Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-5-9

STOCHASTIC SEISMIC RESPONSE ANALYSIS FOR HYSTERETIC SYSTEMS

Chun-Shi Hsu

Dept. of Civil Eng., East-Northern Electric-Power Institute
Jilin, China

SUMMARY

A New degrading bilinear hysteretic model is to proposed in order to simplify stochastic seismic response analysis. Using this model, the response of a SDF hysteretic system subjected to gaussian short noise excitation is examined by equivalent linearization. An ordinary differential equation is derived for the covariance matrix of the response. Comparison with the equation derived by using the Fokker-Planck approach shows that the two approaches lead to completely same equation of covariance matrix. Numerical examples show the availavility of this approach.

INTRODUCTION

In recent years, there has been considerable interest in the problem of calculating the response of hysteretic systems subjected to random excitations. This problem has been approached form many different point of view including linearization (Ref.1), the Fokker-Planck approach (Ref.2), the power balance approach (Ref.3) etc.. A number of different models for hysteretic behavior has also been investigated ranging from the bilinear hysteretic model to model specially formulated to facilitate analysis. However, due to analytical complications, except for the work of Wen (Ref.4), Asano (Ref.5), Ishimaru (Ref.6), there has been relatively little effort directed forward analytical studies of systems exhibiting more general curved hysteretic behaviors.

The paper is to propose a new degrading bilinear hysteretic model for stochastic seismic response analysis. This model is formulated introducing a degrading property parameter into the well known bilinear model. The main purpose of this research is to simplify the stochastic seismic response analysis by using the proposed model to approximate complicated models such as the Clough's model in analysis. The approximation is realized in this paper by determining the value of the degrading property parameter under consideration of energy absorption equivalency of the two models. With introducing an auxiliary variable, the hysteretic restoring force can be expressed into Fourier integral forms so that it becomes possible to use the equivalent linearization or the Fokker-Planck approcah for hysteretic systems with this model.

The differential equation of covariance matrix of state variables is derived directly by using the equivalent linearization approach proposed by Wen (Ref.7) for systems subjected to gaussian short noise excitations. This derived equation shows complete equivalency with the equation derived by the author in another paper for same systems usings Fokker-Planck approach proposed by Kobori et al. (ref.8,9).

THE HYSTERETIC RESTORING FORCE MODEL

The stiffness k1 of proposed model, as shown in Fig.1, is given by (Ref.10)

$$K_1 = K_0 \frac{r\alpha(z^+ + z^-) + \eta_0}{\alpha(z^+ + z^-) + \eta_0} \tag{1}$$

in which, α, z^+ and z^- are degrading property parameter, positive and negative maximum plastic displacements occurring in response, and r , η_0 are the second slop ratio.the maximum elastic displacement, respectively.

Because of the relation to z^+ and z^-, k1 decrease with developing z^+ and z^- in response. In the other hand, the decreasing ratio of k1 is mainly controlled by α. For example, it is clear from Eq. 1 that this model is simply equal to bilinear or peak-oriented model as α is given 0.0 or 0.5. It means that this model can represent different degrading level up to peak-oriented model simply by giving α from 0.0 to 0.5.

Fig. 2, shows comparison between Clough's model (loop OABCD"EFB) and the proposed model (loop OABDEGB). If α is given by

$$\alpha = \begin{cases} (\eta_0 - rz^+)/4\eta_0, & dQ/dx<0 \\ (\eta_0 - rz^-)/4\eta_0, & dQ/dx>0 \end{cases} \tag{2}$$

where Q is restoring force, then energy absorption of the two models are equivaltnt, because of the equivalency of the areas of $\triangle$ BDD" and $\triangle$ EGB to the areas of $\triangle$ BCD" and $\triangle$ EFB, respectively.

As the narrow band response is expectable under earthquake when system damping is small enough, considering energy absorption equivalency, it is desirable simplify the stochastic response analysis by the proposed model.

ANALYTICAL REPRESENTATION OF RESTORING FORCE

Introducing an auxiliary variable y (Ref.10) the hysteretic characteristic of proposed model can be transformed to non-hysteretic characteristic analytically, And y is defined as follows:

$$\dot{y} = L(v, y) = v + 2[\{\alpha(z^+ + z^-) + \eta_0\} Q(y)/\eta_0 - y]\,\delta(v) \tag{3}$$

in which v, Q(y) and $\delta(\ )$ are velocity dx/dt, pseudo-restoring force and Dirac $-\delta$ function, respectively.

For the proposed model (Fig.1) the restoring force Q(x,y) and y can be expressed in the Fourier integral forms finally as

$$Q(x, y) \fallingdotseq rx + (1-r)\eta_0/[\pi i\{\alpha(E[z^+] + E[z^-]) + \eta_0\}] \times \int_{-\infty}^{\infty} (1/\beta^2) \sin[\beta\{\alpha(E[z^+] + E[z^-]) + \eta_0\}] \exp(i\beta y) d\beta \tag{4}$$

$$\dot{y} \fallingdotseq v + 1/(\pi^2 i) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (1/\beta^2) \sin[\beta\{\alpha(E[z^+] + E[z^-]) + \eta_0\}] \times \exp\{i(\beta y + \gamma v)\} d\beta\, d\gamma - (1/\pi)\int_{-\infty}^{\infty} y \exp(i\gamma v) d\gamma \tag{5}$$

In Eq. 4 , 5 an approximate treatment has been used in order to overcome the analytical difficulty, namely expectations of z^+ and z^- has been used instead of z^+ and z^-. Furthermore, in the general cases, the expectations are equal to each other and can be expressed in one symbol as E[z]. The value of E(z) can be evaluated numerically in analysis making use of folowing formulae (Ref.10)

$$E[\dot{z}_i]=A_i/B_i$$

$$A_i=0.5(\sigma_v/\sqrt{2\pi})[1-2\,erf(\bar{\eta}_0/\sigma_x\sqrt{1-\rho_{xv}^2})+\rho_{xv}\exp(-\bar{\eta}_0^2/2\sigma_x^2)\{1+2\,erf(\rho_{xv}\bar{\eta}_0/\sigma_x\sqrt{1-\rho_{xv}^2})\}]$$

$$B_i=1+0.5\,\Delta t[0.5\,\sigma_v\sqrt{1-\rho_{xv}^2}\exp\{-\bar{\eta}_0^2/2\sigma_x^2(1-\sigma_{xv}^2)\}/(2\pi\sigma_x)$$

$$+0.5\,\rho_{xv}\,\bar{\eta}_0\sigma_v\exp(-\bar{\eta}_0^2/2\sigma_x^2)\{1+2\,erf(\rho_{xv}\bar{\eta}_0/\sigma_x\sqrt{1-\rho_{xv}^2})\}/\sqrt{2\pi}\,\sigma_x^2] \quad (6)$$

$$erf(x)=(1/2\pi)\int_0^x\exp(-0.5\,t^2)\,dt$$

$$\bar{\eta}_0=\eta_0+E[z_{i-1}]+0.5\,\Delta t\,E[\dot{z}_{i-1}] \quad (7)$$

$$E[z_i]=E[z_{i-1}]+0.5\,\Delta t\,E[\dot{z}_{i-1}]+0.5\,\Delta t\,E[\dot{z}_i] \quad (8)$$

in which, σx, σv and ρxv are the standard deviations of x,v and the coefficient of correlation of x and v, respectively.

APPLICATION OF EQUIVALENT LINEARIZATION

The dimensionless equation of motion of the single degree of freedom (SDF) hysteretic system may now be expressed as

$$\ddot{x}+2h\omega_0\dot{x}+\omega_0^2Q(x,y)=f(t) \quad (9)$$

$$\dot{y}=L(v,y)$$

in which, f(t) is a base acceleration and h, ω_o are the viscous damping ratio, the natural frequency of the hysteretic system (r=1), respectively.

Making use of the method of equivalent linearization (Ref.11), the nonlinear Eq.9 can be reduced to the following linear set.

$$\ddot{x}+2h\omega_0\dot{x}+C_1\omega_0^2+C_2\omega_0^2y=f(t) \quad (10)$$

$$\dot{y}=C_3v+C_4y$$

The equivalent coefficients C1-C4 can be evaluated assuming gaussian probability distribution of the multidimensional response process. This yields

$$C_1=r$$

$$C_2=2\eta_0(1-r)\times erf\{(2\alpha E[z]+\eta_0]/\sigma_y\}/(2\alpha E[z]+\eta_0) \quad (11)$$

$$C_3=1-\rho_{yv}\sigma_yC_4/\sigma_v$$

$$C_4=-2[1-erf\{(2\alpha E[z]+\eta_0)/\sigma_y\ 1-\rho^2{}_{vy}\}]/\ 2\pi\ \sigma v$$

where σy and ρyv are respectively the standard deviation of y and the coefficient of correlation of v and y. And erf( ) is the error function.

RESPONSE OF COVARIANCE MATRIX

Referring to Ref.11, the covariance matrix of response of a SDF linear system subjected to a gaussian short noise excitation can be simply determined. Introducing the three state variables:x1=x, x2=v. x3=y, Eq. 10 can then be rewritten as a system of first order differential equations as follows:

$$\{x\}=[G]\{x\}+\{F\} \quad (12)$$

in which

$$\{F\}=[0\quad f(t)\quad 0]^T \quad (13)$$

and

$$[G]=\begin{bmatrix} 0 & 1 & 0 \\ -C_1\omega_0^2 & -2h\omega_0^2 & -C_2\omega_0^2 \\ 0 & C_3 & C_4 \end{bmatrix} \tag{14}$$

Assuming the covariance matrix of x to be $[k]$ where $k_{ij}=E[x_i x_j]$ and the earthquake-like random excitation $f(t)$ to be a gaussian short noise process with zero mean, special density $2\pi S_0$, and envelope function $e(t)$, it may be shown that the covariance matrix $[k]$ satisfies the following differential equation

$$[\dot{k}]=[G][k]+[K][G]^T+[B] \tag{15}$$

in which

$$[B]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\pi S_0 e(t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{16}$$

The non-stationary covariance matrix of x can be evaluated by solving Eq.15 numerically on the basis of a step-by-step integration method.

COMPARISON WITH FOKKER-PLANCK APPROACH

The author has reported a solution technique for the random response of hysteretic systems with the proposed restoring force characteristics based on use of the Fokker-Planck approach. The differential equations of the elements of the response covariance matrix corresponding to Eq.15 are given as follows (Ref.10):

$$\begin{aligned}
\dot{k}_{xx} &= 2k_{xv} \\
\dot{k}_{yy} &= 2\xi_y k_{yy} + 2\xi_v k_{vv} \\
\dot{k}_{vv} &= -2\omega_0^2\beta_x k_{xv} - 4\omega_0 h k_{vv} - 2\omega_0^2\beta_y k_{vy} + 2\pi S_0 \\
\dot{k}_{xy} &= k_{xy}\xi_y + \xi_v k_{xv} + k_{yv} \\
\dot{k}_{xv} &= -\omega_0^2\beta_x k_{xx} - 2\omega_0 h k_{xv} - \omega_0^2\beta_y k_{xy} + k_{vv} \\
\dot{k}_{yv} &= -\omega_0^2\beta_x k_{xy} - (2\omega_0 h - \xi_y) k_{vy} - \omega_0^2\beta_y k_{yy} + \xi_v k_{vv}
\end{aligned} \tag{17}$$

in which

$$\beta_x=C_1;\ \beta_y=C_2;\ \xi_v=C_3;\ \xi_y=C_4 \tag{18}$$

A careful comparison of the two sets of Eqs.15,16 and Eqs.17,18 indicates that the equivalent linearization and the Fokker-Planck approaches lead to completely same differential equation of covariance matrix.

The reason of the equivalency of the two approaches for the system mentioned may be as follows: By introducing y the hysteretic system is transformed to non-hysteretic system analytically. And then under gaussian short noise excitations the system response is Markov process. Furthermore the gaussian process assumption of response implies that the Fokker-Planck approach treats the system as linear system. Exactly, the linearization is used implicitly when evaluating the coefficients $\beta_x, \beta_y, \xi_v, \xi_y$.

NUMERICAL EXAMPLES

As an example of the application of the proposed approach, consider system defined by $\eta_0=1$, $\omega_0=1$ and h=0.01. Let the envelope of the excitation be a step function, $e(t)=u(t)$.

AS for α, the following formula is used in order to compare with Clough's model.

$$a = (\eta_0 - rE[z])/4\eta_0 \qquad (19)$$

Monto-Carlo simulation study has also been undertaken for same system but with Clough's model, an ensemble size of 200 was used for this study.

In Figs.3-5 σx, σv, and E[z] are plotted as a function of time t for values of $2\pi S_0$ equal to 0.8, 0.4, and 0.2. Values of r are equal to 0.0, 0.1 and 0.3.

An examination of Figs.3-5 indicates that the simulation and analytical results are in fair good agreement regardless the fact that in the analytical study the proposed model has been used instead of Clough's model.

CONCLUSIONS

A new degrading bilinear hysteretic model is proposed in order to simplify the stochastic seismic response analysis. Comparison of the analytical and simulation results shows that the proposed model can be used to approximate complicated models such as the Clough's model in stochastic seismic analysis. The approximation can be realized by determining with concept of energy absorption equivalency.

Comparison between the equivalent linearization and the Fokker-Planck approaches for the same system shows that the two approaches lead to completely same differential equation of the covariance matrix. Because of the simplicity and the flexibility of the equivalent linearization approach to more general structural systems, it may be said that this approach is more available and powerful to these systems.

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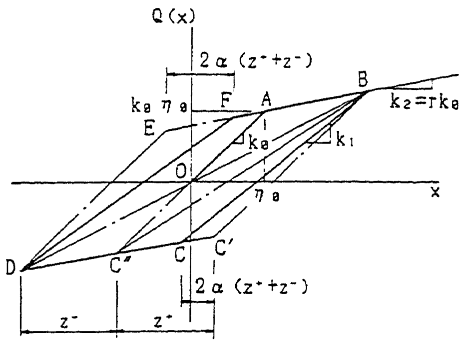

Fig.1

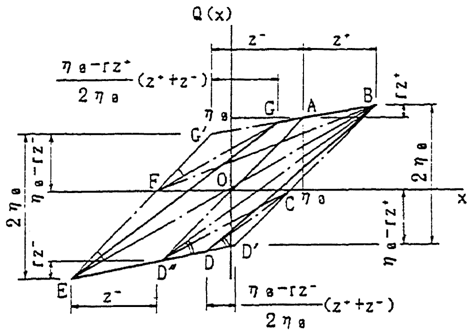

Fig.2

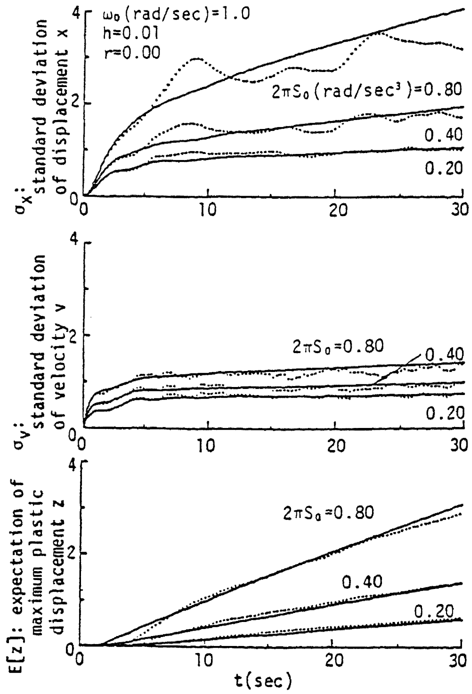

Fig.3 —— Analysis ...... Simulation

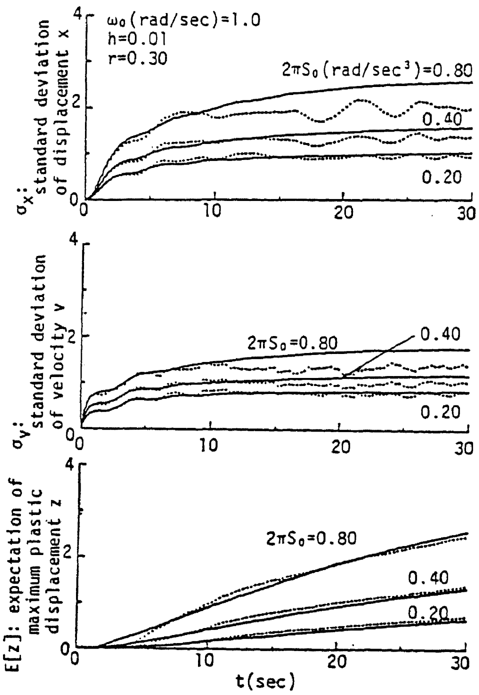

Fig.4 —— Analysis ...... Simulation

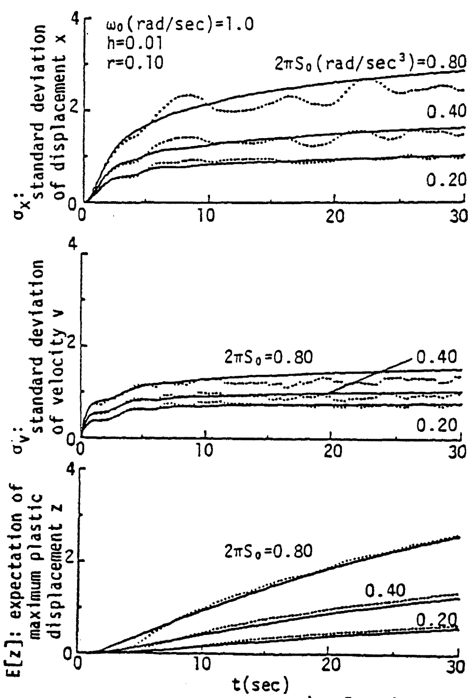

Fig.5 —— Analysis ...... Simulation