of freedom spring-mass system Applicability of the interpolation function method , the new method presented here , is evaluated by means of a SDOF spring-mass system. A random spring constant and mass are normally distributed with a coefficient of variation equal to 0.2 and probabilistically independent. In Fig.1 the standard deviations obtained by the above three methods are shown. In spite of large coefficients of variations , very close agreement among values simulated by these methods is obtained.

A two degree of freedom spring-mass system In order to verify the assumed formulation about the overall mean and variance , the standard deviation responses of a 2DOF spring-mass system under the condition similar to a SDOF spring-mass system are computed by the three methods and shown in Fig.2. The result by this method and that by the second MCS agree very well. It is because the distributions of random variables which are assumed in the proposed method are almost equal to those in the second MCS. Compared with the result simulated by MCS , however , the result by the proposed method has an error in some degree. The reason is considered that the error is caused due to the difference of distributions of random variables in the two methods , that is , random variables in the proposed method and the second MCS are assumed to be distributed only on each variable's axis but those in MCS are assumed to be distributed on the whole variables' planes.

A single-element model Consider a single-element model in order to estimate the applicability of an interpolation function method to FEM. Fig.3 through Fig.5

show standard deviation responses where random values , which have the coefficient of variation equal to 0.2 , are an elasticity , a density and a damping coefficient , respectively. From these figures , it is clear that the interpolation function method proposed here can be applied to FEM.

A practical model The present method is applied to the 20 - layer model which is shown in Fig.6. The model consists of 5 materials. The random values of these materials are probabilisticaly independent each other and are normally distributed with a coefficient of variation equal to 0.2. Standard deviation responses at mid-surface point simulated by the proposed method and MCS are shown in Fig.7. Although the number of samples in MCS is no more than 100 , close agreement between the two methods is obtained.

CONCLUSIONS

An interpolation function method for the stochastic dynamic response analysis has been developed. And it has been founded that this method and MCS agree very well even if the analyzed model has an uncertain parameter of which coefficient of variation is large and even if elasticities and/or densities are regarded as random variables. For further discussion the reader should refer to reference 6) .

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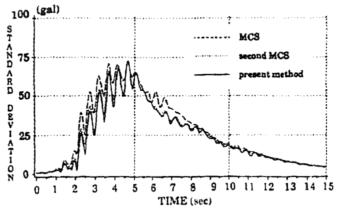

Fig. 1　σ - response by each method.

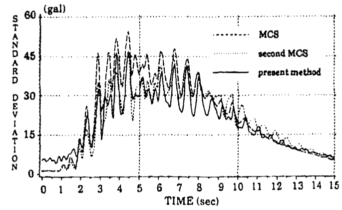

Fig. 2　σ - response by each method.

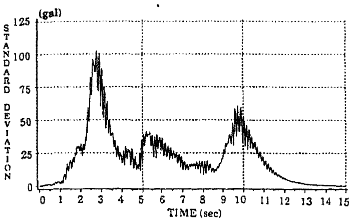

Fig. 3 a)　σ - response by MCS (*r. v.* : E).

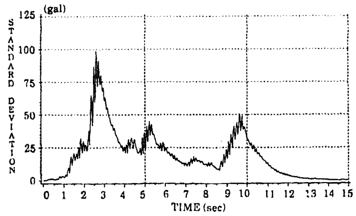

Fig. 3 b)　σ - response by the present method (*r. v.* : E).

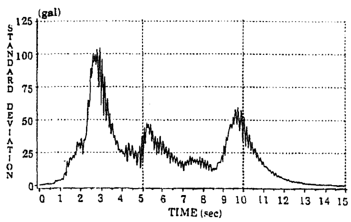

Fig. 4 a)　σ - response by MCS (*r. v.* : *p*).

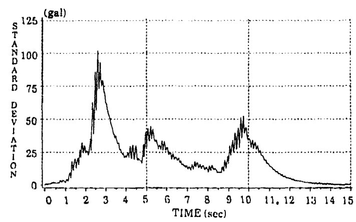

Fig. 4 b)　σ - response by the present method (*r. v.* : *p*).

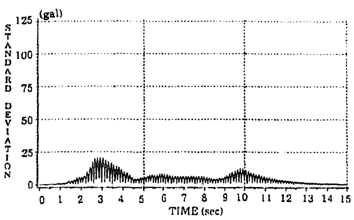

Fig. 5 a)　σ - response by MCS (*r. v.* : *h*).

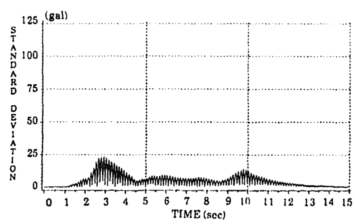

Fig. 5 b)　σ - response by the present method (*r. v.* : *h*).

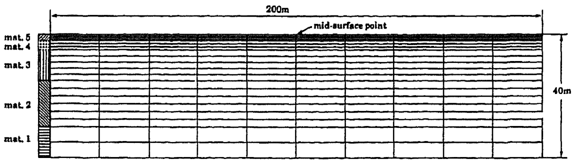

Fig. 6　20 - layer model

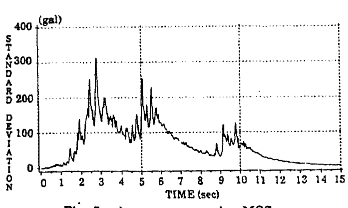

Fig. 7 a)　σ - response by MCS.

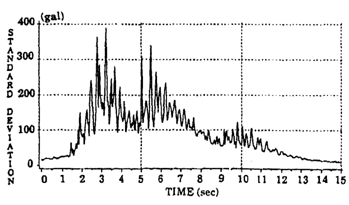

Fig. 7 b)　σ - response by the present method.